Oppgave 1

$$M = \begin{pmatrix} 1 & -2 \\ -3 & 0 \end{pmatrix}$$

$$M^2 = \begin{array}{cc|cc} & & 1 & -2 \\ & & -3 & 0 \\ \hline 1 & -2 & 7 & -2 \\ -3 & 0 & -3 & 6 \end{array} \qquad M^4 = \begin{array}{cc|cc} & & 7 & -2 \\ & & -3 & 6 \\ \hline 7 & -2 & 55 & -26 \\ -3 & 6 & -39 & 42 \end{array}$$

$$2M + M^4 = \begin{pmatrix} 57 & -30 \\ -45 & 42 \end{pmatrix}$$

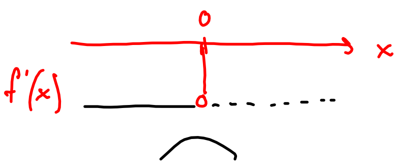
$$\det(2M + M^4) = \begin{vmatrix} 57 & -30 \\ -45 & 42 \end{vmatrix} = 57 \cdot 42 - 30 \cdot 45 = \underline{\underline{1044}}$$

Oppgave 2

$$f(x) = e^{-kx^2}$$

a) $f'(x) = e^{-kx^2} \cdot (-2kx)$

$f'(x) = 0$ gir $x = 0$



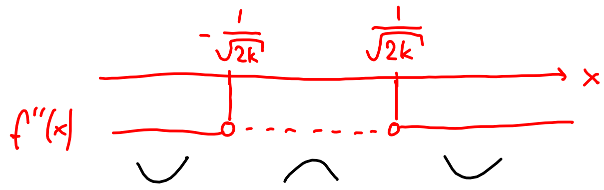
Vokser på $(-\infty, 0]$ og avtar på $[0, \infty)$

Globalt maksimumspunkt $\underline{\underline{x = 0}}$

b) $f''(x) = e^{-kx^2} \cdot 4k^2x^2 + e^{-kx^2}(-2k)$

$= 2ke^{-kx^2}(2kx^2 - 1)$

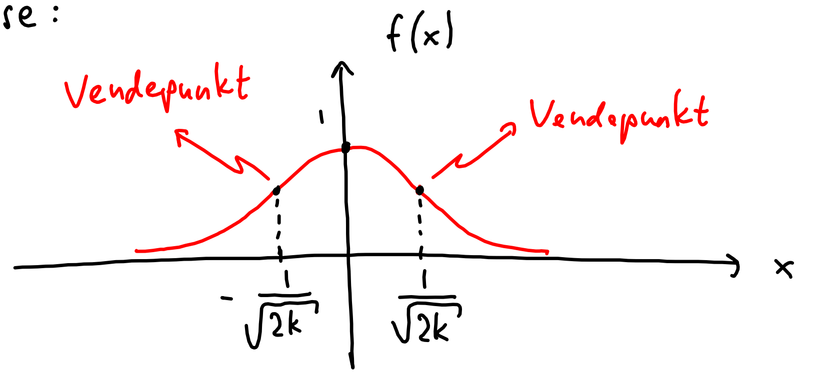
$f''(x) = 0$ gir $2kx^2 - 1 = 0$, dvs. $x = \pm \frac{1}{\sqrt{2k}}$



Konveks på $\left(-\infty, -\frac{1}{\sqrt{2k}}\right]$ og $\left[\frac{1}{\sqrt{2k}}, \infty\right)$

Konkav på $\left[-\frac{1}{\sqrt{2k}}, \frac{1}{\sqrt{2k}}\right]$

Skisse:



c) $V(r) = 2\pi \int_0^r x\, f(x)\,dx = 2\pi \int_0^r x e^{-kx^2}\,dx$

$= 2\pi \int_0^{-kr^2} \cancel{x} e^{u} \left(-\frac{1}{2k\cancel{x}}\right) du = 2\pi \left(-\frac{1}{2k}\right) \int_0^{-kr^2} e^u\,du$

