SNUBLEGRUPPE MAT 1100 02.11.2016

8.4.3.

a) $\int \sqrt{\frac{\arcsin x}{1-x^2}}\,dx$

$= \int \underbrace{\sqrt{\arcsin x} \cdot \frac{1}{\sqrt{1-x^2}}}\,dx$

$= \int u^{\frac{1}{2}}\,du$

$= \frac{1}{\frac{3}{2}} u^{\frac{3}{2}} + C$

$= \frac{2}{3}(\arcsin x)^{\frac{3}{2}} + C$

SUBST.

$u = \arcsin x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

8.4.3. b)

$$\int \sin 2x \, \frac{e^{\cos^2 x}}{e^{\sin^2 x}} \, dx$$

$$= \int \sin 2x \, e^{\cos^2 x - \sin^2 x} \, dx$$

$$= \int \sin 2x \, e^{\cos 2x} \, dx$$

$$= \int -\frac{1}{2} e^u \, du$$

$$= -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{\cos 2x} + C$$

$$e^{i2x} = \cos 2x + i \sin 2x$$

$$(e^{ix})^2 = (\cos x + i \sin x)^2$$

HUSK OPPG 3.3.10

$$\cos^2 x - \sin^2 x = \cos 2x$$

SUBST

$$u = \cos 2x$$

$$du = -\sin 2x \cdot 2 \, dx$$

$$-\frac{1}{2} du = \sin 2x \, dx$$

8.4.3. d)

$$\int \frac{7x-1}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{7x}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{7x}{\sqrt{1-x^2}}\,dx - \underbrace{\int \frac{1}{\sqrt{1-x^2}}\,dx}_{=\arcsin x}$$

$$=-\frac{7}{2}\int \frac{du}{\sqrt{u}} - \arcsin x$$

$$=-\frac{7}{2}\int u^{-\frac{1}{2}}\,du - \arcsin x$$

$$=-\frac{7}{\cancel{2}}\cdot\cancel{2}\,u^{\frac{1}{2}} - \arcsin x + C = -7(1-x^2)^{\frac{1}{2}} - \arcsin x + C$$

