SNUBLEGRUPPE 09.11.2016
MAT 1100

DELVIS

$$(u\cdot v)' = u'v + uv'$$
$$u\cdot v = \int u'v\,dx + \underline{\int uv'\,dx}$$
$$\int uv'\,dx = u\cdot v - \int u'v\,dx$$

- produkter
- ·1-trikset
- flere ganger

SUBST

- Ser en sammensatt funksjon
- Bytte ut med ny variabel
- Invertere substitusjonen
- Grensene blir nye i bestemte integraler.

DELBRØK

- Brukes på rasjonale uttrykk
- Vi deler opp et stort uttrykk i mindre og håndterbare enheter.

9.1.1. b)

$$\int x \ln x \, dx$$

$u \quad v'$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2}x^2 \, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

DELVIS

$u = \ln x \qquad u' = \frac{1}{x}$

$v' = x \qquad v = \frac{1}{2}x^2$

$$\int u v' dx = u \cdot v - \int u' v \, dx$$

9.1.1 e)

$$\int 1 \arctan x \, dx$$

$$= x \arctan x - \int \frac{x}{1+x^2} \, dx$$

$$= x \arctan x - \frac{1}{2}\int \frac{ds}{s}$$

$$= x \arctan x - \frac{1}{2}\ln|s| + C$$

$$= x \arctan x - \frac{1}{2}\ln(1+x^2) + C$$

DELVIS

$u = \arctan x \qquad u' = \frac{1}{1+x^2}$

$v' = 1 \qquad v = x$

SUBST

$s = 1 + x^2$

$\frac{ds}{dx} = 2x$

$\frac{1}{2}ds = x\,dx$

9.1.9. $\int 1 \cdot \sin(\ln x)\,dx$

$$= x\sin(\ln x) - \int \cos(\ln x)\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx$$

$$= x\sin(\ln x) - \left(x\cos(\ln x) - \int -\sin(\ln x)\frac{1}{x}\cdot x\,dx\right)$$

$$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

$$2\int \sin(\ln x)\,dx = x\left(\sin(\ln x) - \cos(\ln x)\right) + C$$

$$\int \sin(\ln x)\,dx = \frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C$$

DELVIS

① $u = \sin(\ln x)$ $\quad u' = \cos(\ln x)\cdot\frac{1}{x}$

$v' = 1$ $\quad v = x$

② $u = \cos(\ln x)$ $\quad u' = -\sin(\ln x)\cdot\frac{1}{x}$

$v' = 1$ $\quad v = x$

9.1.11. $\int \frac{x^2}{1+x^2} \arctan x \, dx$

DELVIS

$u = \arctan x \qquad u' = \frac{1}{1+x^2}$

$v' = \frac{x^2+0}{1+x^2} \qquad v = \int \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \, dx$

$= \int 1 \, dx - \int \frac{1}{1+x^2} \, dx$

$= x - \arctan x$

9.2.1. a) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

SUBST
$u = \sqrt{x}$
$x = u^2$
$\frac{dx}{du} = 2u$
$dx = 2u\,du$

$= \int \frac{\sin u}{\cancel{u}} \cdot 2\cancel{u}\,du$

$= -2\cos u + C$

$= \underline{\underline{-2\cos\sqrt{x} + C}}$

9.2.1. e) $\int e^{\sqrt{x}}\,dx$

SUBST
$s = \sqrt{x}$
$x = s^2$
$\frac{dx}{ds} = 2s$
$dx = 2s\,ds$

DELVIS
$u = s \qquad u' = 1$
$v' = e^s \qquad v = e^s$

$= \int e^s \; 2s\,ds$

$= 2\int e^s \cdot s\,ds$

$= 2\left[se^s - \int e^s ds\right] = 2\left(se^s - e^s\right) + C = \underline{\underline{2\left(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}\right) + C}}$

9.2.9. $\int_0^1 e^{\arcsin x}\,dx$

SUBST

$s = \arcsin x$

$x = \sin s$

$\frac{dx}{ds} = \cos s$

$dx = \cos s\,ds$

$\frac{ds}{dx} = \frac{1}{\sqrt{1-x^2}}$

$s(0) = \arcsin 0 = 0$

$s(1) = \arcsin 1 = \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} e^s \cdot \cos s\,ds$$

$$= \cos s \cdot e^s - \int e^s(-\sin s)\,ds$$

$$= \cos s \cdot e^s + \int e^s \sin s\,ds$$

$$= \cos s \cdot e^s + e^s \sin s - \int e^s \cos s\,ds + C$$

① DELVIS

$u = \cos s \qquad u' = -\sin s$

$v' = e^s \qquad v = e^s$

② $u = \sin s \qquad u' = \cos s$

$v' = e^s \qquad v = e^s$

$$2\int_0^{\frac{\pi}{2}} e^s \cos s\,ds = \Big[e^s(\cos s + \sin s) + C\Big]_0^{\frac{\pi}{2}}$$

