7.) $$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{\sqrt{1+u}}{\cancel{u}}\,2\cancel{u}\,du \qquad \boxed{9.2}$$

$u=\sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\,dx$
$2u\,du = dx$

$$= 2\int \underbrace{\sqrt{1+u}}_{=(1+u)^{\frac{1}{2}}}\,du = 2\cdot\frac{2}{3}(1+u)^{\frac{3}{2}} + C = \frac{4}{3}\left(1+\sqrt{x}\right)^{\frac{3}{2}} + C$$

9.) $$\int_0^1 e^{\arcsin x}\,dx = \int_0^{\frac{\pi}{2}} e^{u}\cos u\,du$$

$x = \sin u$
$dx = \cos u\,du$

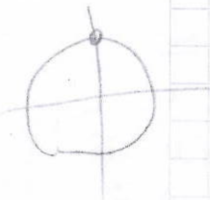
Vil få vekk arcsin: Velger derfor $x=\sin u$, siden $\arcsin(\sin u) = u$

v/ delvis int: $v = \cos u$, $w' = e^u$

$\boxed{19}$

$$= \left[\frac{e^u}{2}\left(\cos(u) + \sin(u)\right)\right]_{u=0}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}e^{\frac{\pi}{2}}\left(\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\right) - \frac{1}{2}\left(\cos 0 + \sin 0\right)$$



$$= \frac{1}{2}e^{\frac{\pi}{2}}(0+1) - \frac{1}{2}(1+0)$$

$$= \frac{1}{2}e^{\frac{\pi}{2}} - \frac{1}{2}$$

15.) $$\int_0^{\sqrt{3}} \frac{1+x}{\sqrt{4-x^2}}\,dx = \int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx + \int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx$$

substitusjon

$$= \int_0^{\sqrt{3}} \frac{1}{\sqrt{4\left(1-\left(\frac{x}{2}\right)^2\right)}}\,dx + \int_4^1 \left(-\frac{1}{2}\right)\frac{1}{\sqrt{u}}\,du$$

$u = 4-x^2$
$du = -2x\,dx$
$\int \frac{x}{\sqrt{4-x^2}}\,dx$
$= \int \frac{x}{\sqrt{u}}\left(-\frac{1}{2x}\right)du$
$x=0 \Rightarrow u=4$
$x=\sqrt{3} \Rightarrow u=1$

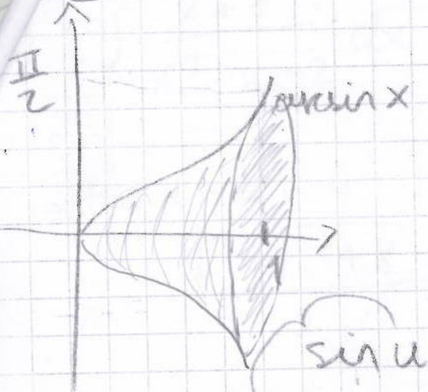
$$= \frac{1}{2}\int_0^{\sqrt{3}} \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx + \frac{1}{2}\int_1^4 u^{-\frac{1}{2}}\,du$$

Bytter int. grenser $\Rightarrow$ må også bytte fortegn

$$= \frac{1}{2}\left[\arcsin\left(\frac{x}{2}\right)2\right]_{x=0}^{x=\sqrt{3}} + \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_{x=1}^{4}$$

$$= \arcsin\frac{\sqrt{3}}{2} - \arcsin 0 + 2 - 1 = \frac{\pi}{3} - 0 + 1 = \frac{\pi}{3} + 1$$

23.) $y = \arcsin x$, $0 \le x \le 1$: Finn volum til omdreinings-legemet om x-aksen:



$$V = \int_0^1 \pi (\arcsin x)^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} u^2 \cos u \, du$$

$\sin u = x$
$\cos u \, du = dx$

$x = 1 \Rightarrow u = \frac{\pi}{2}$
$x = 0 \Rightarrow u = 0$

Vil ha vekk arcsin! Fint å ta $x = \sin u \Rightarrow \arcsin x = u$

$$\underline{M}: \int_0^{\frac{\pi}{2}} u^2 \cos u \, du$$

$$= \left[u^2 \sin u\right]_{u=0}^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} u \sin u \, du$$

$w(u) = u^2$
$v'(u) = \cos u$
$\Downarrow$
$w'(u) = 2u$
$v(u) = \sin u$

$$= \left[u^2 \sin u\right]_{u=0}^{\frac{\pi}{2}} - 2\left(\left[-u \cos u\right]_{u=0}^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos u \, du\right)$$

$\tilde{w}(u) = u$
$\tilde{v}'(u) = \sin u$
$\Downarrow$
$\tilde{v}(u) = -\cos u$
$\tilde{w}'(u) = 1$

$$= \left[u^2 \sin u + 2u \cos u - 2\sin u\right]_{u=0}^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{4} \cdot 1 + 2\frac{\pi}{2} \cdot 0 + 1(-2) - 0 + 0 - 0$$

$$= \frac{\pi^2}{4} - 2$$

Dermed er:

$$V = \pi \int_0^{\frac{\pi}{2}} u^2 \cos u \, du = \frac{\pi^3}{4} - 2\pi$$