LINEAR ALGEBRA

1.1.1. $\vec{a} = (1, -2, 4, -5, 1) \in \mathbb{R}^5$

$\vec{b} = (-3, 5, 5, 0, -3)$

$$\vec{a} + \vec{b} = (1-3, -2+5, 4+5, -5+0, 1-3)$$
$$= \underline{\underline{(-2, 3, 9, -5, -2)}}$$

$$\vec{a} - \vec{b} = (1-(-3), -2-5, 4-5, -5-0, 1-(-3))$$
$$= \underline{\underline{(4, -7, -1, -5, 4)}}$$

$$\mathbb{R} \ni \vec{a} \cdot \vec{b} = 1 \cdot (-3) + (-2) \cdot 5 + 4 \cdot 5 + (-5) \cdot 0 + 1 \cdot (-3)$$
$$= -3 - 10 + 20 + 0 - 3$$
$$= \underline{\underline{4}}$$

1.3.3. $\vec{x} = (1+3i, -2i, 2+3i) \in \mathbb{C}^3$

$\vec{y} = (\underline{2}, \underline{1+2i}, \underline{-1+i})$

$$\overline{\vec{y} \cdot \vec{x}} = \vec{x} \cdot \underline{\vec{y}} = (1+3i) \cdot \underline{\overline{2}} + (-2i) \cdot \underline{\overline{(1+2i)}} + (2+3i)\underline{\overline{(-1+i)}}$$
$$= (1+3i) \cdot 2 + (-2i)(1-2i) + (2+3i)(-1-i)$$
$$= 2 + 6i + (-2i) - 4 + (-2) - 2i - 3i + 3$$
$$= \underline{\underline{-1-i}}$$

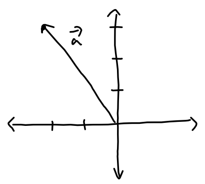
1.1.3. a) $(\vec{x}+\vec{y})\cdot(\vec{x}+\vec{y})$ $\in \mathbb{R}^n$

$= \vec{x}\cdot\vec{x} + \vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{x} + \vec{y}\cdot\vec{y}$

$= \vec{x}\cdot\vec{x} + 2\vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{y}$

$2x$
$2\cdot x$

1.2.1



$\vec{a} = (-2,3)$ $\in \mathbb{R}^2$

$\vec{b} = (4,1)$

$\vec{a}\cdot\vec{b} = -2\cdot 4 + 3\cdot 1$

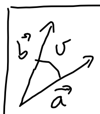
$= \underline{\underline{-5}}$

$|\vec{a}| = \sqrt{(-2)^2+3^2}$

$= \underline{\sqrt{13}}$

$|\vec{b}| = \sqrt{4^2+1^2}$

$= \underline{\sqrt{17}}$



$\boxed{\cos v = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} \qquad |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}}$

$\cos v = \dfrac{-5}{\sqrt{13}\cdot\sqrt{17}}$

Kalk: $\cos^{-1}$

$v = \arccos\left(\dfrac{-5}{\sqrt{13}\cdot\sqrt{17}}\right)$

$\approx 1,91$ (radianer)

$\left(\dfrac{180}{\pi}\right)$ $\approx \underline{\underline{109,65°}}$

1.3.2. $\vec{a} = (3+2i, -1+i) \in \mathbb{C}^2$

$$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{(3+2i)(3-2i)+(-1+i)(-1-i)}$$
$$= \sqrt{3^2+2^2+(-1)^2+1^2}$$
$$= \underline{\underline{\sqrt{15}}}$$

$(a+ib)(a-ib) = a^2+b^2$

$\vec{b} = (i, 2+3i, 2-i) \in \mathbb{C}^3$

$$|\vec{b}| = \sqrt{\vec{b}\cdot\vec{b}} = \sqrt{i(-i)+(2+3i)(2-3i)+(2-i)(2+i)}$$
$$= \sqrt{1+2^2+3^2+2^2+1^2}$$
$$= \underline{\underline{\sqrt{19}}}$$

1.2.3. $\vec{a} = (1,2,3)$ $\vec{b} = (-1,0,1)$ $\in \mathbb{R}^3$

$$\cos v = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$\vec{a}\cdot\vec{b} = 1\cdot(-1) + 2\cdot 0 + 3\cdot 1$$
$$= \underline{2}$$

