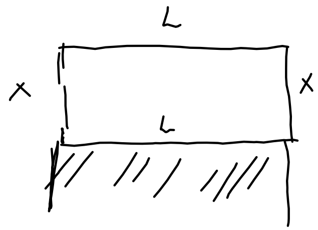
SNUBLEGRUPPE 19.10.2016

7.1.1.



- $50 = x + x + L = 2x + L$; $L = 50 - 2x$
- $A = L \cdot x$

  $A(x) = (50-2x)\cdot x$

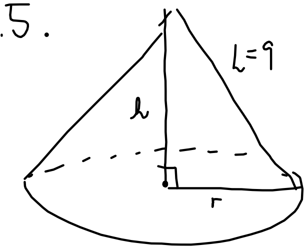
  $= 50x - 2x^2$
- $A'(x) = 50 - 4x = 0$

  $50 = 4x$

  $\underline{x = \frac{25}{2}}$
- $A\left(\frac{25}{2}\right) = 50 \cdot \frac{25}{2} - 2\left(\frac{25}{2}\right)^2$

  $\underline{\underline{= \frac{625}{2} \quad (m^2)}}$

7.15.



skal maksimere volum.

$$V = \frac{\pi}{3} r^2 h$$

$$h = \sqrt{L^2 - r^2} = \sqrt{81 - r^2}$$

$$r^2 = L^2 - h^2 = 81 - h^2$$

$$V(r) = \frac{\pi}{3} r^2 \sqrt{81 - r^2}$$

$$\boxed{\begin{aligned} V(h) &= \frac{\pi}{3}(81 - h^2)h \\ &= \frac{\pi}{3}(81h - h^3) \end{aligned}}$$

- $V'(h) = \frac{\pi}{3}(81 - 3h^2)$

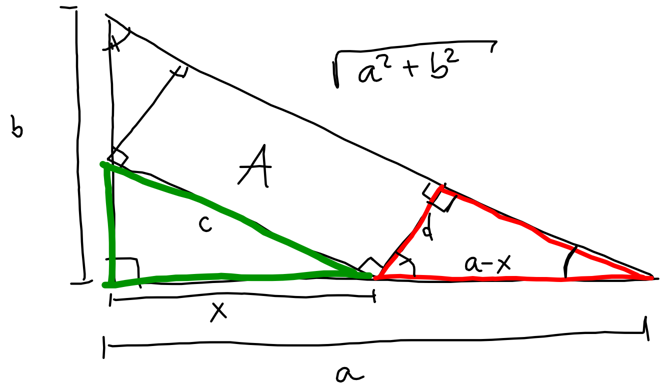
$$= 27\pi - \pi h^2 = 0$$

$$27\cancel{\pi} = \cancel{\pi} h^2$$

$$\underline{h = 3\sqrt{3}} \quad (-3\sqrt{3})$$

- $V(3\sqrt{3}) = \frac{\pi}{3}\left(81 \cdot 3\sqrt{3} - (3\sqrt{3})^3\right) \approx \underline{\underline{59,4}}$

7.1.8.



$$A(x) = \frac{b(a-x)}{\cancel{\sqrt{a^2+b^2}}} \cdot \frac{\cancel{\sqrt{a^2+b^2}}\,x}{a}$$

$$= \frac{b}{a}(a-x)x$$

$$= bx - \frac{b}{a}x^2$$

$$A'(x) = b - \frac{2b}{a}x = 0$$

$$\cancel{b} = \frac{2\cancel{b}x}{a}$$

$$x = \frac{a}{2}$$

① $A(x)$

② Maksimere A, finne x-verdi

$$A(x) = c \cdot d$$

Formlike trekanter

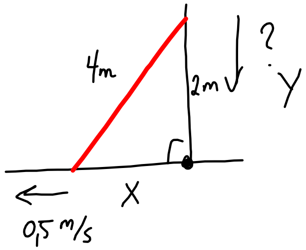
$$\frac{d}{a-x} = \frac{b}{\sqrt{a^2+b^2}}$$

$$d = \frac{b(a-x)}{\sqrt{a^2+b^2}}$$

$$\frac{c}{x} = \frac{\sqrt{a^2+b^2}}{a}$$

$$c = \frac{\sqrt{a^2+b^2}\,x}{a}$$

7.2.1.



