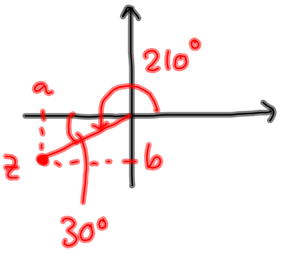
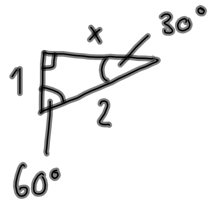
Midtveis Mat 1100 h13

① $z = 2e^{i(7\pi/6)}$

$$\frac{7\pi}{6} = \pi + \frac{\pi}{6} = 180^\circ + 30^\circ$$



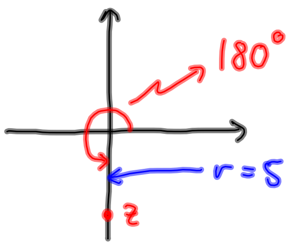


$$x^2 + 1^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

$$z = a + bi = -\sqrt{3} - i$$

$\boxed{C}$

② $z = -5i$



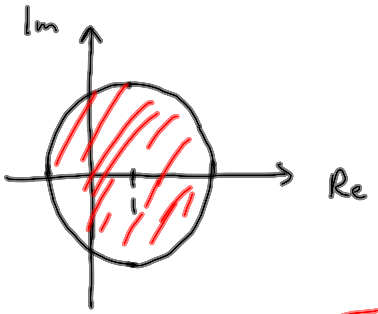
$$180^\circ + 90^\circ = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$z = 5e^{i\left(\frac{3\pi}{2}\right)}$$

$\boxed{C}$

③ $|z-1| < 2$

Skal ha alle punkter z som har avstand mindre enn 2 fra punktet $1 = 1 + 0i$



$\boxed{E}$

④ $z = i$

$i \cdot i^2 = -i$

A) $P(z) = P(i) = i^3 + i^2 + i + 1$

$= -i - 1 + i + 1 = 0$

$\boxed{A}$

5. $$\lim_{n\to\infty} \frac{n^5 - 5n^4}{7+5n^3+3n^5} = \lim_{n\to\infty} \frac{1 - \frac{5}{n}}{\frac{7}{n^5} + \frac{5}{n^2} + 3} = \frac{1}{3}$$

$\frac{5}{n} \to 0$, $\frac{7}{n^5} \to 0$, $\frac{5}{n^2} \to 0$

$\boxed{E}$

6. $$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right) = \lim_{n\to\infty} \frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{(\cancel{n}+1) - \cancel{n}}{\sqrt{n+1}+\sqrt{n}} = 0$$

$\boxed{B}$

7. $$f(x) = e^{x^2} = e^{(x^2)}$$

$$f'(x) = e^{x^2} \cdot 2x$$

$$f''(x) = \left(e^{x^2} \cdot 2x\right) \cdot 2x + e^{x^2} \cdot 2$$

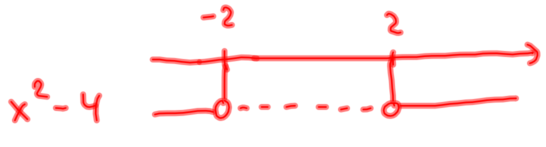
$$= 4x^2 e^{x^2} + 2e^{x^2} = 2e^{x^2}(2x^2+1)$$

$\boxed{C}$

8. $$f(x) = x^4 - 24x^2$$

$$f'(x) = 4x^3 - 48x$$

$$f''(x) = 12x^2 - 48 = 12(x^2 - 4)$$



$\boxed{D}$

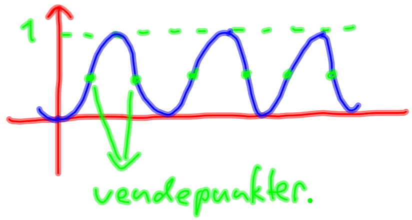
9. $$f(x) = \frac{\sin x}{x}$$

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

$x \to \infty$

$\boxed{E}$

(10) $f(x) = \sin^2 x = (\sin x)^2$



Alternativt:

$$f'(x) = 2\sin x \cdot \cos x = 2 \sin 2x$$

$$\begin{aligned} f''(x) &= 2\cos x \cdot \cos x \\ &\quad + 2 \sin x \cdot (-\sin x) \\ &= 2\cos^2 x - 2\sin^2 x \\ &= 2(\cos^2 x - \sin^2 x) \\ &= 2\cos(2x) \end{aligned}$$

$\boxed{C}$

(11) $f(x) = \sin\left(e^{\cos x}\right)$

$$f'(x) = \cos\left(e^{\cos x}\right) \cdot e^{\cos x} \cdot (-\sin x)$$

$\boxed{B}$

(12)

