SNUBLEGRUPPE 28.09.2016

$\sqrt{4} = 2$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

Funksjon:

$\mathbb{R} \to \mathbb{R}$

D_f

V_f

$f(x) = \sin x$ $\quad D_f = \mathbb{R}$

$V_f = [-1, 1]$

$f(x) = \sqrt{x+1}$ $\quad D_f : [-1, \infty)$ $\quad x+1 \geq 0$

$x \geq -1$

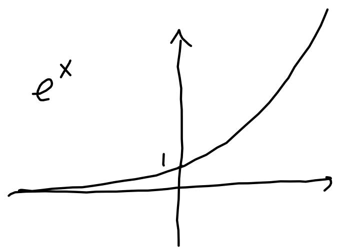
$V_f : [0, \infty)$

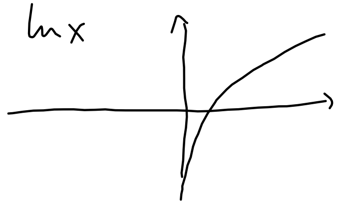
$f(x) = \ln(\underline{\sin x})$

$\sin x > 0$

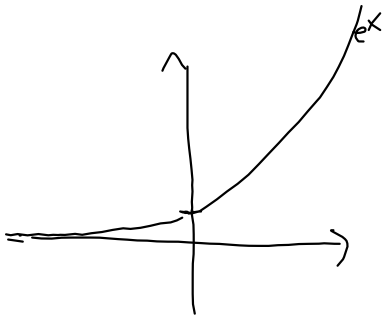
$D_f = \underline{(0, \pi)} + \underline{2n\pi}, \ \underline{n \in \mathbb{Z}}$

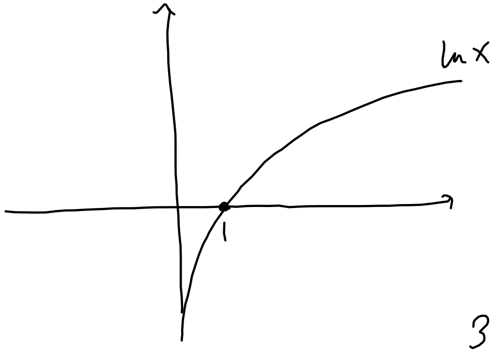
$= \bigcup_{n=-\infty}^{\infty} (2n\pi, (2n+1)\pi)$





$$\boxed{e^{\ln x} = x}$$





$e^{\overset{\ln 1}{0}} = 1$ $\boxed{\ln 1 = 0}$

$e^1 = e$ $\ln e = 1$

$e^{-1} = \frac{1}{e}$ $\ln \frac{1}{e} = -1$

3 Logaritmeregler:

① : $\ln(a \cdot b) = \ln a + \ln b$ ← $\ln(a \cdot a) = 2\ln a$

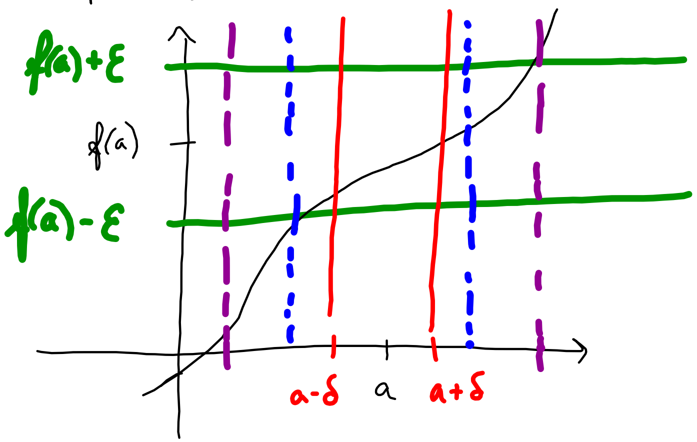
② $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ $\ln(ab^{-1})$

③ $\ln(a^n) = n \ln a$

KONTINUITET:

En funksjon f er kontinuerlig i et punkt $a \in D_f$ dersom følgende gjelder:

For enhver $\varepsilon > 0$ eksisterer en $\delta > 0$ slik at når $x \in D_f$ og $|x-a| < \delta$ så er $|f(x) - f(a)| < \varepsilon$.



5.1.5. a) $f(x) = 2x+1$ er kontinuerlig i $a=2$

Gitt $\varepsilon > 0$

Se på $|x-2| < \delta$

og $|f(x) - f(2)| = |2x+1-(2\cdot 2+1)|$

$= |2x+1-5|$

$= |2x-4|$

$= 2|x-2|$

$< 2\cdot\delta$

$\leq 2\cdot\frac{\varepsilon}{2} = \varepsilon$.

