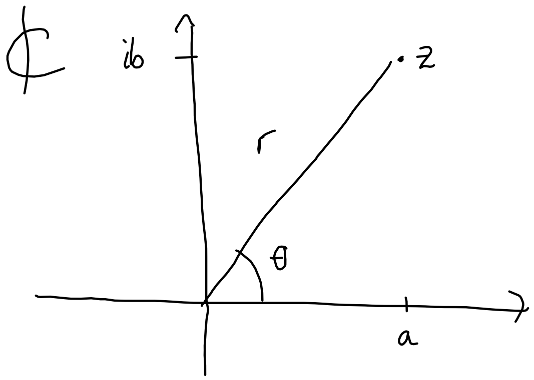
SNUBLEGRUPPE 07.09.2016 MAT1100

$\mathbb{C}$



$$z = a + ib$$
$$= r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$r = \sqrt{a^2 + b^2}$$
$$\cos\theta = \frac{a}{r}$$
$$\sin\theta = \frac{b}{r}$$

$\boxed{3.3.3.}$

$$z = 1 + i\sqrt{3}$$

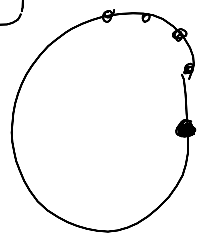
$a = 1$

$b = \sqrt{3}$

$$r = \sqrt{1^2 + (\sqrt{3})^2}$$
$$= \sqrt{1 + 3}$$
$$= \sqrt{4}$$
$$= \underline{\underline{2}}$$

$$\boxed{\boxed{e}^{i\theta} = \cos\theta + i\sin\theta}$$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1



$$\theta : \left.\begin{aligned} \cos\theta &= \frac{1}{2} \\ \sin\theta &= \frac{\sqrt{3}}{2} \end{aligned}\right\} \quad \theta = \frac{\pi}{3}$$

$$z = \underline{\underline{2e^{i\frac{\pi}{3}}}}$$

De Moivres formel

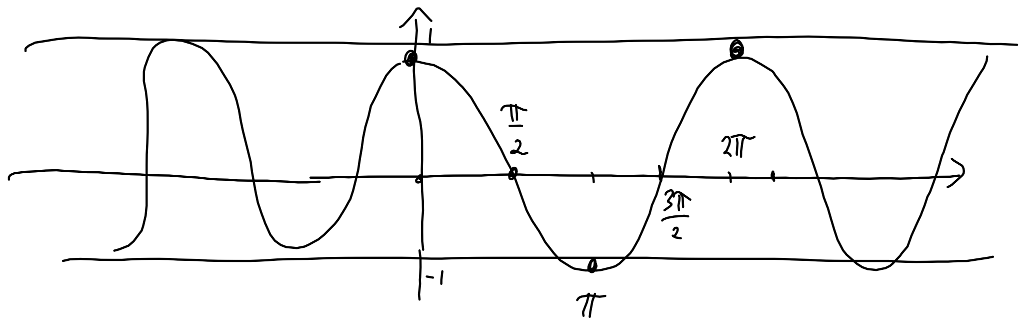
$$\left(e^{i\theta}\right)^n = \left(\cos\theta + i\sin\theta\right)^n$$

$$e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$$

$$\frac{\cos(n\theta)}{\sin(n\theta)}$$

$$(\cos\theta + i\sin\theta)^n$$

$$-1 \leq \underline{\cos}\left(4\underline{3210}\right) \leq 1$$



3.3.10 SUMMEFORMLER FOR cos og sin

$\cos(\underline{z+w})$ $\sin(\underline{z+w})$

$$e^{i(z+w)} = \cos(z+w) + i\sin(z+w)$$

$$e^{iz}\cdot e^{iw} = (\cos z + i\sin z)\cdot(\cos w + i\sin w)$$

$$= \cos z\cdot\cos w + i\cos z\cdot\sin w + i\sin z\cdot\cos w + \underbrace{i\cdot i}_{-1}\sin z\cdot\sin w$$

$i^2 = -1$

- $\cos(z+w) = \cos z\cdot\cos w - \sin z\cdot\sin w$
- $\sin(z+w) = \cos z\cdot\sin w + \sin z\cdot\cos w$

3.3.8 $(1+i)^{804}$

$$\underbrace{(1+i)}_{\parallel}^{804}$$

$$= \left(2^{\frac{1}{2}} \cdot e^{i\frac{\pi}{4}}\right)^{804}$$

$$= \left(2^{\frac{1}{2}}\right)^{\cdot 804} \cdot \left(e^{i\frac{\pi}{4}}\right)^{804}$$

$$= 2^{\frac{804}{2}} \cdot e^{i\pi \cdot \frac{804}{4}}$$

$$= 2^{402} \cdot e^{i\pi \cdot 201}$$

$$= 2^{402} \cdot e^{i\pi}$$

$$= 2^{402} \cdot (-1)$$

$$= -2^{402}$$

$$z = 1 + i$$

$$= \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$= 2^{\frac{1}{2}} e^{i\frac{\pi}{4}}$$

$a = 1$

$b = 1$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\theta: \left.\begin{aligned} \cos\theta &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\theta &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \theta = \frac{\pi}{4}$$

$[0, 2\pi)$

$$201\pi - 2\pi \cdot 100 = \pi$$

$$e^{i\pi} = \cos\pi + i\sin\pi = -1$$

$$e^{i\pi} + 1 = 0$$

RØTTER AV KOMPLEKSE TALL

$w^n = z$ w er nte rot av z

$z = re^{i\theta}$

n n-te røtter

$$w_k = r^{\frac{1}{n}} e^{i\left(\frac{\theta + 2k\pi}{n}\right)}, \quad k = 0, \dots, n-1$$

