SNUBLEGRUPPA MAT1100

- HVORFOR SNUBLEGRUPPA
- HVORDAN LÆRTE JEG TALL
- HVA FØLER DU FOR MATEMATIKK

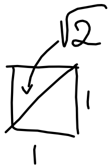
HVORDAN LÆRTE JEG TALL

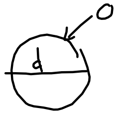
$\mathbb{N}$ $\{1, 2, 3, \dots\}$ De naturlige tallene

$\mathbb{Z}$ $\{\dots, -3, -2, -1, \underline{\underline{0}}, 1, 2, 3, \dots\}$ De hele tallene

$\mathbb{Q}$ $\frac{a}{b}$ $\left\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\right\}$ Brøkene / de rasjonale tallene

Irr



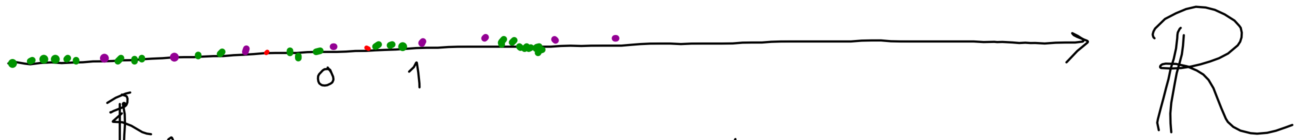


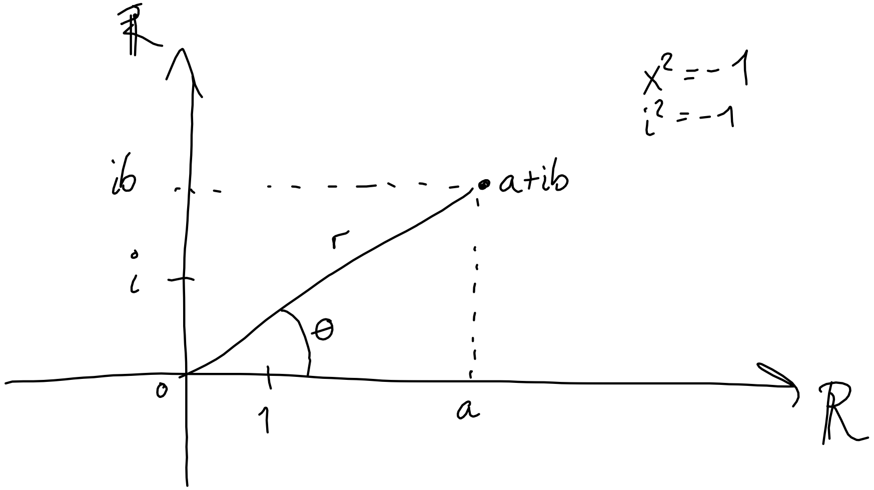
$\frac{O}{d} = \pi$ $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2{,}718281828\dots$ $\varphi = \frac{1+\sqrt{5}}{2}$

De irrasjonale tallene

algebraiske
$x^2 = 2$

transcendentale





$x^2 = -1$

$i^2 = -1$

POTENSREGLER

$(\in \mathbb{N})$

$$\underbrace{a \cdot \ldots \cdot a}_{n \text{ ganger}} = a^n$$

eksponent

POTENS

grunntall $\neq 0$

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{m \cdot n} = a^{n \cdot m} = (a^m)^n$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$\frac{a}{a} = a^{1-1} = \boxed{a^0 = 1}$$

