SNUBLEGRUPPE MAT 1100 05.10.2016

6.1.1. a)

$$f(x) = \underline{\cos x} \cdot \underline{\sin x}$$

$$f'(x) = -\sin x \cdot \sin x + \cos x \cdot \cos x$$

$$= \underline{\underline{-\sin^2 x + \cos^2 x}}$$

g)

$$f(x) = \underbrace{x}_{\text{elementær}} \underbrace{\cos(\ln x)}_{\text{sammensatt}}$$

$$f'(x) = 1 \cdot \cos(\ln x) + x\left(\cos(\ln x)\right)'$$

$$= \cos(\ln x) + \cancel{x} \cdot (-\sin(\ln x)) \cdot \frac{1}{\cancel{x}}$$

$$= \underline{\underline{\cos(\ln x) - \sin(\ln x)}}$$

6.1.3. a) $\boxed{f'(x) = f(x)\, D\left[\ln|f(x)|\right]}$ Logaritmisk derivasjon

$$f(x) = x^2 \cdot \cos^4 x \cdot e^x$$

alltid ≥ 0

$$\ln|f(x)| = \ln|x^2 \cdot \cos^4 x \cdot e^x| = \ln(x^2 \cdot \cos^4 x \cdot e^x)$$

$$\overset{①}{=} \ln x^2 + \ln(\cos^4 x) + \ln e^x$$

$$\overset{③}{=} 2\ln x + 4\ln(\cos x) + x\ln e$$

$$= 2\ln x + 4\ln(\cos x) + x$$

$$D\left[\ln|f(x)|\right] = \frac{2}{x} + \frac{4}{\cos x}(-\sin x) + 1$$

$$= \frac{2}{x} - 4\tan x + 1$$

$$f'(x) = x^2 \cdot \cos^4 x \cdot e^x \left(\frac{2}{x} - 4\tan x + 1\right)$$

6.1.11. a) $f(x) = |x-1|$ Vis at $f'(1)$ ikke eksisterer

Ser på: $\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x-1}$

$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

- $\lim\limits_{x \to 1^+} \dfrac{|x-1| - 0}{x-1} = \lim\limits_{x \to 1^+} \dfrac{(x-1)}{(x-1)} = \underline{\underline{1}}$

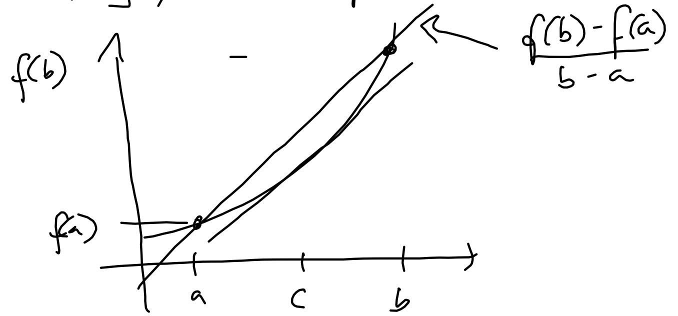
- $\lim\limits_{x \to 1^-} \dfrac{|x-1| - 0}{x-1} = \lim\limits_{x \to 1^-} \dfrac{-(x-1)}{(x-1)} = \underline{\underline{-1}}$

Grensene er forskjellige, så $f'(1)$ eksisterer ikke!

MIDDELVERDISETNINGEN: Anta f kont. på $[a,b]$, deriverbar på (a,b)

Da finnes $c \in (a,b)$ s.a.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



6.2.7. (1) Vis at mellom $\underline{(0,x)}$ finnes en c slik at $\sin x = x\cos c$.

$f(y) = \sin y \qquad f'(c) = \frac{f(x)-f(0)}{x-0}$ ved Middelverdisetn.

$f'(y) = \cos y \qquad \cos c = \frac{\sin x - \sin 0}{x-0} = \frac{\sin x}{x} \quad \Big| \cdot x$

$\longrightarrow \underline{x\cos c = \sin x}$

$$\boxed{\begin{aligned} |x|\cdot\tfrac{1}{2} &= |\sin x| \\ |x| &= 2|\sin x| \geq |\sin x| \end{aligned}}$$

(2) Vise at $|\sin x| \leq |x| \;\forall\; x>0$

$\forall$ = for alle

$|x\cos c| = |\sin x|$

$|x||\cos c| = |\sin x|$ siden $|\cos c| \leq 1$

$\underline{|x| \geq |\sin x|}$

L'HOPITAL Viktig teknikk.

$\frac{"0"}{0}$ $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$ og $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ eksisterer

Da eksisterer $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

MORAL: Deriverer teller for seg og nevner for seg.

