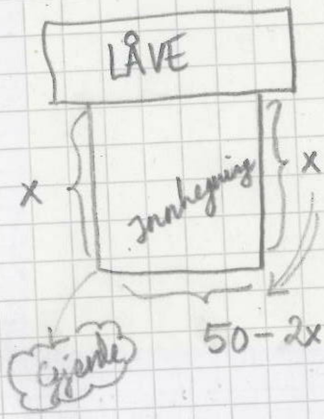
7.1 : Maksimums- og minimumsproblemer

1) Innhegning til hest: Rektangulær.

Har 50 m gjerde. Hva er max areal av innhegning?



Så $A(x) =$ (areal innhegning m/ side x) $= (50-2x)x$

$$\max_x A(x)$$

$$= \max_x (50-2x)x = \max_{x>0} (50x - 2x^2)$$

Deriverer og setter lik 0:

$$50 - 4x = 0$$

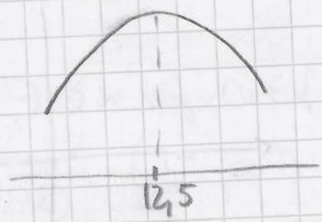
$$x = \frac{50}{4} = \underline{\frac{25}{2}} \quad (>0)$$

Dette er et maksimumspunkt siden:

$A'(x) = 50 - 4x > 0$ for $x < \frac{25}{2}$ og

$A'(x) < 0$ for $x > \frac{25}{2}$, så

f:

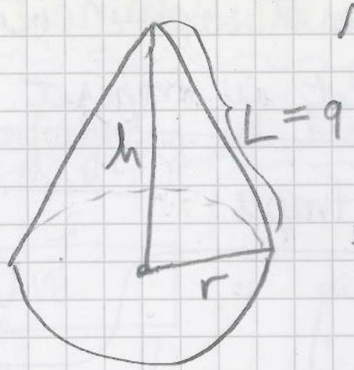


Dermed er det maksimale arealet:

$$A(x^*) = A\left(\frac{25}{2}\right) = 25 \cdot \frac{25}{2} = \underline{\underline{\frac{625}{2}}}$$

5.)

Max volum av kjegla?



Volum kjegle $= V(r) = \dfrac{\pi r^2 h}{3}$

Her: $h = \sqrt{81 - r^2}$

Pythagoras

$$V(r) = \frac{\pi r^2 \sqrt{81 - r^2}}{3}$$

$$V'(r) = \frac{\pi}{3}\left(2r\sqrt{81-r^2} + r^2 \frac{1}{2}\frac{1}{\sqrt{81-r^2}}(-2r)\right)$$

$$= \frac{\pi}{3}\left(2r\sqrt{81-r^2} - \frac{r^3}{\sqrt{81-r^2}}\right)$$

$$V'(r) = 0 \Leftrightarrow \frac{\pi}{3} r \left(2\sqrt{81-r^2} - \frac{r^2}{\sqrt{81-r^2}}\right) = 0$$

$\Leftrightarrow r = 0$ eller $2\sqrt{81-r^2} = \dfrac{r^2}{\sqrt{81-r^2}}$

Kan ikke gi max volum!

$$2(81 - r^2) = r^2$$

$$162 = 3r^2$$

$$r^2 = \frac{162}{3} = 54$$

$$r^* = r = \sqrt{\frac{162}{3}} = \sqrt{54}$$

$$V(r^*) = \frac{\pi r^2 \sqrt{81 - r^2}}{3}$$

$$= \frac{\pi 54 \sqrt{27}}{3} = 18\pi \cdot 3\sqrt{3} = \underline{\underline{54\pi\sqrt{3}}}$$