0^0 ikke def

$$a^{-n} = a^{0-n} = \frac{a^0}{a^n} = \frac{1}{a^n}$$

Kommutativitet av + og ·

$$n \cdot m = m \cdot n$$

$$n + m = m + n$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\underbrace{a^{\frac{1}{n}} \cdot \ldots \cdot a^{\frac{1}{n}}}_{n \text{ ganger}} = a$$

$$(a^{\frac{1}{n}})^n = a^{\frac{n}{n}} = a^1 = a$$

$$a^{\pi} \quad a^e \quad e^{\sqrt{2}} \quad \sqrt{2}^{\pi}$$

Hva med $z = \underline{a} + i\underline{b}$ $\quad e^z$

$$e^z := e^a(\cos b + i \sin b)$$

$$e^z = e^{a+ib} = e^a \cdot e^{ib}$$

LIKNINGER

er lik

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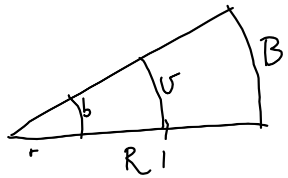
VARIABLER

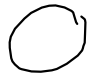
RADIANER OG EKSAKTE VERDIER AV TRIGONOMETRISKE FUNKSJONER



$$\frac{B}{R} = \frac{b}{r} = \frac{v}{1} = v$$

$$\frac{\theta}{2\cdot 1} = \frac{\theta}{2r} = \frac{\theta}{d} = \pi$$

$$\underline{\theta = 2\pi}$$



$360^\circ = 2\pi$

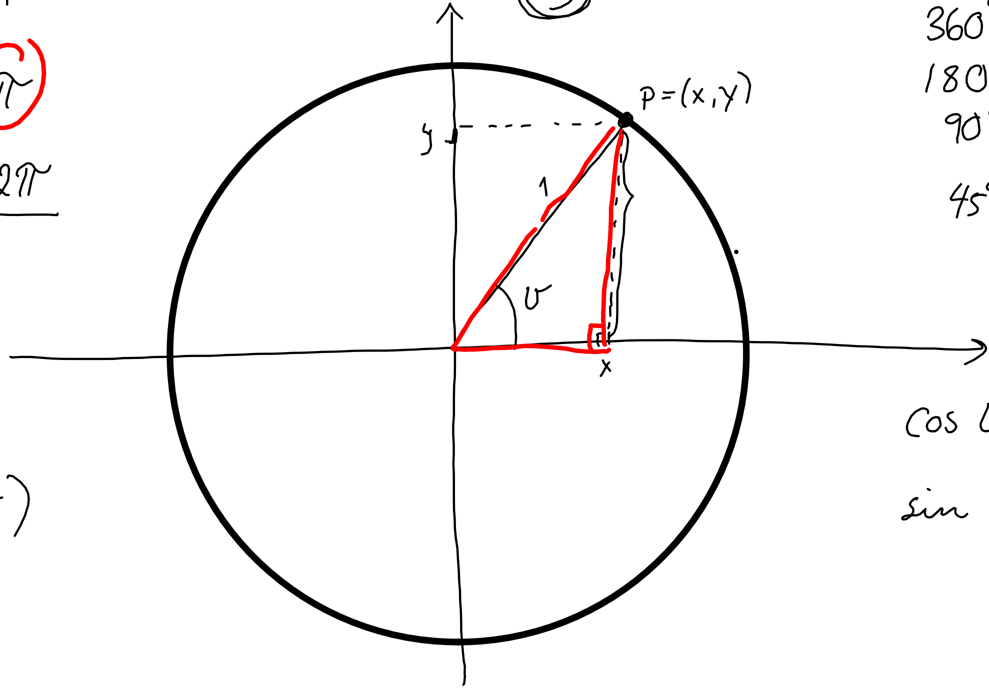
$180^\circ = \pi$

$90^\circ = \frac{\pi}{2}$

