7.4 Oppgaver

1. Beregn integralet ved å bruke den oppgitte substitusjonen.

a) $\int (2+3t)^{11}\,dt, \quad y = 2+3t$

b) $\int t^2\sqrt{1-t^3}\,dt, \quad y = 1-t^3$

2. La $F(x) = \sin^2 x$

a) Finn $F'(x)$. b) Finn $\int 2\sin x\cos x\,dx$

3. La $F(x) = e^{\cos x}$

a) Finn $F'(x)$. b) Finn $\int e^{\cos x}\sin x\,dx$

4. Finn integralene under.

a) $\int x(x^2+9)^{13}\,dx$ b) $\int \frac{x^3}{(7x^4-5)^5}\,dx$

c) $\int (6x+5)^7\,dx$ d) $\int (ax+b)^7\,dx$

e) $\int_0^1 x^2(x^3+1)^9\,dx$ f) $\int_1^{t_0} \frac{k_p\ dt}{(kx-\omega t)^2}$

g) $\int \sqrt{x}\,dx$ h) $\int t\sqrt{t^2+1}\,dt$

i) $\int_1^2 \frac{1}{\sqrt{x}}\,dx$ j) $\int \frac{4y^2}{\sqrt{8-y^3}}\,dy$

5. Finn integralene under. Her er k et positivt naturlig tall.

a) $\int (\sin x)^7\cos x\,dx$ b) $\int \frac{1}{\sin x}\cdot\cos x\,dx$

c) $\int \frac{\cos x}{\sin x}\,dx$ d) $\int e^{x^2}\cdot 2x\,dx$

e) $\int xe^{x^2}\,dx$ f) $\int e^{\sin x}\cos x\,dx$

g) $\int \frac{e^{\ln x}}{x}\,dx$ h) $\int t^2\sin(t^3)\,dt$

i) $\int t^{16}\sin(t^{17})\,dt$ j) $\int t^{k-1}\sin(t^k)\,dt$

6. Finn integralene under.

a) $\int \sqrt{8x+3}\,dx$ b) $\int \sqrt{a_0x+r}\,dx$

c) $\int \frac{x}{\sqrt{x^2+1}}\,dx$ d) $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

e) $\int \frac{x-5}{\sqrt{x}}\,dx$ f) $\int t^3\sqrt{1+t^4}\,dt$

7. Finn integralene. Her er a og b reelle tall ulik 0.

a) $\int \sin(ax)\,dx$ b) $\int \cos(ax)\,dx$

c) $\int \sin(ax+b)\,dx$ d) $\int \cos(ax+b)\,dx$

e) $\int e^{ax}\,dx$ f) $\int \frac{1}{ax}\,dx$

g) $\int e^{ax+b}\,dx$ h) $\int \frac{1}{ax+b}\,dx$

i) $\int e^{e^x}e^x\,dx$ j) $\int e^{e^{ax}}e^{ax}\,dx$

8. Finn integralene under. Her er k et positivt naturlig tall.

a) $\int \sqrt{\sin x}\cos x\,dx$ b) $\int e^{5x}\,dx$

c) $\int e^{kx}\,dx$ d) $\int \cos x(\sin x)^{19}\,dx$

e) $\int \cos x(\sin x)^{23}\,dx$ f) $\int \cos x(\sin x)^k\,dx$

g) $\int \frac{x}{e^{x^2}}\,dx$ h) $\int \frac{x^{k-1}}{e^{x^k}}\,dx$

i) $\int \sin(\sin x)\cos x\,dx$ j) $\int \frac{\tan x}{\cos^2 x}\,dx$

9. Finn:

a) $\int (a+bx)^5\,dx$ b) $\int 2t\sqrt{1-t^2}\,dt$

10. La $n > 1$ være et helt tall. Bruk substitusjonen $u = \sqrt[n]{x}$ til å finne integralene

a) $\int \sqrt[n]{x}\,dx$ b) $\int \frac{1}{\sqrt[n]{x}}\,dx$

11. La $n > 1$ være et helt tall. Finn integralet

$$\int_0^1 x^7(x^8+4)^n\,dx$$

12. Bruk substitusjonen $u = 1-x$ til å vise at

$$\int \frac{1}{(1-x)^2}\,dx = \frac{1}{1-x} + C_1,$$

der C_1 er en integrasjonskonstant. Vis så ved derviasjon av høyre side at

$$\int \frac{1}{(1-x)^2}\,dx = \frac{x}{1-x} + C_2,$$

der C_2 er en integrasjonskonstant. Forklar hvordan dette er mulig.