$u = -kx^2 \qquad \frac{du}{dx} = -2kx$

$du = -2kx\,dx \qquad dx = -\frac{1}{2kx}\,du$

$x = 0$ gir $u = 0$
$x = r$ gir $u = -kr^2$

$= -\frac{\pi}{k}\left[e^u\right]_0^{-kr^2}$

$= -\frac{\pi}{k}\left[e^{-kr^2} - e^0\right]$

$= \underline{\underline{\frac{\pi}{k}\left(1 - e^{-kr^2}\right)}}$

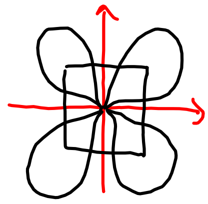
$\lim_{r\to\infty} V(r) = \lim_{r\to\infty} \frac{\pi}{k}\left(1 - \underbrace{e^{-kr^2}}_{\to 0}\right) = \underline{\underline{\frac{\pi}{k}}}$

$-kr^2 \to -\infty$

<u>Grensen eksisterer</u>

Oppgave 3

a) $\vec{F}(x,y,z) = (x^2yz,\ xy^2z,\ xyz^2) = xyz\cdot(x,y,z)$



omtrent...

$$\vec{F}' = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial z} \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2xyz & x^2z & x^2y \\ y^2z & 2xyz & xy^2 \\ yz^2 & xz^2 & 2xyz \end{pmatrix}}}$$

b) $$\begin{vmatrix} 2xyz & x^2z & x^2y \\ y^2z & 2xyz & xy^2 \\ yz^2 & xz^2 & 2xyz \end{vmatrix}$$

$$= 2xyz\begin{vmatrix} 2xyz & xy^2 \\ xz^2 & 2xyz \end{vmatrix} - x^2z\begin{vmatrix} y^2z & xy^2 \\ yz^2 & 2xyz \end{vmatrix} + x^2y\begin{vmatrix} y^2z & 2xyz \\ yz^2 & xz^2 \end{vmatrix}$$

$$= 2xyz(4x^2y^2z^2 - x^2y^2z^2) - x^2z(2xy^3z^2 - xy^3z^2) + x^2y(xy^2z^3 - 2xy^2z^3)$$

$$= 2xyz\cdot 3x^2y^2z^2 - x^2z\cdot xy^3z^2 + x^2y(-xy^2z^3)$$

$$= 6x^3y^3z^3 - x^3y^3z^3 - x^3y^3z^3 = \underline{4x^3y^3z^3}$$

Determinanten er 0 for de punktene (x,y,z) der $x=0$ eller $y=0$ eller $z=0$.

Oppgave 4

$f : (-1, 1) \to \mathbb{R}$ ved $f(x) = \begin{cases} \dfrac{x^2}{1-\sqrt{1-x^2}} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$

Har $f(0) = k$

f kont. i $x = 0$ betyr at $\lim\limits_{x \to 0} f(x) = f(0)$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2}{1-\sqrt{1-x^2}}$$

$$= \lim_{x \to 0} \frac{x^2 \cdot \left(1+\sqrt{1-x^2}\right)}{\left(1-\sqrt{1-x^2}\right)\cdot\left(1+\sqrt{1-x^2}\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2} \cdot \left(1+\sqrt{1-x^2}\right)}{\cancel{1^2} - \left(\cancel{1} - \cancel{x^2}\right)}$$

$$= \lim_{x \to 0} \left(1+\sqrt{1-x^2}\right) = 2$$

Ja, det fins en slik k, nemlig $\underline{\underline{k=2}}$

At f er kont. for $x \neq 0$ fås fordi den er den gitt ved en regneformel bygd opp av kontinuerlige funksjoner.