$45^\circ = \frac{\pi}{4}$

$60^\circ = \frac{\pi}{3}$

$30^\circ = \frac{\pi}{6}$

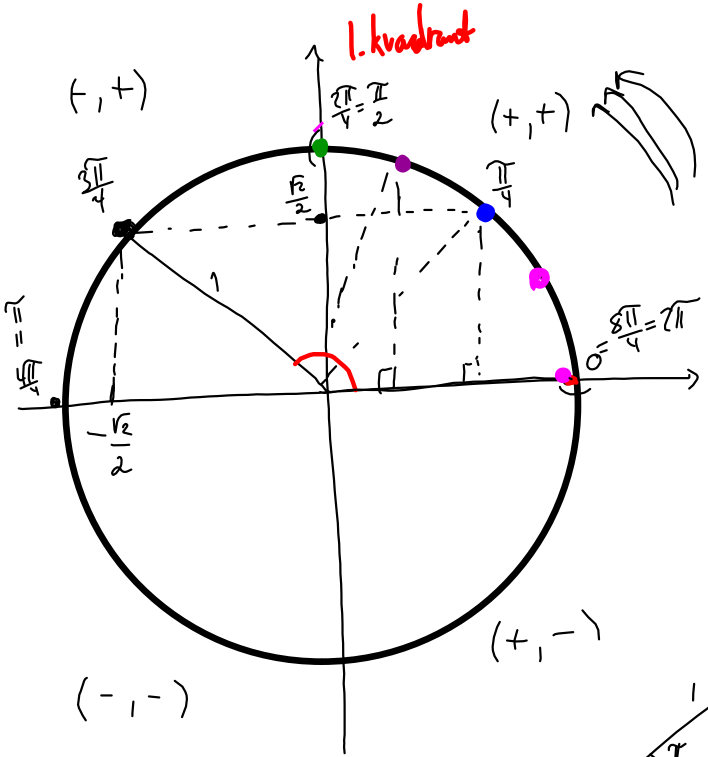


$$P = (\overset{x}{\cos v}, \overset{y}{\sin v})$$

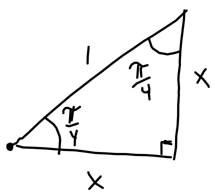
$$\cos v = \frac{x}{1} = x$$

$$\sin v = \frac{y}{1} = y$$

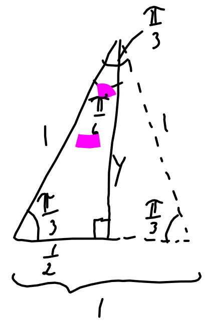
$$v = v + 2k\pi,\ k \in \mathbb{Z}$$



v	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin v$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan v$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—



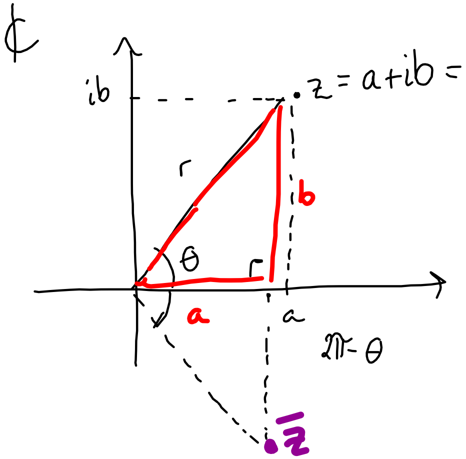
$$x^2 + x^2 = 1^2$$
$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \frac{\sqrt{2}}{2}$$



$$y^2 + \frac{1}{2}^2 = 1^2$$
$$y^2 = 1 - \frac{1}{4} = \frac{3}{4}$$
$$y = \frac{\sqrt{3}}{2}$$

KOMPLEKSE TALL

$\mathbb{C}$



$z = a+ib = re^{i\theta} = r(\cos\theta + i\sin\theta)$

(a,b)
(r,θ)

$$\boxed{\begin{aligned} r &= \sqrt{a^2+b^2} \\ \theta &: \quad \cos\theta = \frac{a}{r} \\ & \quad \sin\theta = \frac{b}{r} \end{aligned}}$$

Trenger begge!

