9.3.1. d) $\int \frac{x+7}{x^2-x-2}\,dx = \int \frac{x+7}{(x-2)(x+1)}\,dx$

$$\frac{x+7}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$\frac{x+7}{(x-2)(x+1)} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)}$$

$$x+7 = Ax + A + Bx - 2B$$

- $1 = A + B$ koeffisientene til x^1
- $7 = A - 2B$ — " — x^0 (konstantledd)

- $A = 1 - B = 1-(-2) = 3$
- $7 = (1-B) - 2B$
  $6 = -3B$
  $B = -2$

I a) x2

- $\int \frac{3}{x-2}\,dx + \int \frac{(-2)}{x+1}\,dx$

$u = x-2$
$du = dx$

$= 3\ln|x-2| - 2\ln|x+1| + C$

9.3.3. a) $\int \frac{2}{x^2+6x+10}\,dx = 2\int \frac{1}{(x+3)^2+1}\,dx$

Prøver
u^2+1

$x^2+6x+10$

$= \underbrace{x^2+2\cdot x\cdot 3+9}+1$

$(x+3)^2+1$

$(a+b)^2 = a^2+2ab+b^2$

$2\int \frac{1}{(x+3)^2+1}\,dx$

$u=(x+3)$
$du = dx$

$= 2\int \frac{1}{u^2+1}\,du$

$= 2\arctan u + C$

$= 2\arctan(x+3) + C$

9.3.3 b) $\int \frac{2x-2}{x^2+4x+8}\,dx$

II ⇒ ②

Prøve oss:
$u = x^2+4x+8$
$du = (2x+4)dx$

$+0$
$= \int \frac{2x+4 \quad -2-4}{x^2+4x+8}\,dx$

$= \int \frac{2x+4}{x^2+4x+8}\,dx - 6\int \frac{1}{x^2+4x+8}\,dx$

$x^2+2x\cdot 2+8$
$= \underbrace{x^2+2x\cdot 2+4}+4$
$= (x+2)^2+4$

$= \int \frac{1}{u}\,du - 6\int \frac{1}{(x+2)^2+4}\,dx$

$= \ln|x^2+4x+8| - \frac{6}{4}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx + C$

$= \ln|x^2+4x+8| - \frac{3}{\cancel{2}}\int \frac{\cancel{2}du}{u^2+1} + C$

$u = \frac{x+2}{2} = \frac{1}{2}x+1$
$du = \frac{1}{2}dx$
$2du = dx$

$$= \ln|x^2+4x+8| - 3\arctan\left(\frac{x+2}{2}\right) + C$$

9.3.13. $\int \frac{2x^3+2x+1}{(x^2+1)^2}\,dx$

$$\frac{2x^3+2x+1}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$$

$$= \frac{(Ax+B)(x^2+1)+Cx+D}{(x^2+1)^2}$$

$$2x^3+0\cdot x^2+2x+1 = Ax^3+Ax+Bx^2+B+Cx+D$$

- $2 = A$
- $0 = B$
- $2 = A + C \Rightarrow C = 0$
- $1 = B + D = D$

I a) (2) II b)

$$= \int \frac{2x}{x^2+1}\,dx + \int \frac{1}{(x^2+1)^2}\,dx \qquad u = x^2+1,\ du = 2x\,dx$$

$$= \int \frac{1}{u}\,du + \underline{I_2 = \frac{1}{2(2-1)}\cdot\frac{x}{(1+x^2)^{2-1}} + \frac{2\cdot 2-3}{2(2-1)} I_1}$$

FORMEL

$$= \ln|x^2+1| + \underline{\frac{1}{2}\cdot\frac{x}{(1+x^2)} + \frac{1}{2}\int \frac{1}{x^2+1}\,dx}$$

$$= \underline{\underline{\ln|x^2+1| + \frac{x}{2(1+x^2)} + \frac{1}{2}\arctan x + C}}$$

9.4.8.

$$\int \sin^3 x \cos^2 x \, dx$$

$$= \int \sin x \left(1-\cos^2 x\right)\cos^2 x \, dx$$

$$= \int \sin x \cos^2 x \, dx - \int \sin x \cdot \cos^4 x \, dx$$

$$= -\int u^2 du + \int u^4 du$$

$$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

$$= \underline{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$$

9.4.10 $\int \sin^4 x \cos^2 x \, dx$

Løses ved trigonometriske id.

