SNUBLEGRUPPE MAT 1100 26.10.2016

7.5.3. a) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} \frac{x\cos x}{\sin x}$

$\overset{\text{"}\frac{0}{0}\text{"}}{\underset{L'H}{=}} \frac{1\cdot\cos x + x(-\sin x)}{\cos x}$

$= \lim_{x\to 0} \frac{\cos x}{\cos x} + \frac{x(-\sin x)}{\cos x}$

$= \lim_{x\to 0} 1 - \frac{x\sin x}{\cos x}$

$= 1$

$$\boxed{\cot x = \frac{\cos x}{\sin x}}$$

$$\tan x = \frac{\sin x}{\cos x}$$

b) $$\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\left(\frac{\pi}{2} - x\right)} \overset{\text{"}\frac{0}{0}\text{"}}{\underset{\text{L'H}}{=}} \lim_{x \to \frac{\pi}{2}} \frac{-\frac{1}{\sin^2 x}}{-1} = \underline{\underline{1}}$$

$$(\cot x)' = \left(\frac{\cos x}{\sin x}\right)'$$

$$= \frac{(-\sin x)\cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \underline{\underline{\frac{-1}{\sin^2 x}}}$$

$$\underline{(\tan x)' = \frac{1}{\cos^2 x}}$$

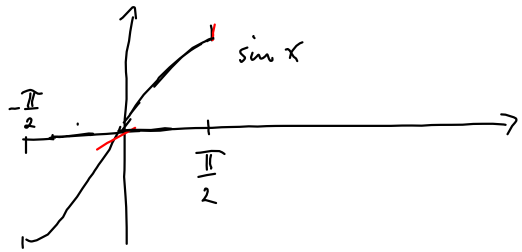
7.6.1.

a) $\arcsin\left(\frac{1}{2}\right) = \arcsin\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$

$\uparrow$ $\sin\frac{\pi}{6}$

$\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\arcsin(\sin x) = x$



e) $\arccos\left(\frac{1}{2}\right) = \arccos\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\uparrow$ $\cos\frac{\pi}{3}$

h) $\arctan(\sqrt{3}) = \arctan\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\uparrow$

$\tan x = \sqrt{3}$

$\frac{\sin x}{\cos x} = \sqrt{3} \Rightarrow x = \frac{\pi}{3}$

7.6.2. a)

$$D[\arcsin\sqrt{x}]$$

+ kjerneregel

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})'$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \underline{\underline{\frac{1}{2\sqrt{x}\sqrt{1-x}}}}$$

$$D[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$D[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$!!! \longrightarrow D[\arctan x] = \frac{1}{1+x^2}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

b) $D[\arctan e^x] = \dfrac{1}{1+(e^x)^2} \cdot e^x$

$$= \underline{\dfrac{e^x}{1+e^{2x}}}$$

INTEGRASJON

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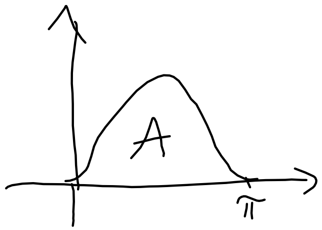
8.3.1. a) $\int_0^{\pi} \sin x \, dx = \Big[-\cos x\Big]_0^{\pi} = (-\cos \pi) - (-\cos 0)$

$= 1 + 1$

$= \underline{\underline{2}}$



b) $\int_0^2 2x^3 \, dx = 2\int_0^2 x^3 \, dx$

$= 2\left[\frac{x^4}{4}\right]_0^2$

$= 2\left[\frac{2^4}{4} - \frac{0^4}{4}\right] = \underline{\underline{8}}$

$\int a \cdot f(x) \, dx = a\int f(x) \, dx$

$(af(x))' = af'(x)$

3

$\int x^a \, dx = \frac{x^{a+1}}{a+1}$

$\boxed{a \neq -1}$

$\int x^{\pi} \, dx = \frac{x^{\pi+1}}{\pi+1}$

c)

$$\int_0^1 e^{-x}\,dx$$

$$= \int_0^1 e^{u}(-du)$$

$$= -\int_{x=0}^{x=1} e^{u}\,du$$

$$= -\left[e^{u}\right]_{x=0}^{x=1}$$

$$= -\left[e^{-x}\right]_0^1$$

$$= -(e^{-1} - e^{0})$$

$$= \underline{\underline{1 - e^{-1}}}$$

SUBST

$u = -x$

$\frac{du}{dx} = u' = -1$

TEKNIKK:

$du = -dx$

$-du = dx$

e^{x}

e^{-x}

$\boxed{e^{-x} \cdot (-1)}$

d) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \Big[\arcsin x\Big]_{-\frac{1}{2}}^{\frac{1}{2}} = \arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right)$