$$\boxed{\begin{aligned} a &= r\cos\theta \\ b &= r\sin\theta \end{aligned}}$$

$$z = a+ib = r\cos\theta + ir\sin\theta = r(\underbrace{\cos\theta + i\sin\theta}_{= e^{i\theta}}) = re^{i\theta}$$

I $\quad \bar{z} = a - ib = re^{-i\theta}$

II $\quad$ Divisjon; vil ikke ha i i nevner; utvide med $\overline{\text{nevner}}$

III $\quad$ Kan trekke røtter: n $\quad$ n-te røtter

$z = re^{i\theta}$

$z^{\frac{1}{n}}: \quad \underline{w_k = r^{\frac{1}{n}} e^{i\left(\frac{\theta + 2k\pi}{n}\right)}}, \quad k = 0, \dots, n-1$

$w_0 \cdot \left(e^{\frac{i2\pi}{n}}\right)^k$

$\underbrace{r^{\frac{1}{n}} e^{\frac{i\theta}{n}}}_{} \cdot \underbrace{e^{\frac{i2k\pi}{n}}}_{}$

3.1.1. c) $2i + 3(4+i) = \underline{2i} + 12 + \underline{3i}$

$= 12 + i(2+3)$

$= 12 + 5i$

$i^2 = -1$

3.1.5 b) $(1+i)\underline{z} + 3 = 1 - i \quad | -3$

$(1+i)z = 1 - 3 - i$

$(1+i)z = -2 - i \quad | : (1+i)$

$$z = \frac{-2-i}{1+i} = \frac{(-2-i)\cdot(1-i)}{(1+i)\,(1-i)} = \frac{-2(1-i) - i(1-i)}{1\cdot(1-i) + i(1-i)} = \frac{-2 + 2i - i + \overset{=-1}{i^2}}{1 - i + i - i^2}$$

$$= \frac{-2 + i - 1}{1 - (-1)}$$

$$= \frac{-3+i}{2} = -\frac{3}{2} + \frac{1}{2}i$$

3.2.5. b) $z = 1\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \underline{\underline{\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}}}$

3.2.13. $z = 1 + i\sqrt{3}$ $\quad w = 1 + i$

a)
$$\begin{aligned} z\cdot w &= (1+i\sqrt{3})(1+i) \\ &= 1\cdot(1+i) + i\sqrt{3}(1+i) \\ &= 1 + i + i\sqrt{3} + \sqrt{3}\cdot i^2 \\ &= 1 + i + i\sqrt{3} - \sqrt{3} \\ &= \underline{\underline{(1-\sqrt{3}) + i(1+\sqrt{3})}} \end{aligned}$$

$$\begin{aligned} \frac{z}{w} &= \frac{(1+i\sqrt{3})}{(1+i)} = \frac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)} \\ &= \frac{1\cdot(1-i) + i\sqrt{3}(1-i)}{2} \\ &= \underline{\underline{\frac{(1+\sqrt{3})}{2} + \frac{(-1+\sqrt{3})}{2}i}} \end{aligned}$$

b) $z = 1 + i\sqrt{3}$ $\quad a = 1$, $b = \sqrt{3}$

$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \underline{2}$

θ: $\left.\begin{aligned} \cos\theta &= \frac{1}{2} \\ \sin\theta &= \frac{\sqrt{3}}{2} \end{aligned}\right\}$ $\theta = \frac{\pi}{3}$

$\underline{\underline{z = 2e^{i\frac{\pi}{3}}}}$

$\underline{\underline{w = \sqrt{2}\, e^{i\frac{\pi}{4}}}}$

c)

$$\frac{z}{w} = \frac{2e^{i\frac{\pi}{3}}}{\sqrt{2}\,e^{i\frac{\pi}{4}}} = \sqrt{2}\,e^{i\frac{\pi}{12}} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

Sammenlikne med a)

$$\sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2} \qquad \sqrt{2}\sin\frac{\pi}{12} = \frac{(-1+\sqrt{3})}{2}$$