Seksjon 7.3 $\boxed{1}$ a) $s = \frac{1}{2}gt^2$ b) 1.3 sekunder $\boxed{2}$ $B = v_0^2/2a$ $\boxed{3}$ $s^2h/3$ $\boxed{4}$ 1.5 liter $\boxed{6}$ $V = \int_0^h A\,dx = Ah$ $\boxed{7}$ b) $V = \int_0^h a(x)dx = \frac{1}{3}Ah$ $\boxed{8}$

Seksjon 7.4 $\boxed{1}$ a) $\frac{1}{36}(2+3t)^{12}+C$ b) $-\frac{2}{9}(1-t^3)^{3/2}+C$ $\boxed{2}$ a) $2\sin x\cos x$ b) $\sin^2 x + C$ $\boxed{3}$ a) $-e^{\cos x}\sin x$ b) $e^{\cos x}+C$ $\boxed{4}$ a) $\frac{1}{28}(x^2+9)^{14}+C$ b) $\frac{-1}{112}(7x^4-5)^{-4}+C$ c) $\frac{1}{48}(6x+5)^8+C$ d) $\frac{1}{8a}(ax+b)^8+C$ e) 34.1 f) $(k_p/\omega)[(kx-\omega t_0)^{-1}-(kx-\omega)^{-1}]+C$ g) $\frac{2}{3}(\sqrt{x})^3+C$ h) $\frac{1}{3}(\sqrt{t^2+1})^3+C$ i) $2\sqrt{2}-2$ j) $-\frac{8}{3}\sqrt{8-y^2}+C$ $\boxed{5}$ a) $\frac{1}{8}(\sin x)^8+C$ b) $\ln|\sin x|+C$ c) $\ln|\sin x|+C$ d) $e^{x^2}+C$ e) $(1/2)e^{x^2}+C$ f) $e^{\sin x}+C$ g) $e^{\ln x}+C$ h) $-(1/3)\cos(t^3)+C$ i) $-(1/17)\cos(t^{17})+C$ j) $-(1/k)\cos(t^k)+C$ $\boxed{6}$ a) $\frac{1}{12}(\sqrt{8x+3})^3+C$ b) $\frac{2}{3a_0}(\sqrt{a_0x+r})^3+C$ c) $\sqrt{x^2+1}+C$ d) $\frac{1}{3}(x^2-2)\sqrt{1+x^2}+C$ e) $(\frac{2}{3}x-2)\sqrt{x}+C$ f) $\frac{1}{6}(\sqrt{1+t^4})^3+C$ $\boxed{7}$ a) $-(1/a)\cos(ax)+C$ b) $(1/a)\sin(ax)+C$ c) $-(1/a)\cos(ax+b)+C$ d) $(1/a)\sin(ax+b)+C$ e) $(1/a)e^{ax}+C$ f) $(1/a)\ln|ax|+C$ g) $(1/a)e^{ax+b}+C$ h) $(1/a)\ln|ax+b|+C$ i) $e^{e^x}+C$ j) $(1/a)e^{e^{ax}}+C$ $\boxed{8}$ a) $\frac{2}{3}(\sin x)^{3/2}+C$ b) $(1/5)e^{5x}+C$ c) $(1/k)e^{kx}+C$ d) $(1/20)(\sin x)^{20}+C$ e) $(1/24)(\sin x)^{24}+C$ f) $\frac{1}{k+1}(\sin x)^{k+1}+C$ g) $-(1/2)e^{-x^2}+C$ h) $-(1/k)e^{-x^k}+C$ i) $-\cos(\sin x)+C$ j) $(1/2)\tan^2 x+C$ $\boxed{9}$ a) $(a+bx)^6/6b+C$ b) $-\frac{2}{3}(1-t^2)^{3/2}+C$ $\boxed{10}$ a) $\frac{n}{n+1}(\sqrt[n]{x})^{n+1}+C$ b) $\frac{n}{n-1}(\sqrt[n]{x})^{n-1}+C$ $\boxed{11}$ $\frac{1}{8(n+1)}[5^{n+1}-4^{n+1}]$ $\boxed{12}$ $\frac{1}{1-x}-\frac{x}{1-x}=\frac{1-x}{1-x}=1$, dvs. de to svarene skiller seg kun med en konstant