$$\int x^{-\frac{1}{2}}\,dx = \frac{1}{-\frac{1}{2}+1}\,x^{-\frac{1}{2}+1} + C$$

$$= 2x^{\frac{1}{2}} + C$$

SUBST

$u = 1-x^2$

$du = -2x\,dx$

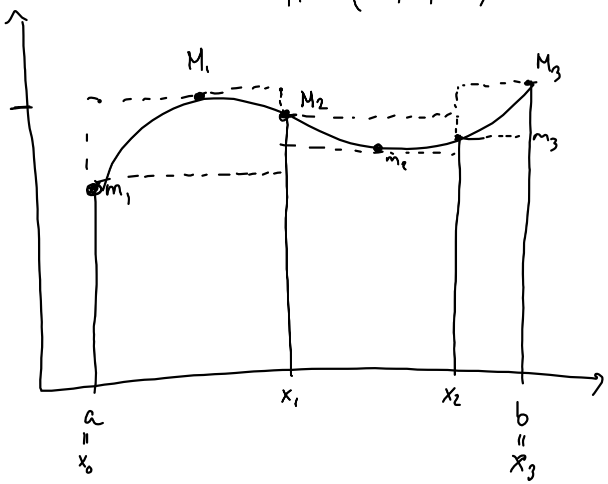
$-\frac{1}{2}\,du = x\,dx$

$u = \sqrt{1-x^2}$

SUMMER OG INTEGRALER

DARBOUX

$\Pi = \{x_0, x_1, \dots, x_n)$



$Ø(\Pi) = M_1 \cdot (x_1 - x_0) + M_2(x_2 - x_1) + M_3(x_3 - x_2)$

$N(\Pi) = m_1(x_1 - x_0) + m_2(x_2 - x_1) + m_3(x_3 - x_2)$

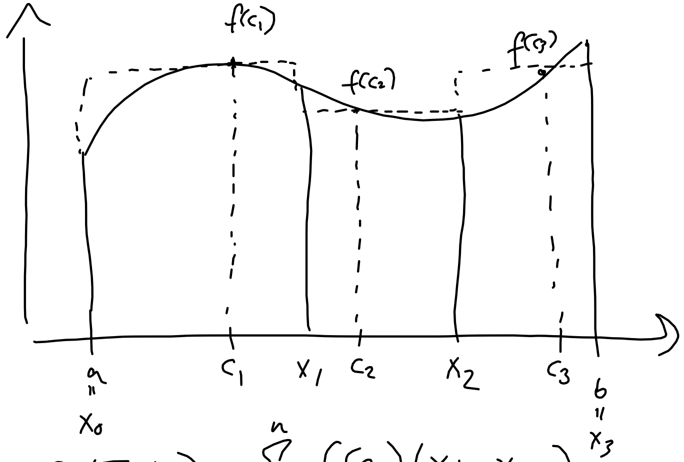
$\overline{\int_a^b} f(x)\,dx$ = den minste $Ø(\Pi)$ (||) $= \int_a^b f(x)\,dx$ &

$\underline{\int_a^b} f(x)\,dx$ = den største $N(\Pi)$

RIEMANN

$\Pi = \{x_0, x_1, \dots, x_n\}$

$U = \{c_1, \dots, c_n\} \quad c_i \in [x_{i-1}, x_i]$



$R(\Pi, U) = \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$

$\lim_{n \to \infty} R(\Pi_n, U_n) = \alpha = \int_a^b f(x)\,dx$

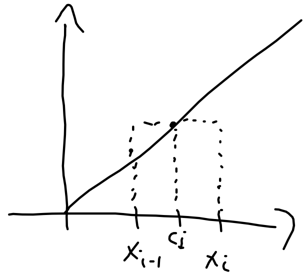
Ekvivalent def.

8.5.2.

$$f(x) = x$$

$$\Pi_n = \{0, x_1, \dots, x_n = a\}$$

$$U_n = \left\{c_i = \frac{x_i + x_{i-1}}{2}\right\}$$



$$\int_0^a x\,dx = \frac{a^2}{2}$$

$$R(\Pi_n, U_n) = \sum_{i=1}^{n} \underbrace{f(c_i)}_{= c_i}(x_i - x_{i-1})$$

$$= \sum_{i=1}^{n} \frac{(x_i + x_{i-1})}{2}(x_i - x_{i-1})$$

$$= \sum_{i=1}^{n} \frac{x_i^2 - x_{i-1}^2}{2}$$

$$= \frac{1}{2}\sum_{i=1}^{n}(x_i^2 - x_{i-1}^2)$$

$$= \frac{1}{2}\left[\cancel{x_1^2} - x_0^2 + \cancel{x_2^2} - \cancel{x_1^2} + x_3^2 - \cancel{x_2^2} + \dots + x_n^2 - x_{n-1}^2\right]$$

$$= \frac{1}{2}\left[x_n^2 - x_0^2\right]$$

$$= \frac{1}{2}(a^2 - 0^2)$$

$$= \frac{a^2}{2}$$

8.5.4.

$$\underbrace{\lim_{n\to\infty} \frac{1}{n^{3/2}} \left(\sum_{i=1}^{n} \sqrt{i} \right)}_{\text{som Rieman-sum for } \int_0^1 \sqrt{x}\,dx} = \frac{2}{3}$$

$$\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{\frac{1}{2}}\,dx = \left[\frac{1}{\frac{3}{2}} x^{\frac{3}{2}} \right]_0^1 = \frac{2}{3}.$$

$R(\Pi_n, U_n)$

$\Pi_n = \left\{ 0 = x_0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, x_n = 1 \right\}$

$U_n = \left\{ \frac{1}{n}, \frac{2}{n}, \dots, 1 \right\}$ $\quad c_i = \frac{i}{n}$

$f(c_i) = \sqrt{\frac{i}{n}}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{i}{n}} (x_i - x_{i-1})$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sqrt{i}}{\sqrt{n}} \left(\frac{i}{n} - \frac{i-1}{n} \right)$$