Velg $\delta = \frac{\varepsilon}{2}$

v/ Def. av kontinuitet er $f(x)$ kontinuerlig i $a=2$.

5.1.5. b) $f(x) = x^2$ i $a = 3$

TRIKS: $h = x - a$ (3)

$x = h + 3$

Gitt $\varepsilon > 0$ $\quad |h| = |x-3| < \delta$

$$|f(x) - f(3)| = |x^2 - 9|$$
$$= |(h+3)^2 - 9|$$
$$= |h^2 + 6h + \cancel{9-9}|$$
$$= |h^2 + 6h|$$
$$= |h||h+6|$$
$$< |h| \cdot 7$$
$$= \frac{\varepsilon}{7} \cdot 7$$
$$= \varepsilon$$

TRIKS 2:

$|h| < \delta < 1$

$|h+6| < 7$

$|h| < \delta < 2$

$|h+6| < 8$

$< |h| \cdot 8$

Velg $\delta = \frac{\varepsilon}{8}$ $\quad = \frac{\varepsilon}{\cancel{8}} \cdot \cancel{8}$

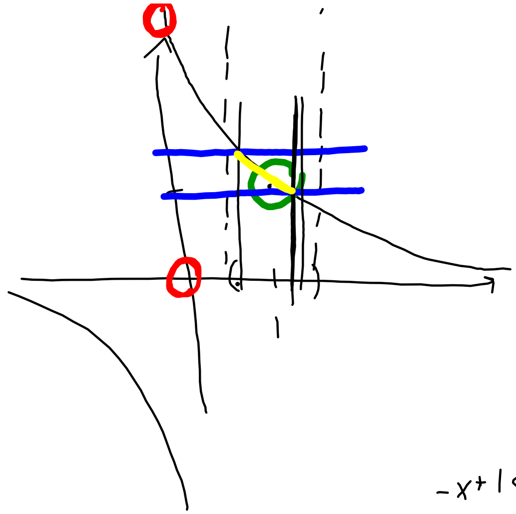
$= \varepsilon$

Velg $\delta \leq \frac{\varepsilon}{7}$

Velg $\delta = \min\{\frac{\varepsilon}{7}, 1\}$

Da er $f(x)$ kont i $a=3$.

5.1.5. e) $f(x) = \frac{1}{x}$ i pkt $a = 1$



Gitt $\varepsilon > 0$.

$$|f(x) - f(1)| = \left|\frac{1}{x} - 1\right|$$
$$= \left|\frac{1-x}{x}\right|$$
$$= \left|\frac{x-1}{x}\right|$$
$$= \frac{|x-1|}{|x|}$$
$$< \frac{|x-1|}{\frac{1}{2}}$$
$$= |x-1| \cdot 2$$
$$< \frac{\varepsilon}{2} \cdot 2 = \underline{\varepsilon}$$

$-x+1 < \frac{1}{2}$
$-x < -\frac{1}{2}$
$\underline{x > \frac{1}{2}}$

(1) Holde x unna 0.
Velg $\delta_1 = \frac{1}{2}$
$|x-1| < \frac{1}{2}$
$\frac{1}{2} < |x|$

$x - 1 < \frac{1}{2}$ eller $-(x-1) < \frac{1}{2}$
$x < \frac{3}{2}$ eller $x > \frac{1}{2}$

(2) Velg $\delta_2 = \frac{\varepsilon}{2}$
(?) Velg $\delta = \min\left\{\frac{\varepsilon}{2}, \frac{1}{2}\right\}$

5.1.5. g) $f(x) = \sqrt{x}$ i $a = 4$ $|x-4| < \delta$

Gitt $\varepsilon > 0$

$$\underline{|\sqrt{x} - \sqrt{4}|} = |\sqrt{x} - 2| = \left|\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}+2)}\right|$$

$$= \left|\frac{x-4}{\sqrt{x}+2}\right|$$

① Velg $\delta_1 = 4$, da er $x > 0$ og $|\sqrt{x}+2| > 2$

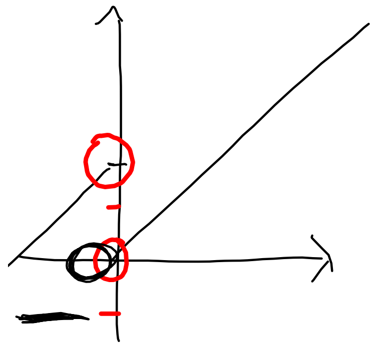
$$\leq \frac{|x-4|}{2}$$

② Velg $\delta_2 = 2\varepsilon$

$$< \frac{\cancel{2}\varepsilon}{\cancel{2}} = \underline{\underline{\varepsilon}}.$$

③ <u>Velg $\delta = \min\{2\varepsilon, 4\}$.</u>

5.1.6. a)



$$f(x) = \begin{cases} x+1 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$$ diskontinuerlig i $x=0$.