$\|$

$$\underbrace{w_0,\ w_0 \cdot w_1,\ w_0 \cdot (w_1)^2,\ \dots,\ w_0 (w_1)^{n-1}}_{n \text{ nte røtter}}$$

$$w_0 = r^{\frac{1}{n}} e^{i\frac{\theta}{n}}$$

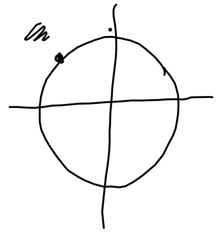
$$w_1 = e^{i\frac{2\pi}{n}}$$

3.48. a) $w^{3} = \underbrace{-1+i}_{}$ (z)

$a = -1$, $b = 1$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\theta:\ \cos\theta = -\frac{1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$, $\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\Big\}\ \theta = \frac{3\pi}{4}$



$w^3 = \sqrt{2}\cdot e^{i\frac{3\pi}{4}}$

x

$w_0 = \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}}\cdot e^{i\frac{3\pi}{4\cdot 3}}$

$= 2^{\frac{1}{6}}\cdot e^{i\frac{3\pi}{12}}$

$w_+ = e^{i\frac{2\pi}{3}}$

$e^{i\frac{2\pi}{n}}$

$\frac{3}{12} + \frac{2\cdot 4}{3\cdot 4} = \frac{11}{12}$

$w_1 = w_0\cdot w_+$

$= 2^{\frac{1}{6}}\cdot e^{i\frac{3\pi}{12}}\cdot e^{i\frac{2\pi}{3}}$

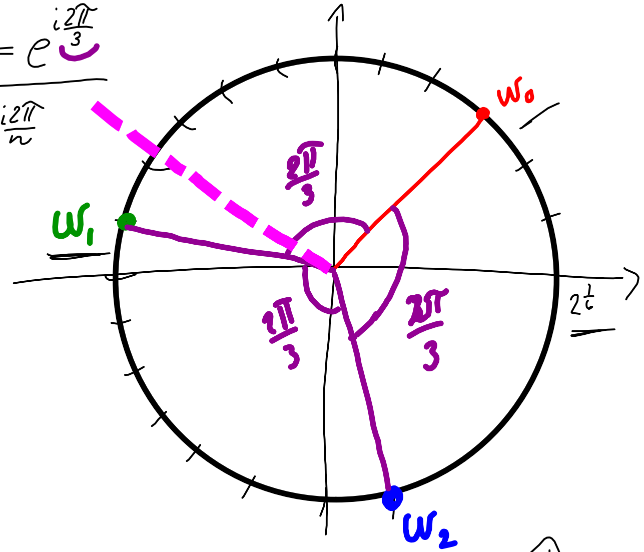
$= 2^{\frac{1}{6}}\cdot e^{i\frac{\pi\cdot 11}{12}}$

$\frac{11}{12} + \frac{2\cdot 4}{3\cdot 4} = \frac{19}{12}$

$w_2 = w_1\cdot w_+$

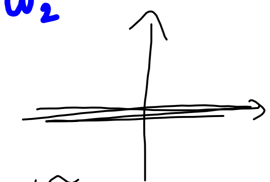
$= 2^{\frac{1}{6}}\cdot e^{i\pi\frac{11}{12}}\cdot e^{i\frac{2\pi}{3}}$

$= 2^{\frac{1}{6}}\cdot e^{i\pi\frac{19}{12}}$



b) $(w_1)^n \in \mathbb{R}$

$\left(2^{\frac{1}{6}} e^{i\pi\cdot\frac{11}{12}}\right)^n = 2^{\frac{n}{6}}\cdot e^{i\pi\cdot\frac{n11}{12}}$



$e^{i\pi},\ e^{i2\pi}$ $\quad e^{ik\pi}$ $\quad k\in\mathbb{Z}$

$\cancel{\pi}\,\frac{n\cdot 11}{12} = k\cdot\cancel{\pi}$

$n\cdot 11 = k\cdot 12$

$k = 11$

$\underline{n = 12}$

$(w_1)^{12} = 2^2\cdot e^{i\pi\cdot 11}$

$= \underline{-2^2}$

3.4.15.

$$z^3 + iz^2 + z = 0$$

$$z(z^2 + iz + 1) = 0$$ Reell faktorisering

$$\underline{z = 0} \qquad z^2 + iz + 1 = 0$$

$$z = \frac{-i \pm \sqrt{i^2 - 4\cdot 1\cdot 1}}{2\cdot 1}$$

$$z = \frac{-i \pm \sqrt{-5}}{2}$$

$$z = \frac{-i \pm i\sqrt{5}}{2}$$

$$z = i\left(\frac{-1 \pm \sqrt{5}}{2}\right)$$

algebraens fundamentalteorem

kompleks faktorisering

$$z\left(z - i\left(\frac{-1+\sqrt{5}}{2}\right)\right)\left(z - i\left(\frac{-1-\sqrt{5}}{2}\right)\right)$$

$$\sqrt{-1} = \begin{cases} i \\ -i \end{cases}$$

Hva betyr $\sqrt{z}$; tar den roten som ligger $[0, \pi)$

$\sqrt{2}$

$x^2 = 2, \quad x = \pm\sqrt{2}$

- $az^2 + bz + c = 0, \quad a, b, c \in \mathbb{C}$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- $a, b, c \in \mathbb{R}$

$\rightarrow b^2 - 4ac$

$= 0$ en reell rot ←

> 0 to reelle røtter

< 0 "$\pm i\sqrt{4ac - b^2}$" to komplekse røtter

$$\sqrt{b^2 - 4ac} = \sqrt{-1(4ac - b^2)}$$