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2}$$
$$= \underline{\sqrt{14}}$$

$$|\vec{b}| = \sqrt{(-1)^2 + 0^2 + 1^2}$$
$$= \underline{\sqrt{2}}$$

$$\cos v = \frac{2}{\sqrt{14}\cdot\sqrt{2}}$$

$$v \approx 1{,}18$$
$$\approx \underline{\underline{67{,}8^\circ}}$$

1.2.5.

$$\vec{a} = (4, 3, 1, 2) \in \mathbb{R}^4$$

$$\vec{b} = (-1, 3, 2, 0)$$

$$\vec{a} \cdot \vec{b} = 4(-1) + 3 \cdot 3 + 1 \cdot 2 + 2 \cdot 0 = \underline{7}$$

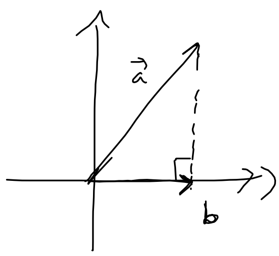
$$|\vec{a}| = \sqrt{4^2 + 3^2 + 1^2 + 2^2} = \underline{\sqrt{30}}$$

$$|\vec{b}| = \sqrt{\vec{b} \cdot \vec{b}} \qquad |\vec{b}|^2 = \vec{b} \cdot \vec{b}$$

$$|\vec{b}| = \sqrt{(-1)^2 + 3^2 + 2^2 + 0^2} = \underline{\sqrt{14}}$$

$$\cos v = \frac{7}{\sqrt{30} \cdot \sqrt{14}}$$

$$v \approx 1{,}22 \approx \underline{\underline{70{,}0^\circ}}$$

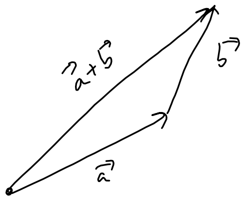


$$\vec{P}_{a,b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \vec{b}$$

$$= \frac{7}{14} (-1, 3, 2, 0)$$

$$= \underline{\underline{\left(-\tfrac{1}{2}, \tfrac{3}{2}, 1, 0\right)}}$$

1.2.13. Per påstår $|\vec{a}| = 3$, $|\vec{b}| = 2$ og $|\vec{a} + \vec{b}| = 7$



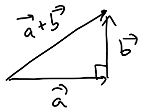
- TREKANTULIKHETEN:

  $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

  $7 > 3 + 2$ motsier trekantulikheten, så Per tar feil.

- PYTHAGORAS' Hvis $\vec{a}$ står normalt på $\vec{b}$ $(\vec{a} \cdot \vec{b} = 0)$

  $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{a} + \vec{b}|^2$



- SCHWARZ' ULIKHET For alle $\vec{a}, \vec{b} \in \mathbb{R}^n$ er $|\vec{a} \cdot \vec{b}| \le |\vec{a}| \cdot |\vec{b}|$

1.2.15. Skal vise $\big| |\vec{x}| - |\vec{y}| \big| \le |\vec{x} - \vec{y}|$ (Den omvendte trekantulikheten)

- $|\vec{x}| \underset{+\vec{0}}{=} |(\vec{x} - \vec{y}) + (\vec{y})| \underset{\text{TREKANT ULIKHET}}{\le} |\vec{x} - \vec{y}| + |\vec{y}|$

$$|\vec{x}| - |\vec{y}| \le |\vec{x} - \vec{y}|$$

- $|\vec{y}| = |\vec{y} - \vec{x} + \vec{x}| \le |\vec{y} - \vec{x}| + |\vec{x}|$

$|\vec{y} - \vec{x}|^2 = |\vec{y}|^2 - 2\vec{y}\cdot\vec{x} + |\vec{x}|^2$
$= |\vec{x} - \vec{y}|^2$

$$|\vec{y}| - |\vec{x}| \le |\vec{x} - \vec{y}|$$

$$\big| |\vec{x}| - |\vec{y}| \big| \le |\vec{x} - \vec{y}|$$

1.2.17. $|\vec{x} + \vec{y}|^2 + |\vec{x} - \vec{y}|^2 = 2|\vec{x}|^2 + 2|\vec{y}|^2$ $\quad \vec{x}, \vec{y} \in \mathbb{R}^n$