$$= e^{\frac{\pi}{2}} \cdot 1 - e^0 \cdot 1$$

$$\int_0^{\frac{\pi}{2}} e^s \cos s\,ds = \frac{e^{\frac{\pi}{2}} - 1}{2}$$

$$e^s(\cos s + \sin s) + e^s(-\sin s + \cos s)$$

$$= 2e^s \cos s.$$

DELBRØKS OPPSPALTNING

$\int \frac{P(x)}{Q(x)}\,dx \leftarrow$ Polynomer $\quad \deg Q(x) > \deg P(x)$

Hvis $\deg P(x) > \deg Q(x)$: Polynomdivisjon

$Q(x)$ kan faktoriseres over $\mathbb{R}$

I $\quad (x-r_i)^{n_i}$

$$\frac{C_1}{x-r_i} + \frac{C_2}{(x-r_i)^2} + \cdots + \frac{C_{n_i}}{(x-r_i)^{n_i}}$$

- $C_1 \ln|x-r_i|$
- $C_k \cdot \frac{1}{1+k}(x-r_i)^{1+k}$, $k=2,\ldots,n_i$

II $\quad (x^2+a_jx+b_j)^{m_j}$

$$\frac{A_1x+B_1}{(x^2+a_jx+b_j)} + \frac{A_2x+B_2}{(x^2+a_jx+b_j)^2} + \cdots + \frac{A_{m_j}x+B_{m_j}}{(x^2+a_jx+b_j)^{m_j}}$$

$m=1$

$A_1=0 \quad B_1\int\frac{1}{(x^2+a_jx+b_j)}dx$

- fullføre kvadrat $\frac{1}{1+u^2}$
- $\arctan u$

$A_1 \neq 0 \quad \int\frac{A_1x+B_1}{(x^2+a_jx+b_j)}dx$

- Manipulere til vi har derivert av nevner i teller $\rightarrow$ substitusjon

$m>1$

- $$I_m = \int\frac{du}{(1+u^2)^m} = \frac{1}{2(m-1)}\frac{u}{(1+u^2)^{m-1}} + \frac{2m-3}{2(m-1)} I_{m-1}$$

9.3.1. d)

$$\int \frac{x+7}{x^2-x-2}\,dx = \int \frac{x+7}{(x-2)(x+1)}\,dx$$

$$\frac{x+7}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$x+7 = Ax + A + Bx - 2B$$

$$1\cdot x + 7 = (A+B)x + (A-2B)$$

$$1 = A + B \qquad 7 = A - 2B$$

$$A = 1 - B \qquad 7 = (1-B) - 2B$$

$$6 = -3B$$

$$A = 3 \qquad B = -2$$

$$\int \frac{3}{x-2} + \frac{-2}{x+1}\,dx = 3\ln|x-2| - 2\ln|x+1| + C$$

9.3.3 b)

$$\int \frac{2x-2+0}{x^2+4x+8}\,dx$$

$$= \int \frac{2x+4-2-4}{x^2+4x+8}\,dx$$

$$= \int \frac{2x+4}{x^2+4x+8}\,dx - 6\int \frac{1}{x^2+4x+8}\,dx$$

$$= \int \frac{1}{u}\,du - 6\int \frac{1}{(x+2)^2+4}\,dx$$

$$= \ln|x^2+4x+8| \underbrace{- \frac{6}{4}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx}_{\parallel}$$

$$- \frac{6}{4}\cdot 2\int \frac{1}{u^2+1}\,du$$

$$= \ln|x^2+4x+8| - 3\arctan\left(\frac{x+2}{2}\right) + C$$

$$\boxed{\begin{aligned} u &= x^2+4x+8 \\ \frac{du}{dx} &= 2x+4 \end{aligned}}$$

$$(x+a)^2 = x^2 + 2\cdot a\cdot x + a^2$$

- Fullføre kvadrat

$$x^2+4x+8$$

$$= \underbrace{x^2 + 2\cdot 2\cdot x + 4}\ + 4$$

$$= (x+2)^2 + 4$$

- $\frac{1}{1+u^2}$ $= 4\left(\left(\frac{x+2}{2}\right)^2 + 1\right)$

- SUBST : $u = \frac{x+2}{2}$

$$du = \frac{1}{2}dx$$

$$2\,du = dx$$

9.3.13