21.) a) $$\int \frac{u+2}{u^2+2u+5}\,du = \frac{1}{2}\int \frac{2u+4}{u^2+2u+5}\,du$$

9.3

Kan ikke faktoriseres reelt! (21.)

$$= \frac{1}{2}\int \frac{2u+2}{u^2+2u+5}\,du + \frac{2}{2}\int \frac{1}{u^2+2u+5}\,du$$

$$= \frac{1}{2}\int \frac{1}{v}\,dv + \int \frac{1}{(u+1)^2+4}\,du$$

$v = u^2+2u+5$

$dv = (2u+2)\,du$

$$= \frac{1}{2}\ln(u^2+2u+5) + \frac{1}{4}\int \frac{1}{\left(\frac{u+1}{2}\right)^2+1}\,du$$

$$u^2+2u+5 = u^2+2u+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2+5$$
$$= u^2+2u+1+4$$
$$= (u+1)^2+4$$

$$= \frac{1}{2}\ln(u^2+2u+5) + \frac{2}{4}\arctan\left(\frac{u+1}{2}\right) + C$$

$$= \frac{1}{2}\ln(u^2+2u+5) + \frac{1}{2}\arctan\left(\frac{u+1}{2}\right) + C$$

b)

$$\frac{1}{u(u^2+2u+5)} = \frac{A}{u} + \frac{Bu+C}{u^2+2u+5}$$

$$1 = A(u^2+2u+5) + (Bu+C)u$$
$$= u^2(A+B) + u(2A+C) + 5A$$

$A+B=0$, $2A+C=0$, $5A=1$

$B = -\frac{1}{5}$, $-\frac{2}{5} = C \Leftarrow A = \frac{1}{5}$

c) Regn ut:

$$\int \frac{\tan x}{\cos^2 x + 2\cos x + 5}\,dx = \int \frac{\sin x}{\cos x(\cos^2 x + 2\cos x + 5)}\,dx$$

$$= -\int \frac{1}{u(u^2+2u+5)}\,du \overset{b)}{=} -\frac{1}{5}\int \frac{1}{u}\,du + \frac{1}{5}\int \frac{u+2}{u^2+2u+5}\,du$$

$u = \cos x$

$du = -\sin x\,dx$

$$\overset{a)}{=} -\frac{1}{5}\ln|u| + \frac{1}{10}\ln(u^2+2u+5) + \frac{1}{10}\arctan\left(\frac{u+1}{2}\right) + C$$

$$= \frac{1}{10} \ln(\cos^2 x + 2\cos x + 5) + \frac{1}{10} \arctan\left(\frac{\cos x + 1}{2}\right)$$

$$- \frac{1}{5} \ln|\cos x| + C$$

(Tror fortegnsfeil i fasit)

25.) a) $2+i$ er rot i $z^3 - 11z + 20 = 0$ fordi:

$$(2+i)^3 - 11(2+i) + 20 = (4+4i-1)(2+i)$$

$$-22 - 11i + 20 = \cancel{8} + \cancel{4i} + \cancel{8i} - \cancel{4} - \cancel{2} - \cancel{i} - \cancel{2} - \cancel{11i}$$

$$= 0$$

Reelt polynom $\Rightarrow$ Røttene kommer i kompleks-konjugerte par $\Rightarrow$ $2-i$ er rot og:

$$z^3 - 11z + 20 = (z-(2-i))(z-(2+i))(z-?)$$

$$(z-(2-i))(z-(2+i)) = z^2 - z(2+i) - (2-i)z + (2-i)(2+i)$$

$$= z^2 - z(2+\cancel{i}+2-\cancel{i}) + 5$$

$$= z^2 - 4z + 5$$

Polynomdivisjon:

$$z^3 - 11z + 20 : z^2 - 4z + 5 = z + 4$$

$$-(z^3 - 4z^2 + 5z)$$

$$4z^2 - 16z + 20$$

$$-(4z^2 - 16z + 20)$$

$$0$$

$\Downarrow$

Røttene er $2+i$, $2-i$ og -4

fasit sier 4 men v/ innsetting ser man at -4 er rett