Seksjon 7.5 $\boxed{1}$ a) $x\sin x+\cos x+C$ b) $-xe^{-x}-e^{-x}+C$ c) $-x\cos x+\sin x+C$ d) $-(1/2)te^{-4t}-(1/8)e^{-4t}+C$ e) $\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C$ f) $\frac{1}{25}(1+4e^5)$ g) $1-2e^{-1}$ h) $\frac{1}{3}(x^3e^{x^3}-e^{x^3})+C$ i) $\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$ j) $t^3e^t-3t^2e^t+6te^t-6e^t+C$ k) $x\ln x-x+C$ l) $\frac{1}{\ln 10}[x\ln x-x]+C$ $\boxed{2}$ c) $(1/2)e^x\sin x-(1/2)e^x\cos x+C$ $\boxed{3}$ a) $(1/2)e^x\sin x+(1/2)e^x\cos x+C$ b) $-(1/5)e^{2t}\cos 4t+(1/10)e^{2t}\sin 4t+C$ $\boxed{4}$ b) $x^4e^x-4x^3e^x+12x^2e^x-24xe^x+24e^x+C$ $\boxed{5}$ $x(\ln x)^4-4x(\ln x)^3+12x(\ln x)^2-24x\ln x+24x+C$

Seksjon 7.6 $\boxed{1}$ a) $f'(x)=\sqrt{2}x^{\sqrt{2}-1}$ b) $f'(x)=\frac{1}{2}x^{-1/2}$ c) $f'(x)=\frac{1}{5}x^{-4/5}$ d) $f'(x)=\frac{1}{2}(2x^2+5)^{-1/2}\cdot 4x$ e) $f'(x)=-\frac{1}{2}x^{-3/2}$ f) $f(x)=e^{x\ln x}$ gir at $f'(x)=x^x(1+\ln x)$ $\boxed{2}$ a) Vokser på hele $[0,\infty\rangle$. Minimumspunkt $x=0$. Tilhørende minimumsverdi: $f(0)=0$ b) Konkav på $[0,1]$, konveks på $[1,\infty\rangle$. Vendepunkt $x=1$ $\boxed{3}$ $(4\sqrt{2})/3$ $\boxed{4}$ 2/5

Blandede oppgaver til kapittel 7 $\boxed{1}$ a) 0.90 b) −0.69 c) 0 $\boxed{2}$ a) 27 b) 9 $\boxed{3}$ $\int_1^4 f(x)dx$ $\boxed{4}$ a) $\frac{1}{3}x^3+C$ b) $\frac{1}{4}x^4+C$ c) $\frac{5}{3}x^3-\frac{3}{4}x^4+C$ d) $\frac{1}{2}t^2+C$ e) $-\frac{1}{r}+C$ $\boxed{5}$ a) $\frac{2}{3}x^{3/2}+C$ b) $2\ln|x|+C$ c) $\frac{1}{2}x^2+\ln|x|+C$ d) $\frac{1}{2}e^{2x}+C$ e) $\frac{u}{a}e^{at}+C$ f) $\frac{2}{3}u^{3/2}+C$ $\boxed{6}$ $(1/2)\ln|1+2t|+C$ $\boxed{7}$ a) $\frac{1}{3}(\ln t)^3+C$ b) $\ln|1+at|+C$ c) 9.18 d) $2\ln 2=1.38$ e) $(e^{at_0}-1)/a$ $\boxed{8}$ a) $\sin x-x\cos x+C$ b) π $\boxed{9}$ a) $\cos x+x\sin x+C$ b) -2 $\boxed{10}$ a) $2\cos x+2x\sin x-x^2\cos x+C$ b) $\frac{1}{2}e^x(\sin x+\cos x)+C$ $\boxed{11}$ a) $(x-1)e^x+C$ b) $e^x(x^2-2x+2)+C$ c) $\sin\alpha-\alpha\cos\alpha+C$ d) $(\pi/2)^2-2\approx 0.47$ e) $e^{at}(a\cos\omega t+\omega\sin\omega t)/(a^2+\omega^2)-C$ f) $a(e^{2\pi a/\omega}-1)/(a^2+\omega^2)$ g) $\frac{8}{5}\sqrt{3}-\frac{4}{15}\sqrt{2}\approx 2.39$ $\boxed{12}$ $-e^{t^2}$ $\boxed{14}$ $k^{-1}e^{-ka}$ $\boxed{15}$ $S=Ab(1-e^{-ah/b})$ $\boxed{16}$ $27.40\le\int_0^{10} f(x)dx\le 29.06$ $\boxed{17}$ $k-a$ $\boxed{18}$ a) $\sin x+C$ b) $4\tan x+C$ c) $-\frac{1}{\omega}\cos\omega t+C$ $\boxed{19}$ $\frac{\sin t}{t^2+1}$ $\boxed{20}$ b) $V(t)=V(0)+\int_0^t c\cos(\frac{2\pi t}{T})dt$. 31 m^3 $\boxed{21}$ b) 0.19 liter $\boxed{22}$ a) I ett område dominerer første ledd, i et annet område dominerer annet ledd b) 76.6 mm c) 102.6 mm $\boxed{23}$ $y=2(x-\pi/4)+1$ $\boxed{24}$ Kun $x=0$. For hvis $f(x)=2x-\arctan x$, har vi $f(0)=0$ og $f'(x)\ge 1$ for alle x $\boxed{25}$ a) Nullpunkt $x=0$. Asymptote $y=\pi/2$ b) Voksende på hele intervallet $[-1,\infty\rangle$. $\boxed{26}$ Makspunkter $x=3\pi/2+2k\pi$, der er et k helt tall. Minpunkter $x=\pi/2+2k\pi$, der er et k helt tall. $\boxed{27}$ Mellom 75 og 85 $\boxed{28}$ a) $f'(t)=t^3$ $\boxed{29}$ $A_b=2-2\sqrt{b}$, og $\lim_{b\to 0}A_b=2$ $\boxed{30}$ $(m_0/k)\arcsin(kx)+C$ $\boxed{31}$ $A_b=2\arctan b$, og $\lim_{b\to\infty}A_b=\pi$ $\boxed{32}$ $\frac{1}{2}\tan(x^2)+C$ $\boxed{33}$ a) 39 liter b) $(2/3)\sqrt{6}$ min $\boxed{34}$ $65\pi/2\approx 102.5$ liter $\boxed{35}$ 8 $\boxed{36}$ $V(T)=V(t_0)+\int_{t_0}^{t_1}(v_1(t)-v_2(t))dt=15+\int_0^T(2-t/(t^2+1))dt$, $24-(1/2)\ln 50$.