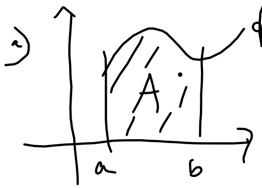
$$= \lim_{n\to\infty} \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \sqrt{i} \left(\frac{1}{n} \right)$$

$$= \lim_{n\to\infty} \frac{1}{n^{\frac{3}{2}}} \sum_{i=1}^{n} \sqrt{i}$$

ANVENDELSER AV INTEGRALER

1) "Arealet under en graf"

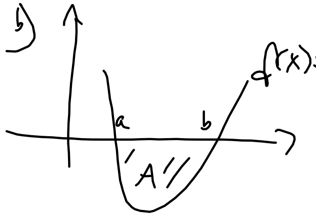
a)



$f(x) \geq 0$ på $[a,b]$

$$A = \int_a^b f(x)\,dx$$

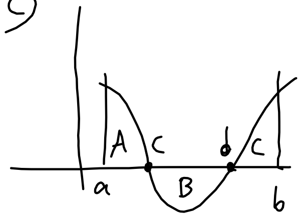
b)



$f(x) \leq 0$ på $[a,b]$

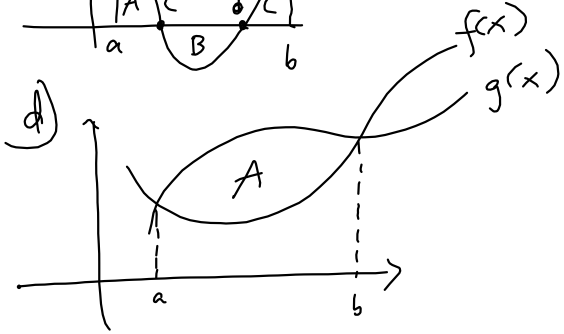
$$A = -\int_a^b f(x)\,dx$$

c)



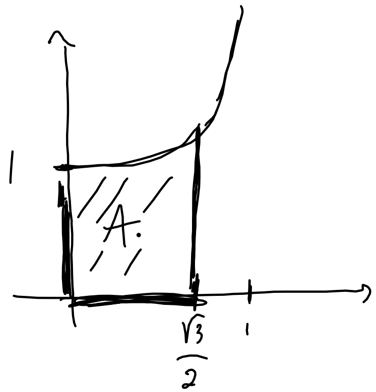
$$= \int_a^c f(x)\,dx - \int_c^d f(x)\,dx + \int_d^b f(x)\,dx$$

d)



$$A = \int_a^b g(x) - f(x)\,dx$$

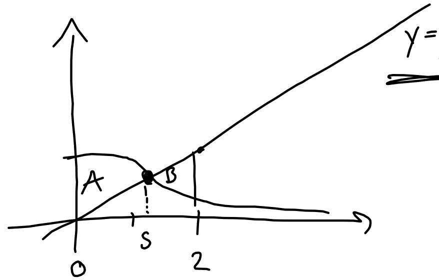
8.6.1. e)



$$y = \frac{1}{\sqrt{1-x^2}}$$

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \Big[\arcsin x\Big]_0^{\frac{\sqrt{3}}{2}} = \frac{\pi}{3}$$

f)



$y = \frac{x}{2}$ $\qquad$ $y = \frac{1}{1+x^2}$

$$\frac{x}{2} = \frac{1}{1+x^2}$$

$$x + x^3 = 2$$

$$x^3 + x - 2 = 0$$

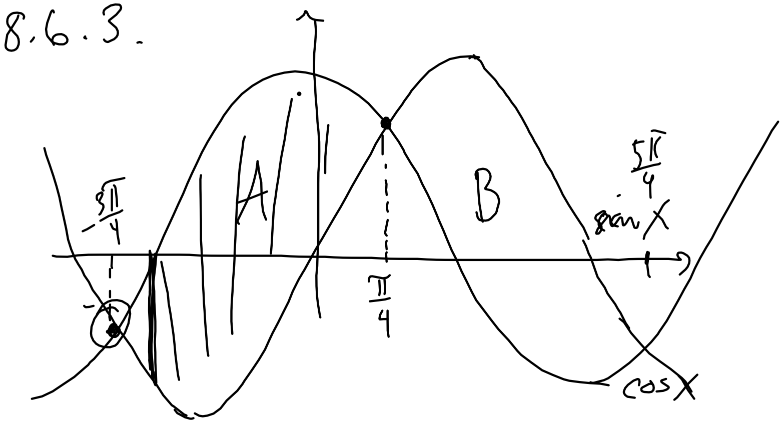
$$x = 1$$

$$A = \int_0^1 \frac{1}{1+x^2} - \frac{x}{2}\,dx = \Big[\arctan x - \frac{1}{4}x^2\Big]_0^1 = \left(\arctan 1 - \frac{1}{4}\right) - \left(\arctan 0 - 0\right)$$