$x(t)$

$y(t)$

$x'(t) = 0{,}5$

$x^2 + y^2 = 4^2$

Vi skal finne $y'(t_0)$ når $y(t_0) = 2$

$$x(t)^2 + y(t)^2 = 4^2$$

Deriverer mhp t: $2x \cdot x' + 2y \cdot y' = 0$

$$y'(t) = -\frac{x(t)\, x'(t)}{y(t)}$$

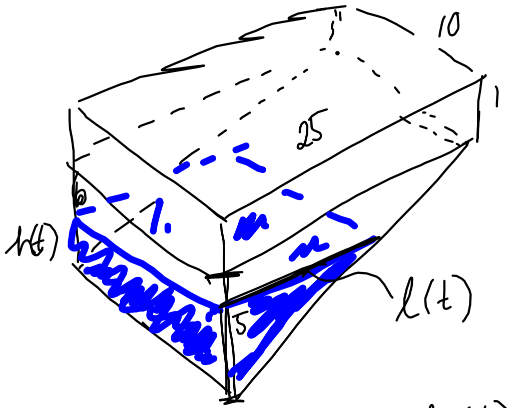
$$y'(t_0) = -\frac{x(t_0)\, x'(t_0)}{y(t_0)}$$

$$x(t_0)^2 = 16 - y(t_0)^2 = 16 - 2^2 = 12$$

$$x(t_0) = 2\sqrt{3} \quad (-2\sqrt{3})$$

$$y'(t_0) = \frac{-2\sqrt{3} \cdot 0{,}5}{2} = -\frac{\sqrt{3}}{2}$$

7.2.9.



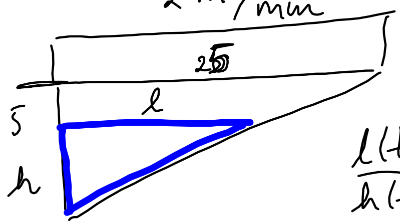
Høyden på vannet i dyp ende av bassenget

$h(t)$

$h'(t_0)$ når $h(t_0) = 3$

$V'(t) = 2000\ l/min = 2000\ dm^3/min$

$= 2\ m^3/min$



$\frac{\ell(t)}{h(t)} = \frac{25}{5} = 5$

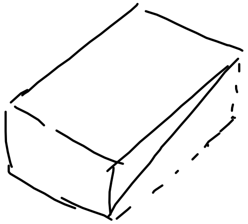
$\ell(t) = 5h(t)$

$h \leq 5$

$V(t) = \frac{1}{2} \cdot \ell(t) \cdot h(t) \cdot 10$

$= 5\,\ell(t) h(t)$

$= 5 \cdot 5\, h(t)^2$



$V'(t) = 25 \cdot 2\, h(t) \cdot h'(t)$

$h'(t) = \frac{V'(t)}{50\, h(t)}$

$h'(t_0) = \frac{V'(t_0)}{50 \cdot h(t_0)} = \frac{2}{50 \cdot 3} = \frac{2}{150}\ m/min$

$\frac{200}{150}\ cm/min = \frac{4}{3}$

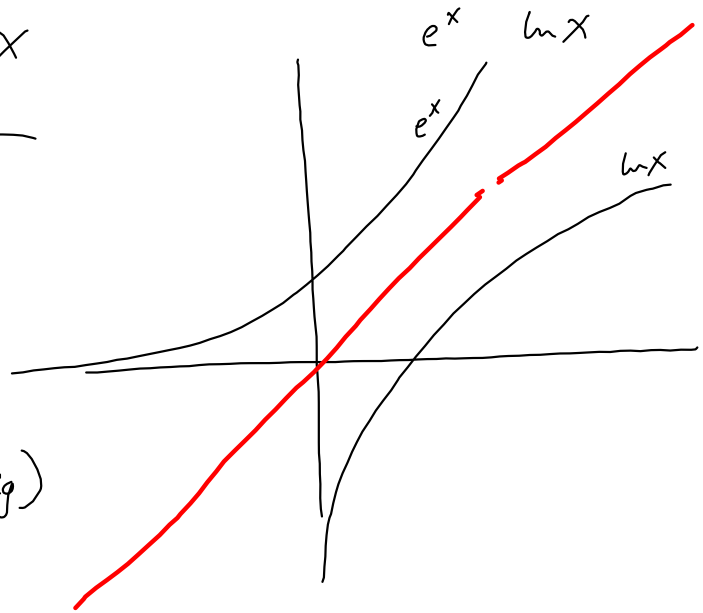
OMVENDTE FUNKSJONER

$f(x)$ $\qquad g(y)$

$f(g(y)) = y \qquad g(f(x)) = x$

EKS: e^{x} $\qquad \ln y$

$e^{\ln y} = y \qquad \ln e^{x} = x$



VIKTIG:
- f må være injektiv (kontinuerlig)
  - strengt avtagende
  - strengt voksende
- $D_f = V_g \qquad D_g = V_f$

TEOREM 7.4.6.