Seksjon 8.1 $\boxed{2}$ b) $\sqrt{2}$ på begge c) $3\pi/4$ og $5\pi/4$ d) $z=\sqrt{2}e^{i(3\pi/4)}$, $w=\sqrt{2}e^{i(5\pi/4)}$ $\boxed{3}$ b) $\operatorname{Re} z=-1$, $\operatorname{Im} z=-\sqrt{3}$, $z=(-1)+(-\sqrt{3})i$ $\boxed{4}$ $e^{i(\pi/4)}=(1/\sqrt{2})+(1/\sqrt{2})i$, $e^{i(\pi/3)}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $e^{i(2\pi/3)}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $e^{i\pi}=-1$, $\frac{1}{2}e^{i(3\pi/2)}=-\frac{1}{2}i$, $2e^{i(\pi/3)}=1+\sqrt{3}i$ $\boxed{5}$ $z+w=9+i$, $zw=26+22i$ $\boxed{6}$ $2e^{i(-\pi/3)}=1-\sqrt{3}i$, $5e^{i\cdot 1}=5\cos 1+(5\sin 1)i\approx 2.7+(4.2)i$ $\boxed{7}$ $-2+(-2)i=\sqrt{8}e^{i(5\pi/4)}$, og $3+4i=5e^{i\theta}$, der $\theta=\arccos(3/5)\approx 0.927$

Seksjon 8.2 $\boxed{1}$ a) $6-2i$ b) $-3-4i$ c) $1+3i$ d) $3+8i$ e) $4+12i$ f) $2+4i$ g) 2 h) $-19-13i$ i) $\frac{11}{10}-\frac{1}{5}i$ j) $-i$ k) $\frac{4}{25}+\frac{3}{25}i$ l) $-5i$ m) $-2+2i$ n) i o) $-1-i$ p) $-(7/625)-(24/625)i$ q) $1+4i$ $\boxed{2}$ a) $6e^{i(3\pi/4)}$ b) $2e^{i(\pi/6)}$ $\boxed{3}$ a) $\overline{4+3i}=4-3i$, $\overline{-2-2i}=-2+2i$ b) $z=re^{i\theta}$ gir $\overline{z}=re^{i(-\theta)}=re^{-i\theta}$