Så;

$$f^{-1}(y) = -\frac{3}{2} + \frac{\sqrt{1+4y}}{2}, \quad D_{f^{-1}} = V_f = [-\tfrac{1}{4}, \infty)$$

c) $f(x) = \dfrac{x-1}{x+2}$;

$$f'(x) = \frac{1}{x+2} + (x-1)(-1)\frac{1}{(x+2)^2}$$

$$= \frac{\cancel{x}+2-\cancel{x}+1}{(x+2)^2} = \underline{\underline{\frac{3}{(x+2)^2}}} > 0 \text{ overalt,}$$

men ikke definert i $x=2$.

Så f er injektiv på (f.eks.) $(-2,\infty)$, som inneholder 0.

$V_f = (-\infty, 1]$; Finner f^{-1} v/ å løse $y = f(x)$ for x

$$\lim_{x\to-\infty} \frac{x-1}{x+2} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1+\frac{2}{x}} = 1$$

Vet dette er største verdi siden f er injektiv; spes avtagende fra over

$$\lim_{x\to-2} \frac{x-1}{x+2} = -\infty$$

$$y = \frac{x-1}{x+2}$$

$$y(x+2) = x-1$$

$$yx - x = -2y-1$$

$$x(y-1) = -2y-1$$

$$x = \frac{-2y-1}{y-1}$$

$$\Downarrow$$

$$\underline{\underline{f^{-1}(y) = \frac{-2y-1}{y-1}}}, \quad D_{f^{-1}} = V_f = (-\infty, 1]$$

7.5: Cotangens

1.) a) $\cot\frac{\pi}{6} = \dfrac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \dfrac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \underline{\underline{\sqrt{3}}}$

lsolutte verdier:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
inx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

b) $\cot\frac{\pi}{4} = \dfrac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \underline{\underline{1}}$

c) $\cot\frac{\pi}{3} = \dfrac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} = \dfrac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} = \underline{\underline{\dfrac{1}{\sqrt{3}}}} = \dfrac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \underline{\underline{\dfrac{\sqrt{3}}{3}}}$

2) a) $D[\cot(x^2)] = -\dfrac{1}{\sin^2(x^2)}2x = \underline{\underline{-\dfrac{2x}{\sin^2(x^2)}}}$

b) $D[\cot^2 x] = 2\cot(x)\left(-\dfrac{1}{\sin^2(x)}\right) = \underline{\underline{-\dfrac{2\cot(x)}{\sin^2(x)}}}$

c) $D[x^2\cot(\sqrt{x}) + \sin(x)] = 2x\cot(\sqrt{x}) + x^2\left(-\dfrac{1}{\sin^2(\sqrt{x})}\right) \cdot$

$\cdot \frac{1}{2}x^{-\frac{1}{2}} + \cos x$

$= 2x\cot(\sqrt{x}) - \dfrac{x^2}{2\sqrt{x}\sin^2(\sqrt{x})} + \cos x$

$= \underline{\underline{2x\cot(\sqrt{x}) - \dfrac{x^{\frac{3}{2}}}{2\sin^2(\sqrt{x})} + \cos x}}$

$2-\frac{1}{2} = \frac{4-1}{2} = \frac{3}{2}$

d) $D[e^x\cot(\ln x)] = e^x\cot(\ln x)$

$+ e^x\left(-\dfrac{1}{\sin^2(\ln x)}\right)\dfrac{1}{x}$

$= \underline{\underline{e^x\left(\cot(\ln x) - \dfrac{1}{x\sin^2(\ln x)}\right)}}$