9.3.13.

$$\int \frac{2x^3+2x+1}{\underbrace{(x^2+1)^2}} dx$$

x^2+1 irredusibelt

$$\int \frac{2x^3+2x+1}{(x^2+1)^2(x-3)} dx$$

$$\frac{A}{x-3} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\to \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} = \frac{2x^3+2x+1}{(x^2+1)^2}$$

$$\frac{(Ax+B)(x^2+1)+(Cx+D)}{(x^2+1)^2} = \frac{2x^3+2x+1}{(x^2+1)^2}$$

Se på tellere $Ax^3 + Ax + Bx^2 + B + Cx + D = 2x^3 + 2x + 1 + 0 \cdot x^2$

- $A = 2$
- $A + C = 2$ $\quad C = 0$
- $B = 0$
- $B + D = 1$ $\quad D = 1$

$$I_m = \int \frac{du}{(1+u^2)^m} = \frac{1}{2(m-1)} \frac{u}{(1+u^2)^{m-1}} + \frac{2m-3}{2(m-1)} I_{m-1}$$

$$= \int \frac{2x}{x^2+1} dx + \int \frac{1}{(x^2+1)^2} dx$$

$$\int \frac{2x}{x^2+1} dx \qquad u = x^2+1, \quad du = 2x\,dx$$

$$= \int \frac{1}{u} du$$

$$= \ln|x^2+1|$$

$$= \ln(x^2+1)$$

I_2 $\quad m=2$

$$\int \frac{1}{(x^2+1)^2} dx = \frac{1}{2(2-1)} \frac{x}{(1+x^2)^{2-1}} + \frac{2\cdot 2-3}{2(2-1)} I_{2-1}$$

$$= \frac{1}{2} \cdot \frac{x}{(1+x^2)} + \frac{1}{2} \int \frac{1}{x^2+1} dx$$

$$= \frac{x}{2(1+x^2)} + \frac{1}{2}\arctan x$$

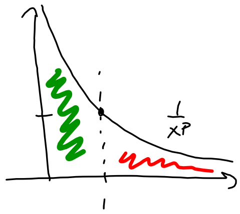
$$= \ln(x^2+1) + \frac{x}{2(1+x^2)} + \frac{1}{2}\arctan x + C$$

LITT OM UEGENTLIGE INTEGRALER

$$\int_0^\infty e^{-x}\,dx = \lim_{b\to\infty}\int_0^b e^{-x}\,dx = \lim_{b\to\infty}\left[-e^{-x}\right]_0^b = \lim_{b\to\infty}\left(-e^{-b}\right)-\left(-e^{0}\right) = 0+1 = \underline{\underline{1}}$$

- ∞

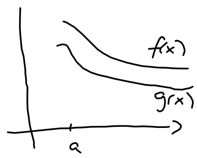
$\int_1^\infty \frac{1}{x^p}\,dx$ konvergerer for $p > 1$
divergerer for $p \leq 1$



$\int_0^1 \frac{1}{x^p}\,dx$ konvergerer for $p < 1$
divergerer for $p \geq 1$

2 VIKTIGE TEKNIKKER

- SAMMENLIGNINGSKRITERIET



La $f,g:[a,\infty)\to\mathbb{R}$ være kontinuerlige & positive funksjoner.
Anta $f(x) \geq g(x) \quad \forall\ x\in[a,\infty)$
  - Hvis $\int_a^\infty f(x)\,dx$ konvergerer, så vil også $\int_a^\infty g(x)$ konvergere
  - Hvis $\int_a^\infty g(x)\,dx$ divergerer, så vil også $\int_a^\infty f(x)$ divergere

- GRENSESAMMENLIGNINGSKRITERIET <u>som over</u> i første linje
  - Hvis $\int_a^\infty f(x)\,dx$ konvergerer OG $\lim_{x\to\infty}\frac{g(x)}{f(x)} < \infty$ så vil $\int_a^\infty g(x)\,dx$ konvergere
  - Hvis $\int_a^\infty f(x)\,dx$ divergerer OG $\lim_{x\to\infty}\frac{g(x)}{f(x)} > 0$ så vil $\int_a^\infty g(x)\,dx$ divergere

9.5.1 a) $\int_0^\infty \frac{dx}{1+x^2} = \underbrace{\int_0^1 \frac{1}{1+x^2}\,dx}_{} + \int_1^\infty \frac{1}{1+x^2}\,dx$

$\int_1^\infty \frac{1}{x^2}\,dx$ konvegerer

$\int_0^1 \frac{1}{x^2}\,dx$ divergere

• $\int_0^1 \frac{1}{1+x^2}\,dx$ På $[0,1]$ så er $1 \geq \frac{1}{1+x^2}$

$\int_0^1 1\,dx$ vs. $\int_0^1 \frac{1}{1+x^2}\,dx = [\arctan x]_0^1$

V/SK: Siden $\int_0^1 1\,dx$ konvegerer, vil $\int_0^1 \frac{1}{1+x^2}\,dx$ konvegere.

