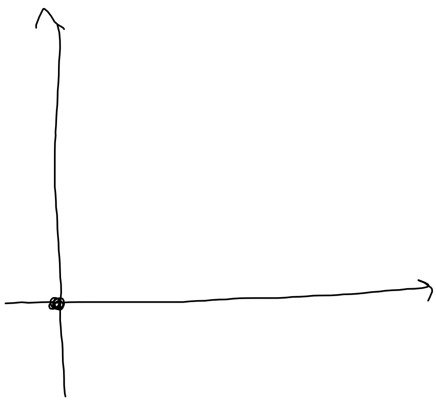
MIDTVEIS H2015

SNUBLE~~GRUPPE~~ ORAKEL MAT1100 12.10.2016

1\.



A) 0

B) $\pi + i$

C) $\frac{1}{2} i$

$\boxed{\text{D) JA}}$

E) i

2\. $z = -1 + i\sqrt{3}$

$$= \underline{\underline{2e^{i\frac{2\pi}{3}}}}$$

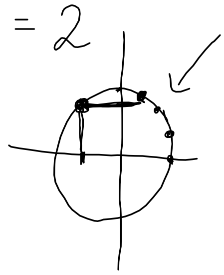
C

$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

θ: $\cos\theta = \frac{-1}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{2\pi}{3}$

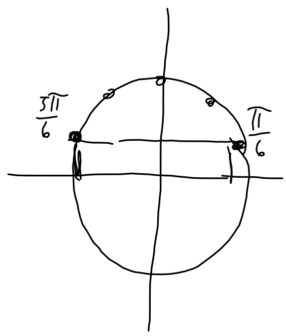


3\. $z = 2e^{i\frac{5\pi}{6}} = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 2\left(-\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right)$

$= \underline{\underline{-\sqrt{3} + i}}$

A



4. $z^2 - 2z + (1-2i) = 0$

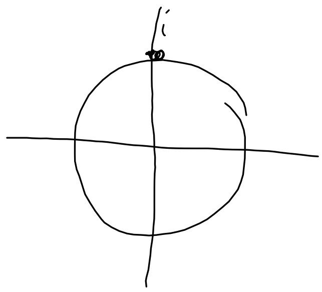
$$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (1-2i)}}{2 \cdot 1}$$

$$z = \frac{2 \pm \sqrt{4 - 4 + 8i}}{2}$$

$$z = \frac{2 \pm \sqrt{8i}}{2}$$

$$z = \frac{2 \pm 2\sqrt{2i}}{2}$$

$$z = 1 \pm \sqrt{2i}$$



$$\sqrt{2i} = \sqrt{2e^{i\frac{\pi}{2}}} = \sqrt{2}\sqrt{e^{i\frac{\pi}{2}}} = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$z = 1 \pm \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$z = 1 \pm \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$z = 1 \pm \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$z = 1 \pm (1+i)$$

$z = 2 + i$

$z = -i$

B

4 alternativ løsning: $z^2 - 2z + (1-2i) = 0$

$z = -i$

$$(-i)^2 - 2(-i) + (1-2i)$$
$$= -1 + 2i + 1 - 2i$$
$$= \underline{\underline{0}}$$

Tester A: $z = 1 - i$

$$(1-i)^2 - 2(1-i) + (1-2i)$$
$$= 1 - 2i + i^2 - \cancel{2} + \cancel{2i} + \cancel{1} - \cancel{2i}$$
$$= -2i - 1 \neq 0$$

så A passer ikke!

Tester B: $z = 2+i$

$$(2+i)^2 - 2(2+i) + (1-2i)$$
$$= \cancel{4} + \underline{4i} + i^2 - \cancel{4} \underline{-2i} + 1 \underline{-2i}$$
$$= -1 + 1$$
$$= \underline{\underline{0}}$$

B

z^3

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+(-b))^3$$

5. T. 4.3.9 s 211:

En $\underbrace{\text{monoton, begrenset}}$ følge er $\underbrace{\text{konvergent}}$

$\Longrightarrow$

B

6. $$\lim_{x\to\infty} \frac{\ln(4x^2+1)}{\ln(x+1)} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{1}{4x^2+1}\cdot(8x)}{\frac{1}{x+1}\cdot 1}$$

$$= \lim_{x\to\infty} \frac{8x}{4x^2+1} \cdot \frac{x+1}{1}$$

$$= \lim_{x\to\infty} \frac{8x^2+8x}{4x^2+1}$$

$$= \lim_{x\to\infty} \frac{8+\frac{8}{x}}{4+\frac{1}{x^2}} = \underline{\underline{2}} \quad E$$

7. $$\lim_{x \to 0} \frac{(\sin x)^2}{5x + x^2} \overset{\text{"}\frac{0}{0}\text{"}}{\underset{L'H}{=}} \lim_{x \to 0} \frac{2 \sin x \cdot \cos x}{5 + 2x} = \frac{0}{5} = \underline{\underline{0}}$$

C

8. D

$w = c + id$
$\bar{w} = c - id$

$w = c$
$\bar{w} = c$

$$az^2 + bz + c = 0$$

$$\overline{az^2 + bz + c} = \overline{0}$$

$$\bar{a}\bar{z}^2 + \bar{b}\bar{z} + \bar{c} = 0 \quad , a, b, c \in \mathbb{R}$$

$$a\bar{z}^2 + b\bar{z} + c = 0$$

så z og $\bar{z}$ er løsninger.