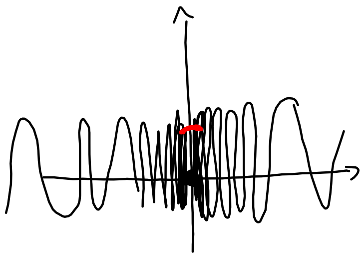
- La $0 < \varepsilon < 1$
- OBS: $f(x) = x+1 > (\varepsilon - 1) + 1 = \varepsilon$

  $\underline{x \in (\varepsilon - 1, 0)}$
- $|f(x) - f(0)| = f(x) > \varepsilon$ i intervallet $(\varepsilon - 1, 0)$

  Uansett valg av δ.

  Dermed er $f(x)$ diskont i $x=0$.

5.1.6. b) $f(x) = \begin{cases} \cos\frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$ er diskont. i $x = 0$



- da $0 < \varepsilon < 1$
- OBS: Vi vet at $\left|\cos\frac{1}{x}\right| = 1 > \varepsilon$ for alle $x = \frac{1}{k\pi}$, $\underline{k \in \mathbb{Z}}$

$$\cos\frac{1}{\frac{1}{k\pi}} = \cos k\pi$$

- $\left|\cos\frac{1}{x} - 0\right| > \varepsilon$

Vansett hvor liten vi velger δ, finnes en k

s.a. $\left|\frac{1}{k\pi} - 0\right| < \delta$

og samtidig $\left|\cos\frac{1}{x} - 0\right| > \varepsilon$.

5.1.7. d)

$f(x) = \cos\left[\ln\left|\sin\left(e^{x^2}\right)\right|\right]$ er kont. i $x = 0$

- x^2 er kont i 0 $\quad 0^2 = 0$
- e^u er kont i 0 $\quad e^0 = \underline{1}$
- $\sin(v)$ er kont. i 1 $\quad \sin 1 > 0$
- $|w|$ er kont i $\sin 1$. fordi kontinuerlig på $\mathbb{R}$
- $|w|$ er kont i $\sin 1$ fordi $\sin 1 \in D_{fns}$. $D_g = (0, \infty)$
- $g(s) = \ln s$ er kont i $\sin 1$ fordi
- $\cos t$ er kont i $\ln(\sin 1)$.

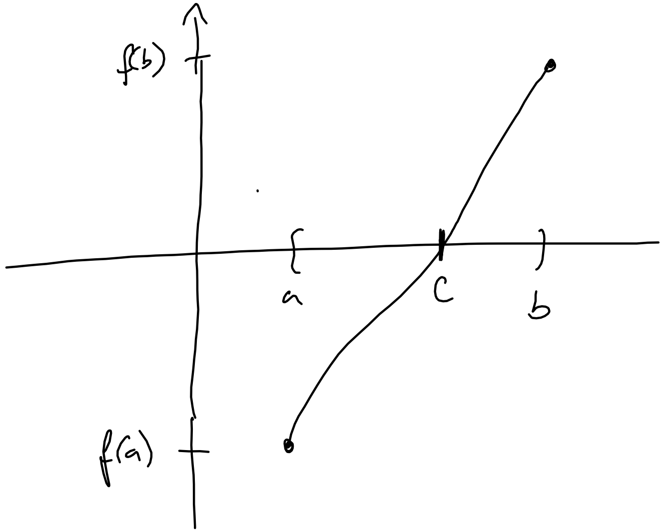
Dermed er $f(x)$ kontinuerlig i $x = 0$.

SKJÆRINGSSETN:

Anta $f:[a,b] \longrightarrow \mathbb{R}$ er en kontinuerlig funksjon.

der $f(a)$ og $f(b)$ har motsatt fortegn.

Da finnes en $c \in [a,b]$ slik at $f(c) = 0$.



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5.2.1. b)

$f(x) = e^x - x - 2$ på $[0,2]$

$f(0) = 1 - 0 - 2 = -1$

$f(2) = e^2 - 2 - 2 > 0$

siden f kont, kan jeg bruke skjæringssetningen, og f har nullpkt på $[0,2]$.