$\arcsin\left(-\frac{1}{2}\right) = \sin\left(-\frac{\pi}{6}\right)$

$\overset{7.6.1a)}{=} \dfrac{\pi}{6} - \left(-\dfrac{\pi}{6}\right)$

$= \underline{\underline{\dfrac{\pi}{3}}}$

$a \neq -1 \quad \displaystyle\int x^a\,dx = \frac{x^{a+1}}{a+1} + C$

$\boxed{\displaystyle\int x^{-1}\,dx = \ln|x| + C}$

e) $\displaystyle\int_1^e \frac{1}{x}\,dx = \Big[\ln|x|\Big]_1^e$

$= \ln e - \ln 1$

$= 1 - 0$

$= \underline{\underline{1}}$

f) $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \Big[\arctan x\Big]_1^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(1)$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \underline{\underline{\frac{\pi}{12}}}$

$\tan x = 1$ når $x = \frac{\pi}{4}$

KAN ALLTID SJEKKE

g) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x} = \Big[-\cot x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left[\frac{-\cos x}{\sin x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} - \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \underline{\underline{-\frac{1}{\sqrt{3}} + \sqrt{3}}}$

h)

$$\int_1^9 (\sqrt{x})^3 \, dx = \int_1^9 x^{\frac{3}{2}} \, dx = \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_1^9 = \left(\frac{9^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1^{\frac{5}{2}}}{\frac{5}{2}} \right)$$

$$= \frac{2 \cdot 3^5}{5} - \frac{2}{5}$$

$$= \frac{2}{5}\left(\underbrace{3^5 - 1}_{242}\right)$$

$$9^{\frac{5}{2}} = \left(9^{\frac{1}{2}}\right)^5 = 3^5$$

8.4.1.

a) $\int \frac{dx}{x+3}$

SUBST.

$u = x+3$ $\qquad 1 \cdot x$

$du = dx$ $\qquad \frac{du}{dx} = 1$

$= \int \frac{du}{u}$

$= \int u^{-1} du$

$= \ln |u| + C$

$= \underline{\underline{\ln |x+3| + C}}$

b) $\int 7x + 3x^{\frac{1}{2}} - \cos x \, dx$

$= \frac{7}{2}x^2 + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \sin x + C$

$= \underline{\frac{7}{2}x^2 + 2x^{\frac{3}{2}} - \sin x + C}$

c) $\int \frac{dx}{1+2x^2}$

$= \int \frac{1}{1+(\sqrt{2}x)^2} \, dx$

$2 = (\sqrt{2})^2$

SUBST:

$u = \sqrt{2}\,x$

$du = \sqrt{2}\,dx$

$\frac{1}{\sqrt{2}} du = dx$

$= \frac{1}{\sqrt{2}} \int \frac{du}{1+u^2}$

$= \frac{1}{\sqrt{2}} \arctan u + C$

$= \frac{1}{\sqrt{2}} \arctan(\sqrt{2}x) + C$

d)

$$\int 8e^{7x} + \frac{1}{\sqrt{x}}\,dx$$

$$8\int e^{7x}dx + \int x^{-\frac{1}{2}}dx$$

$$\| \qquad \qquad \|$$

$$\frac{8}{7}\int e^{u}du + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\frac{8}{7}e^{7x} + 2x^{\frac{1}{2}} + C$$

SUBST

$u = \underline{7}x$

$du = 7\,dx$

$\frac{1}{7}du = dx$

e)

$$\int \frac{4}{\sqrt{7-x^2}}\,dx$$

$$= 4\int \frac{1}{\sqrt{7(1-\frac{1}{7}x^2)}}\,dx$$

$$= \frac{4}{\sqrt{7}}\int \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{7}}\right)^2}}\,dx$$

$$= \frac{4\cancel{\sqrt{7}}}{\cancel{\sqrt{7}}}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= 4\arcsin u + C$$

$$= \underline{4\arcsin\left(\frac{x}{\sqrt{7}}\right) + C}$$

SUBST:

$$u = \frac{x}{\sqrt{7}}$$

$$du = \frac{1}{\sqrt{7}}\,dx$$

$$\sqrt{7}\,du = dx$$

$$7 = \left(\sqrt{7}\right)^2$$

8.4.2.

a) $\int \frac{42}{\sin^2(7x)}\,dx$

SUBST

$u = 7x$

$du = 7\,dx$

$\frac{1}{7}du = dx$

$= \frac{42}{7}\int \frac{1}{\sin^2 u}\,du$

$= -6\cot u + C$

$= -6\cot(7x) + C$

$-6 \cdot \frac{-1}{\sin^2(7x)} \cdot 7$

d) $\int \frac{dx}{\sqrt{x}\cos^2(\sqrt{x})}$

SUBST
$u = \sqrt{x} = x^{\frac{1}{2}}$
$du = \frac{1}{2} x^{-\frac{1}{2}} dx$
$2\,du = \frac{1}{\sqrt{x}} dx$

$= 2\int \frac{du}{\cos^2 u}$

$= 2\tan u + C$

$= 2\tan\sqrt{x} + C$

e)

$$\int \frac{1+x}{1+x^2}\,dx$$

$$= \int \frac{1}{1+x^2} + \frac{x}{1+x^2}\,dx$$

$$= \arctan x + \int \frac{x}{1+x^2}\,dx$$

$$= \arctan x + \frac{1}{2}\int \frac{du}{u}$$

$$= \arctan x + \frac{1}{2}\ln(1+x^2) + C$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$