Oppgave 5

$P(z) = z^3 - 11z^2 + 36z - 26$

$P(1) = 1 - 11 + 36 - 26 = 0$ $\quad z = 1$ rot

$$\left(z^3 - 11z^2 + 36z - 26\right) : (z-1) = z^2 - 10z + 26$$

$$\begin{array}{l}
z^3 - z^2 \\
\hline
\quad -10z^2 + 36z \\
\quad -10z^2 + 10z \\
\hline
\qquad\quad 26z - 26 \\
\qquad\quad 26z - 26 \\
\hline
\qquad\qquad\quad 0
\end{array}$$

Dvs:

$$P(z) = \left(z^2 - 10z + 26\right)\cdot(z-1)$$

$$z^2 - 10z + 26 = 0 \quad \text{gir}$$

$$z = \frac{10 \pm \sqrt{100 - 4\cdot 26}}{2} = \frac{10 \pm \sqrt{-4}}{2}$$

$$= \frac{10 \pm \sqrt{(-1)\cdot 4}}{2} = \frac{10 \pm \sqrt{-1}\cdot 2}{2} = \frac{10 \pm 2i}{2} = 5 \pm i$$

Kompleks faktorisering:

$$P(z) = (z-1)\cdot\big(z-(5+i)\big)\cdot\big(z-(5-i)\big)$$

Oppgave 6

$$\frac{2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$2 = A(x^2+1) + (Bx+C)(x+1)$$

$x = -1$ gir $2 = A\cdot 2 + 0$, $A = 1$

$x = 0$ gir da $2 = 1\cdot(0+1) + C\cdot 1$

$2 = 1 + C$, dvs. $C = 1$

x^2-ledd gir $A + B = 0$, dvs. $B = -1$

$$\int \frac{2}{(x+1)(x^2+1)}\,dx = \int \frac{1}{x+1}\,dx + \int \frac{1-x}{x^2+1}\,dx$$

$$= \ln|x+1| + \int \frac{1}{1+x^2}\,dx - \int \frac{x}{x^2+1}\,dx$$

$$= \ln|x+1| + \arctan x - \int \frac{\cancel{x}}{u}\cdot\frac{1}{2\cancel{x}}\,du$$

$$\boxed{\begin{array}{ll} u = x^2+1 & \frac{du}{dx} = 2x \\ du = 2x\,dx & dx = \frac{1}{2x}\,du \end{array}}$$

$$= \ln|x+1| + \arctan x - \frac{1}{2}\ln|u| + C$$

$$= \ln|x+1| + \arctan x - \frac{1}{2}\ln(1+x^2) + C$$

Oppgave 7

$f:\underbrace{(0,\infty)}_{D_f} \to \mathbb{R}$ ved $f(x) = (x+7)e^{2/x}$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (\underset{\to 0}{x}+7)\underbrace{e^{\overset{\to +\infty}{2/x}}}_{\to +\infty} = +\infty$$

Vertikal asymptote $x=0$

Ingen vertikale asymptoter for $x>0$, for f er kontinuerlig der.

Tester for skråasymptote:

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{(x+7)e^{2/x}}{x}$$

$$= \lim_{x\to\infty} \left(e^{\overset{\to 0}{2/x}} + \frac{7e^{\overset{\to 1}{2/x}}}{x}\right) = 1 + 0 = 1$$

$$b = \lim_{x\to\infty} \left[f(x) - ax\right] = \lim_{x\to\infty} \left[(x+7)e^{2/x} - x\right]$$

$$= \lim_{x\to\infty} \left[(xe^{2/x} - x) + \underbrace{7e^{\overset{\to 0}{2/x}}}_{\to 7}\right]$$

$$\overset{[\infty-\infty]}{=} \lim_{x\to\infty} x\left(e^{2/x} - 1\right) + 7$$

$$\overset{[\infty\cdot 0]}{=} \lim_{x\to\infty} \frac{e^{2/x} - 1}{\frac{1}{x}} + 7$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to\infty} \frac{e^{2/x}\left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}} + 7 = \lim_{x\to\infty} 2e^{\overset{\to 1}{2/x}} + 7$$

$$= 9$$

Skråasymptote: $y = x + 9$