Anta f kontinuerlig, strengt monoton funksjon, deriverbar i x og med $f'(x) \neq 0$.

Da er $g = f^{-1}$ deriverbar i $y = f(x)$ og

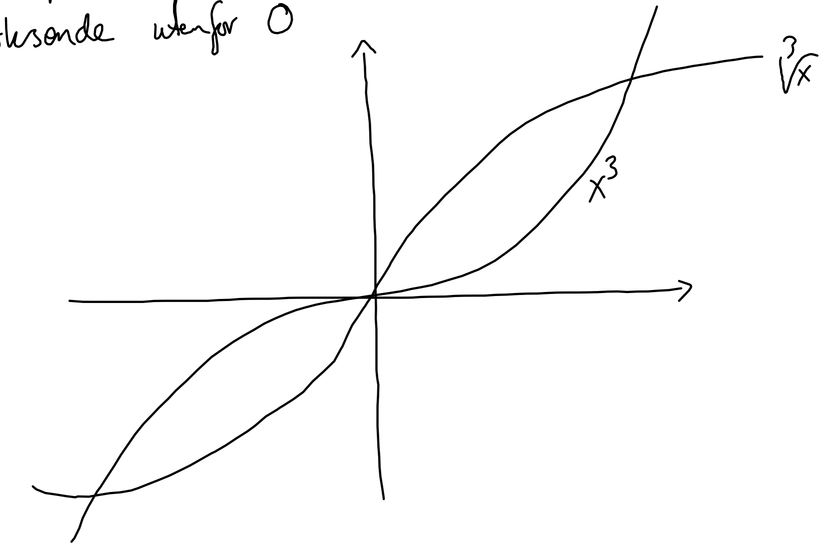
$$g'(y) = \frac{1}{f'(x)}$$

7.4.1. a) $f(x) = x^3$ $\quad f'(x) = 3x^2 \geqslant 0$ $\quad = 0$ for $x = 0$

$f(x)$ er strengt voksende utenfor 0

$f(x) = y = x^3$

$\sqrt[3]{y} = x = g(y)$

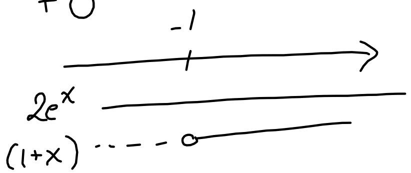


7.4.3.

$$f(x) = 2xe^x + 1 \qquad x \geq -1$$

① Injektiv

$$f'(x) = 2e^x + 2xe^x + 0$$
$$= 2e^x(1+x)$$



$f(x)$ er strengt voksende på $(-1, \infty)$ og dermed injektiv for $x \geq -1$, så vi har g, den omvendte funksjonen til f.

② $g'(1)$

$$1 = f(x) = 2xe^x + 1$$
$$\underline{x = 0}$$

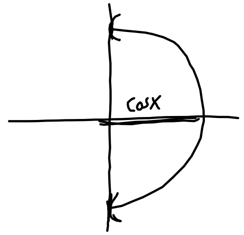
$$g'(y) = \frac{1}{f'(x)}$$

$$g'(1) = \frac{1}{f'(0)} = \frac{1}{2e^0 + 2 \cdot 0 e^0}$$
$$= \underline{\underline{\frac{1}{2}}}$$

$$f(x) = e^x$$
$$y = e^x$$
$$\ln y = \ln e^x$$
$$\ln y = x = g(y)$$

7.4.7.

$f(x) = \tan x$ $\quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$f'(x) = \frac{1}{\cos^2 x} > 0$

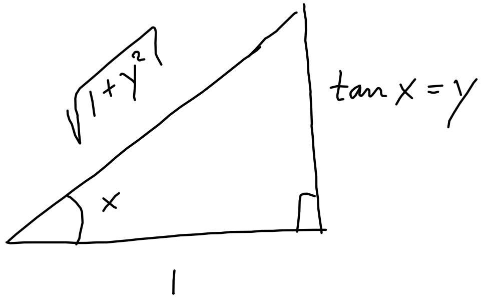
så $f(x)$ er injektiv på intervallet.

$g(y) = \arctan y$ $\qquad g(\tan x) = \arctan(\tan x) = x$

$$g'(y) = \frac{1}{f'(x)} = \frac{1}{\frac{1}{\cos^2 x}}$$

$$= \cos^2 x$$

$$= \frac{1}{1+y^2}$$



$\cos x = \frac{1}{\sqrt{1+y^2}}$