• $\int_1^\infty \frac{1}{1+x^2}\,dx$ $\frac{1}{x^2} \geq \frac{1}{1+x^2}$ positive og kontinuerlige

V/SK: Siden $\int_1^\infty \frac{1}{x^2}\,dx$ konvegerer, vil $\int_1^\infty \frac{1}{1+x^2}\,dx$ konvegere.

• $\int_0^\infty \frac{1}{1+x^2}\,dx = \lim_{b\to\infty} [\arctan x]_0^\infty = \lim_{b\to\infty} \arctan b - \arctan 0 = \frac{\pi}{2} - 0 = \underline{\underline{\frac{\pi}{2}}}$

9.5.3. c) $\displaystyle\int_0^1 \frac{1}{\sqrt{x+x^3}}\,dx$

SK for $(0,1]$:
La $f, g : (0,1] \to \mathbb{R}$ være to positive & kontinuerlige funksjoner
og anta $f(x) \geq g(x)$ på $(0,1]$.
① Hvis $\displaystyle\int_0^1 f(x)\,dx$ konvergerer, så vil $\displaystyle\int_0^1 g(x)\,dx$ konverger
② Hvis $\displaystyle\int_0^1 g(x)\,dx$ divergerer, så vil $\displaystyle\int_0^1 f(x)\,dx$ diverger.

$$\underbrace{\frac{1}{\sqrt{x}} \geq \frac{1}{\sqrt{x+x^3}}}_{\text{positive \& kontinuerlige}} \quad \text{på } (0,1]$$

$\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 \frac{1}{x^{\frac{1}{2}}}\,dx$ konvergerer ved $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ konverger for $p < 1$.

Så ved SK må $\displaystyle\int_0^1 \frac{1}{\sqrt{x+x^3}}\,dx$ konvergere.

$x + x^3 > x \qquad (0,1]$

$\sqrt{x+x^3} > \sqrt{x}$

$\displaystyle\frac{1}{\sqrt{x}} > \frac{1}{\sqrt{x+x^3}}$

LINEÆR ALGEBRA

n-tuppel

$\vec{a} = (a_1, \dots, a_n) \qquad \vec{b} = (b_1, \dots, b_n) \qquad \mathbb{R}^n \;,\; \mathbb{C}^n$

Liten vri

$\vec{a} \pm \vec{b} = (a_1 \pm b_1, \dots, a_n \pm b_n)$

s skalar: $s \cdot \vec{a} = (sa_1, \dots, sa_n)$

$\mathbb{R}^n$: $\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + \dots + a_n \cdot b_n = |\vec{a}| \cdot |\vec{b}| \cdot \cos \nu = \vec{b} \cdot \vec{a}$

$|\vec{a}| = \sqrt{a_1^2 + \dots + a_n^2} \qquad \underbrace{|\vec{a}|^2 = \vec{a} \cdot \vec{a}}$

$\mathbb{C}^n$: $\vec{a} \cdot \vec{b} = a_1 \cdot \bar{b}_1 + \dots + a_n \cdot \bar{b}_n = \overline{\vec{b} \cdot \vec{a}}$

$|\vec{a}| = \sqrt{a_1 \bar{a}_1 + \dots + a_n \bar{a}_n} = \sqrt{|a_1|^2 + \dots + |a_n|^2}$

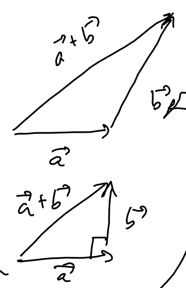
ORTOGONALE VEKTORER: $\vec{a} \cdot \vec{b} = 0$

TRE SETNINGER

① Pytagoras: hvis $\vec{a}, \vec{b} \in \mathbb{C}^n$ ortogonale, så er: $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{a} + \vec{b}|^2$

② Schwartz' ulikhet
For alle $\vec{a}, \vec{b} \in \mathbb{C}^n$ $\quad |\vec{a} \cdot \vec{b}| \le |\vec{a}| \cdot |\vec{b}|$

③ Trekantulikhet $\quad |\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$



VEKTORPRODUKT i $\mathbb{R}^3$

$$\underset{(a_1, a_2, a_3)}{\vec{a}} \times \underset{(b_1, b_2, b_3)}{\vec{b}} = (a_2 b_3 - a_3 b_2,\; a_3 b_1 - a_1 b_3,\; a_1 b_2 - a_2 b_1)$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

1.3.1.