$\frac{"0"}{0}$ $\frac{"\infty"}{\infty}$ $\frac{"-\infty"}{\infty}$ $"0\cdot\infty"$ OMSKRIVES $\underbrace{"1^{\infty}" \quad "0^{0}" \quad "\infty^{0}"}_{\text{TRIKS: } e^{\ln b} = b}$

6.1.3. d) $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} \underset{L'H}{\overset{"\frac{0}{0}"}{=}} \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{-1} = \underline{\underline{1}}$

6.3.3. a)

$$\lim_{x\to 0^+} x^2 \ln x \overset{"0\cdot(-\infty)"}{=} \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} \underset{L'H}{\overset{"\frac{-\infty}{\infty}"}{=}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^+} -\frac{x^{\cancel{3}\,2}}{2\cancel{x}} = \lim_{x\to 0^+} -\frac{x^2}{2} = \underline{\underline{0}}$$

$$\frac{\frac{1}{x}}{-2x^{-3}} = \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \frac{1}{x} \cdot \frac{x^3}{-2} =$$

$$\frac{x^{-1}}{-2x^{-3}} = \frac{x^3}{-2x} =$$

$$= -\frac{1}{2} x^{-1-(-3)} =$$

6.3.9. a)

$$\lim_{x\to 0} (e^x + \sin x)^{\frac{1}{x}} \overset{"1^{\infty}"}{=} \lim_{x\to 0} e^{\ln\left[(e^x+\sin x)^{\frac{1}{x}}\right]}$$

$$= e^{\lim_{x\to 0}\left(\ln\left[(e^x+\sin x)^{\frac{1}{x}}\right]\right)} = e^2$$

$$\lim_{x\to 0}\left(\ln\left[(e^x+\sin x)^{\frac{1}{x}}\right]\right) \overset{3}{=} \lim_{x\to 0} \frac{1}{x}\cdot \ln(e^x+\sin x)$$

$$\overset{"\frac{0}{0}"}{\underset{L'H}{=}} \lim_{x\to 0} \frac{\frac{1}{(e^x+\sin x)}\cdot(e^x+\cos x)}{1 \;"(x)'"}$$

$$= \lim_{x\to 0} \frac{e^x+\cos x}{e^x+\sin x}$$

$$= \frac{2}{1} = 2$$

6.3.17

$$\lim_{x\to\infty}\left(\frac{ax+1}{ax}\right)^x = \sqrt{e} = e^{\frac{1}{2}}$$

Skal finne a.

$$\text{"}1^{\infty}\text{"}: \lim_{x\to\infty}\frac{ax+1}{ax} = \lim_{x\to\infty}\frac{\frac{a\cancel{x}}{\cancel{x}}+\frac{1}{x}}{\frac{a\cancel{x}}{\cancel{x}}} = 1$$

$e^{\ln}$-triks:
$$\lim_{x\to\infty} e^{\ln\left(\frac{ax+1}{ax}\right)^x} \overset{③}{=} \lim_{x\to\infty} e^{x\ln\left(\frac{ax+1}{ax}\right)} = e^{\lim_{x\to\infty} x\ln\left(\frac{ax+1}{ax}\right)}$$

$$\frac{1}{a} = \lim_{x\to\infty} x\ln\left(\frac{ax+1}{ax}\right)$$

eksponent:
$$\lim_{x\to\infty} x\ln\left(\frac{ax+1}{ax}\right) \overset{\text{"}\infty\cdot 0\text{"}}{=} \lim_{x\to\infty}\frac{\ln\left(\frac{ax+1}{ax}\right)}{x^{-1}} \underset{L'H}{\overset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to\infty}\frac{\frac{1}{\left(\frac{ax+1}{ax}\right)}\cdot\left(\frac{ax+1}{ax}\right)'}{-x^{-2}}$$

$$\lim_{x\to\infty} -\frac{x^2\cdot\left[\frac{a\cdot ax - a(ax+1)}{(ax)^2}\right]}{\left(\frac{ax+1}{ax}\right)}$$

$$= \lim_{x\to\infty}\frac{-\left(\frac{\cancel{a^2x^3}-\cancel{a^2x^3} - ax^2}{(ax)^2}\right)}{\left(\frac{ax+1}{ax}\right)} = \lim_{x\to\infty}\frac{ax^2}{(ax)^2}\cdot\frac{ax}{(ax+1)} = \lim_{x\to\infty}\frac{x}{ax+1}$$

$$= \lim_{x\to\infty}\frac{1}{a+\frac{1}{x}} = \frac{1}{a} = \frac{1}{2}$$

Så $a = 2$

ASYMPTOTER

- Vertikale $\quad x = a \qquad \frac{\quad}{x-a} \qquad \lim_{x \to a^+, a^-} f(x) = \begin{cases} \infty \\ -\infty \end{cases}$

- Skrå $\quad y = ax + b$

(1) $\lim_{x \to \infty} \frac{f(x)}{x} = a$

(2) $\lim_{x \to \infty} \left[f(x) - ax \right] = b$

(3) $y = ax + b$