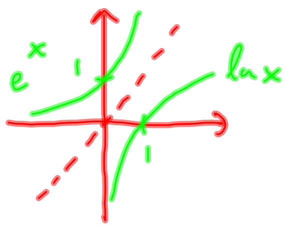
$$\lim_{x\to 0} \frac{\sin x}{x + x^2 + \sin x} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x \to 0} \frac{\underbrace{\cos x}_{\to 1}}{1 + 2x + \underbrace{\cos x}_{\to 1}}$$

$$= \frac{1}{1+0+1} = \frac{1}{2}$$

$\boxed{C}$

(13)

$$\lim_{x \to 0^+} \underbrace{x}_{\to 0}\, \underbrace{e^{\overbrace{1/x}^{\to +\infty}}}_{\to \infty} \overset{[0\cdot\infty]}{=} \lim_{x\to 0^+} \frac{e^{1/x}}{\frac{1}{x}}$$



$$\overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x\to 0^+} \frac{e^{1/x} \cdot \cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\left(\frac{-1}{x^2}\right)}} = +\infty$$

$\boxed{E}$

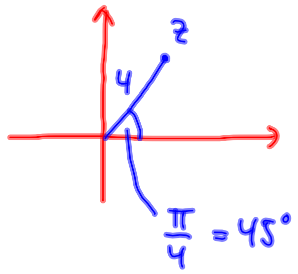
(14) $\boxed{B}$

(15) $z = 4e^{i(\frac{\pi}{4})}$

$$w_0 = \sqrt[4]{4}\, e^{i(\pi/16)}$$

$$= \sqrt{2}\, e^{i(\pi/16)}$$

Dette stemmer med $\boxed{E}$



$$\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2} = 2\cdot 2 = 4$$

$$w_+ = e^{i(2\pi/4)} = e^{i(\pi/2)}$$

$$= e^{i(8\pi/16)}$$

(16) $f(x) = x + \dfrac{\cos x}{x}$ $\quad y = ax+b$ skråasymptote

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty}\left(1 + \frac{\cos x}{x^2}\right) = 1$$

$$b = \lim_{x\to\infty}\left[f(x) - ax\right] = \lim_{x\to\infty}\left[f(x) - x\right]$$

$$= \lim_{x\to\infty} \frac{\cos x}{x} = 0 \qquad \underline{\underline{y = x}} \text{ skråasymp.}$$

$\boxed{B}$

(17) $f(x) = \sqrt{\ln x}$ Må ha $\ln x \geq 0$

dvs. $x \geq 1$. $D_f = [1, \infty)$. $\boxed{E}$

(18)
$$\lim_{h\to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0^+} \frac{\sqrt{\ln(1+h)} - 0}{h}$$

$$\left[\tfrac{0}{0}\right] = \lim_{h\to 0^+} \frac{\frac{1}{2\sqrt{\ln(1+h)}}\cdot\frac{1}{1+h}\cdot 1}{1} = +\infty$$

dvs. f er ikke (ensidig) deriverbar i $x = 1$.
Men den er kontinuerlig i $x = 1$. $\boxed{A}$

19 $f(x) = x e^{x^2-2x}$

$$f'(x) = 1 \cdot e^{x^2-2x} + x \cdot e^{x^2-2x} \cdot (2x-2)$$

$$= e^{x^2-2x}\left[1 + x(2x-2)\right]$$

$$= \underbrace{e^{x^2-2x}}_{\text{alltid positiv}}\left[1 + 2x^2 - 2x\right]$$

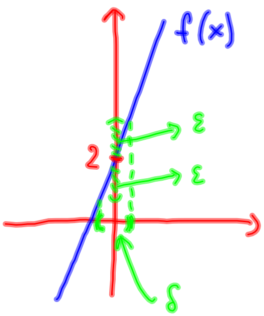
$2x^2 - 2x + 1 = 0$ gir $x = \dfrac{2 \pm \sqrt{4 - 4 \cdot 2 \cdot 1}}{2 \cdot 2}$

dvs. $[1 + 2x^2 - 2x]$ er også alltid positiv.

Så $f'(x) > 0$ for alle $x \in \mathbb{R}$. $\boxed{A}$

20 $f(x) = 3x + 2$

La $\varepsilon > 0$ være gitt. Hvilken δ er slik at $|x| < \delta$ medfører $|f(x) - f(0)| < \varepsilon$, uansett størrelse av ε?



$f(0) = 2$

$\delta = \frac{\varepsilon}{3}$ pga stigningstall 3

dvs. $\boxed{A}$

Med regning:

$$|f(x) - f(0)| = |(3x+2) - 2| = |3x|$$

$\boxed{\begin{array}{l} x = 0 + h \\ \text{dvs. } x = h \end{array}}$ $= |3h| < \varepsilon$ hvis $|h| < \frac{\varepsilon}{3}$

Så $\delta = \frac{\varepsilon}{3}$.