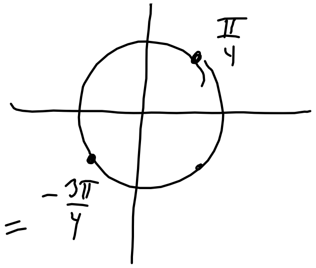
$$= \frac{\pi}{4} - \frac{1}{4} = \frac{\pi - 1}{4}$$

$$B = \int_1^2 \frac{x}{2} - \frac{1}{1+x^2}\,dx$$

8.6.3.



$\cos x - \sin x \geq 0$
på $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$



$\frac{5\pi}{4} = -\frac{3\pi}{4}$

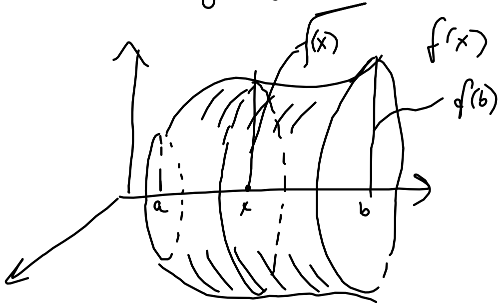
$$A = \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \cos x - \sin x \, dx$$

$$= \left[\sin x + \cos x\right]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} = \dots = \underline{\underline{2\sqrt{2}}}$$

$$B = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$$

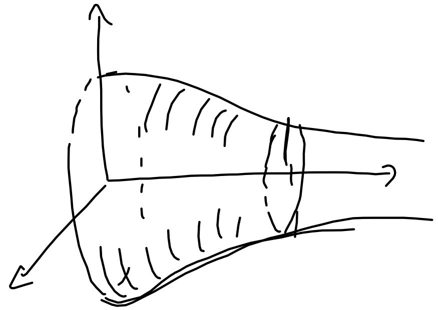
$$= \int_{\frac{5\pi}{4}}^{\frac{\pi}{4}} \cos x - \sin x \, dx$$

2 Omdreiningslegeme om x-aksen



$$\int_a^b \pi f(x)^2 \, dx$$

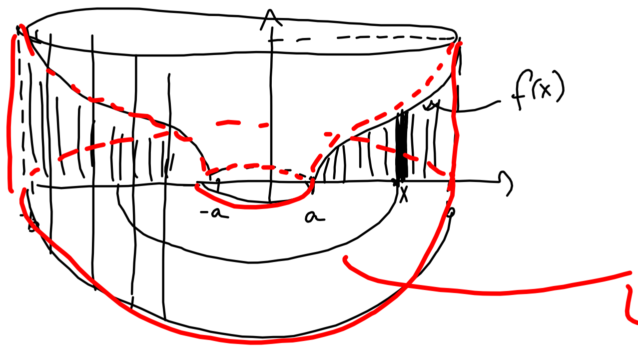
8.6.5 c) $y = \frac{1}{\sqrt{1+x^2}}$



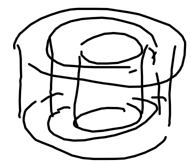
$$V = \int_0^1 \pi \left(\frac{1}{\sqrt{1+x^2}}\right)^2 dx$$

$$= \int_0^1 \pi \cdot \frac{1}{1+x^2} \, dx = \Big[\pi \arctan x\Big]_0^1 = \underline{\underline{\frac{\pi^2}{4}}}$$

③ Omdreiningslegme om y-aksen



$V =$



$A = 2\pi r \cdot f(x)$

$V = \int_a^b 2\pi x \cdot f(x)\, dx$