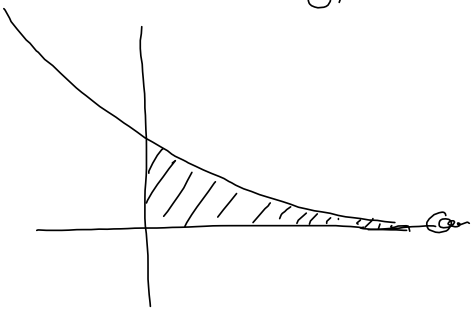
$1 = \sin^2 x + \cos^2 x$

$u = \cos x$

$du = -\sin x \, dx$

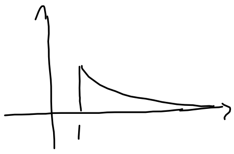
9.5. UEGENTLIGE INTEGRALER

- Kan man beregne integraler hvis den ene grensen er ∞? JA
  (Del opp hvis begge er $\pm\infty$)

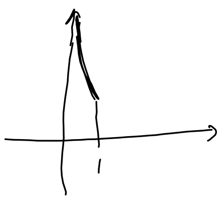


- $$\int_0^\infty e^{-x}\,dx = \lim_{b\to\infty}\int_0^b e^{-x}\,dx = \lim_{b\to\infty}\left[-e^{-x}\right]_0^b = \lim_{b\to\infty}\left(-e^{-b}+1\right) = 0+1 = \underline{\underline{1}}$$

- $\int_1^\infty \frac{1}{x^p}\,dx$ konvergerer for $p>1$, divergerer for $p\leq 1$



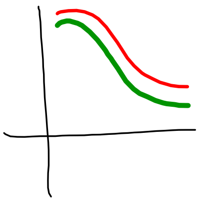
- $\int_0^1 \frac{1}{x^p}\,dx$ konvergerer for $p<1$, diverergerer for $p\geq 1$



- SAMMENLIKNINGSKRITERIET

La $f, g : [a, \infty) \to \mathbb{R}$ kontinuerlige & positive funksjoner

Anta $f(x) \geq g(x)$ $\forall$ $x \in [a, \infty)$.



- Hvis $\int_a^\infty f(x)\,dx$ konvergerer, så vil $\int_a^\infty g(x)\,dx$ konvergere.
- Hvis $\int_a^\infty g(x)\,dx$ divergerer, så vil $\int_a^\infty f(x)\,dx$ divergere.

- GRENSESK. Antakelser som over. (ikke $f(x) \geq g(x)$)
  - Hvis $\int_a^\infty f(x)\,dx$ konvergerer og $\lim_{x\to\infty} \frac{g(x)}{f(x)} < \infty$,
    da konvergerer $\int_a^\infty g(x)\,dx$.
  - Hvis $\int_a^\infty f(x)\,dx$ divergerer og $\lim_{x\to\infty} \frac{g(x)}{f(x)} > 0$
    da vil $\int_a^\infty g(x)\,dx$ divergere

9.5.3 c) $\int_0^1 \frac{1}{\sqrt{x+x^3}}\,dx$

$g(x) = \frac{1}{\sqrt{x+x^3}}$

$f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}}$ $p = \frac{1}{2}$

Ha at $\int_0^1 \frac{1}{x^{\frac{1}{2}}}\,dx$ konvergerer. og $g(x) \leq f(x)$ på $[0,1]$

så ved S.K. konvergerer $\int_0^1 \frac{1}{\sqrt{x+x^3}}\,dx$

g og f er positive og kontinuerlige.