$s\vec{x} + t\vec{y}$ $\quad s = i$, $t = 1 + 2i$ $\quad \vec{x} = \begin{pmatrix} -4i \\ 2-i \end{pmatrix}$ $\quad \vec{y} = \begin{pmatrix} 2+i \\ 2i \end{pmatrix}$

$$s\vec{x} = \begin{pmatrix} -4i \cdot i \\ (2-i) \cdot i \end{pmatrix} = \begin{pmatrix} 4 \\ 2i + 1 \end{pmatrix}$$

$$t\vec{y} = \begin{pmatrix} (1+2i) \cdot (2+i) \\ (1+2i) \cdot 2i \end{pmatrix} = \begin{pmatrix} 2 + i + 4i - 2 \\ 2i - 4 \end{pmatrix} = \begin{pmatrix} 5i \\ 2i - 4 \end{pmatrix}$$

$$s\vec{x} + t\vec{y} = \begin{pmatrix} 4 + 5i \\ 2i + 1 + 2i - 4 \end{pmatrix} = \underline{\begin{pmatrix} 4 + 5i \\ 4i - 3 \end{pmatrix}}$$

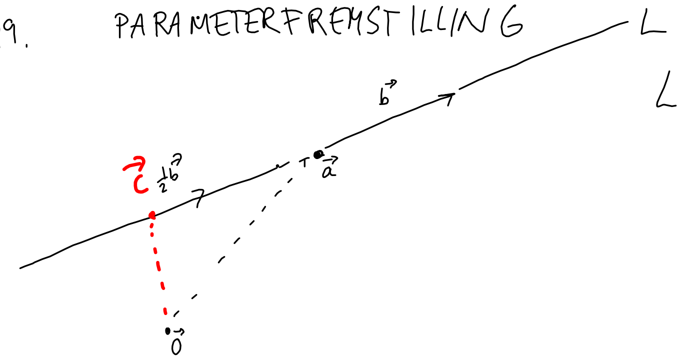
1.3.4. $\vec{x}, \vec{y} \in \mathbb{C}^n$

① $|\vec{x}-\vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2 - 2\,\mathrm{Re}(\vec{x}\cdot\vec{y})$

$$\begin{aligned} &\parallel \\ &(\vec{x}-\vec{y})\cdot(\vec{x}-\vec{y}) \overset{e)\ s.33}{=} \vec{x}\cdot\vec{x} - \vec{x}\cdot\vec{y} - \vec{y}\cdot\vec{x} + \vec{y}\cdot\vec{y} \\ &\qquad = |\vec{x}|^2 - (a+ib) - (a-ib) + |\vec{y}|^2 \\ &\qquad = |\vec{x}|^2 + |\vec{y}|^2 - 2\,\mathrm{Re}(\vec{x}\cdot\vec{y}) \end{aligned}$$

$$\boxed{\begin{aligned} \vec{x}\cdot\vec{y} &= a+ib \\ \vec{y}\cdot\vec{x} &= \overline{\vec{x}\cdot\vec{y}} = a-ib \end{aligned}}$$

1.2.19. PARAMETERFREMSTILLING



$L = \vec a + t\vec b, \quad t \in \mathbb{R}$

$\vec a = (-3,-2,5,8)$

$\vec b = (1,-2,-1,3)$

$$L = (-3,-2,5,8) + t(1,-2,-1,3), \quad t \in \mathbb{R}$$

$$= \begin{pmatrix} -3+t \\ -2-2t \\ 5-t \\ 8+3t \end{pmatrix}, \quad t \in \mathbb{R}$$

Er punktet $\begin{pmatrix} 1 \\ -6 \\ 3 \\ 14 \end{pmatrix}$ på L?

$$\begin{pmatrix} 1 \\ -6 \\ 3 \\ 14 \end{pmatrix} = \begin{pmatrix} -3+t \\ -2-2t \\ 5-t \\ 8+3t \end{pmatrix}$$

$t = 4 \neq$

$-4 = -2t \quad t=2$

Nei. $\underline{\underline{p \notin L}}$