D

9 $a_{n+1} = \sqrt[3]{\frac{(a_n^3+1)}{2}}$

Anta $\lim_{n\to\infty} a_n = A$

$$A = \sqrt[3]{\frac{(A^3+1)}{2}}$$

$$A^3 = \frac{A^3+1}{2} \quad \Big| \cdot 2$$

$$2A^3 = A^3 + 1 \quad \Big| -A^3$$

$$A^3 = 1$$

$$\underline{A = 1}$$

B eller E

$a_1 = 1 \qquad a_2 = \sqrt[3]{\frac{1^3+1}{2}} = 1, \rightarrow \underline{a_n = 1}$

Dette motsier E!

10. $a_n = e^{-n}(\cos n + (-2)^n)$

$$\lim_{n\to\infty} \frac{\cos n + (-2)^n}{e^n}$$

$$= \lim_{n\to\infty}\left(\frac{\cos n}{e^n} + \frac{(-2)^n}{e^n}\right)$$

$e \simeq 2{,}718$

$\frac{\cos n}{e^n} \to 0$

$\frac{(-2)^n}{e^n} = \left(\frac{-2}{e}\right)^n \to 0$ siden $\left|\frac{-2}{e}\right| < 1$

$= 0$ B

11. D

12. $f(x) = \sin(\sin x) + e^x \ln(2x+1)$

$f'(x) = \cos(\sin x) \cdot \cos x$ B eller E

$+ e^x \cdot \ln(2x+1) + e^x \frac{1}{2x+1} \cdot 2$

$= \cos(\sin x) \cdot \cos x$

$+ e^x \ln(2x+1) + \frac{2e^x}{2x+1}$

B

13. $f(x) = \frac{x^2}{x-1}$

I: nevner $\neq 0$

$x = 1$ er asymptote

B eller C

II: ① $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x}{x-1}$

$= \lim_{x\to\infty} \frac{1}{1 - \frac{1}{x}}$

$y = ax + b$

$= 1$, dvs. $a = 1$

② $\lim_{x\to\infty} \left(\frac{x^2}{x-1} - x \right) = \lim_{x\to\infty} \left(\frac{x^2 - x(x-1)}{x-1} \right)$

$f(x) - ax$

$= \lim_{x\to\infty} \frac{x}{x-1} = 1$ dvs $b = 1$

③ $y = x + 1$

C

14. $\lim_{x \to 0} (1-x)^{\frac{1}{x}} \overset{"1^{\infty}"}{\underset{e\text{-triks}}{=}} \lim_{x \to 0} e^{\ln (1-x)^{\frac{1}{x}}}$

$\boxed{e^{\ln b} = b}$

③ $\ln a^n = n \ln a$

③ $= \lim_{x \to 0} e^{\frac{1}{x} \ln(1-x)}$

$= e^{\lim_{x \to 0} \frac{1}{x} \ln(1-x)}$

Se på eksp: $\lim_{x \to 0} \frac{1}{x} \ln(1-x) \overset{"\frac{0}{0}"}{\underset{L'H}{=}} \lim_{x \to 0} \frac{\frac{1}{1-x} \cdot (-1)}{1}$

$= \underline{-1}$

$= \underline{\underline{e^{-1}}}$ D

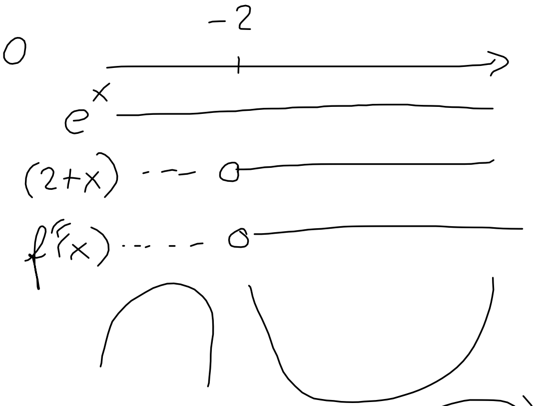
15. $f(x) = xe^x$

$f'(x) = e^x + xe^x$

$f''(x) = e^x + e^x + xe^x$

$= 2e^x + xe^x$

$= \underline{e^x(2+x)} = 0$



strengt konveks $(-2, \infty)$ D

16. $f(x) = \ln(a + \ln(b + \ln(c+x)))$

$$f'(x) = \frac{1}{a + \ln(b + \ln(c+x))} \cdot (a + \ln(b + \ln(c+x)))'$$

$$= \frac{1}{\sim\sim} \cdot \frac{1}{b + \ln(c+x)} \cdot (b + \ln(c+x))'$$

$$= \frac{1}{a + \ldots} \cdot \frac{1}{b + \ldots} \cdot \frac{1}{c+x} (c+x)'$$

E

17. MVT er oppfylt!

$\exists\, c \in (a,b)$ s.a. $f'(c) = \frac{f(b)-f(a)}{b-a}$

det eksisterer

$$f'(c)(b-a) = f(b) - f(a)$$

$$f(b) = f(a) + f'(c)(b-a)$$

C

18. $f(x) = xe^{7/x}$

① $\lim_{x\to\infty} e^{7/x} = 1$ $\qquad \underline{a=1}$

② $\lim_{x\to\infty} (xe^{7/x} - x) = \lim_{x\to\infty} x(e^{7/x} - 1)$

$\overset{\text{"}\infty\cdot 0\text{"}}{=} \lim_{x\to\infty} \frac{e^{7/x}-1}{\frac{1}{x}} \underset{L'H}{\overset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to\infty} \frac{e^{7/x}\cdot\left(\frac{7}{x}\right)'}{\left(\frac{1}{x}\right)'}$

$= \lim_{x\to\infty} e^{7/x}\cdot 7 = \underline{\underline{7}}$ $\qquad \underline{b=7}$

③ $\underline{y = x + 7}$

A