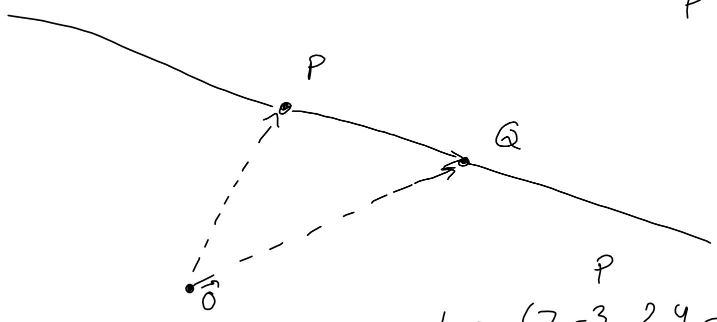
Se oppg. 1.1.3.

$(\vec{x} + \vec{y})\cdot(\vec{x} + \vec{y}) + (\vec{x} - \vec{y})\cdot(\vec{x} - \vec{y})$

$= \vec{x}\cdot\vec{x} + \underline{2\vec{x}\cdot\vec{y}} + \vec{y}\cdot\vec{y} + \vec{x}\cdot\vec{x} \underline{- 2\vec{x}\cdot\vec{y}} + \vec{y}\cdot\vec{y}$

$= \underline{2|\vec{x}|^2 + 2|\vec{y}|^2}$

1.2.21. Linja gjennom $\underset{P}{(7,-3,2,4,-2)}$ og $\underset{Q}{(2,1,-1,-1,5)} \in \mathbb{R}^5$



$$L = P + t\cdot\vec{PQ} \quad , \ t \in \mathbb{R}$$

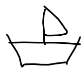
$$\vec{PQ} = \vec{Q} - \vec{P} = (-5, 4, -3, -5, 7)$$

$$L = \underset{P}{(7,-3,2,4,-2)} + t(-5,4,-3,-5,7)$$
$$= \underline{(7-5t, -3+4t, 2-3t, 4-5t, -2+7t)}, \ t \in \mathbb{R}$$

NB! Her blir det mange muligheter

1.2.25.

Ship A  $A(t) = \overset{t=0}{(0,4)} + (3,4)\cdot 15t \qquad t \geq 0$

Ship B  $B(s) = (39,14) + (-12,5)\cdot 13s \qquad s \geq 0$

a) Hvor krysser kursene?

$$A(t) = B(s)$$

$$\longrightarrow \quad (45t,\ 4+60t) = (39-12\cdot 13s,\ 14+5\cdot 13s)$$

- $45t = 39 - 156s$
- $4 + 60t = 14 + 65s$

$$45t + 156s = 39$$

$$60t - 65s = 10$$

Løsning: $\boxed{t = \frac{1}{3} \qquad s = \frac{2}{13}}$

$$A\left(\tfrac{1}{3}\right) = \left(45\cdot\tfrac{1}{3},\ 4+60\cdot\tfrac{1}{3}\right)$$

$$= \underline{\underline{(15, 24)}}$$

b) Vil skipene kollidere?

Nei skipene kollider ikke, for kursene møtes for ulike tider.

1.2.7.

$\vec{a} = (4,3) = \vec{b} + \vec{c}$ $\qquad \vec{d} = (1,2)$

$\vec{b} \parallel \vec{d} \iff \vec{b} = t\vec{d}$ for an $t \in \mathbb{R}$

$\vec{c} \perp \vec{d} \iff \vec{c} \cdot \vec{d} = 0$

$\underline{\vec{b} = t(1,2)}$

$\vec{c} = (c_1, c_2)$ $\qquad \vec{c} \cdot \vec{d} = (c_1, c_2)(1,2) = c_1 + 2c_2 = 0$

$= (-2c_2, c_2)$ $\qquad c_1 = -2c_2$

$\underline{= (-2,1)c_2}$

$(4,3) = t(1,2) + c_2(-2,1)$

- $4 = t - 2c_2$
- $3 = 2t + c_2$

$\underline{t = 2}$

$\underline{c_2 = -1}$

$\vec{a} = 2\vec{b} + (-1)\cdot\vec{c}$

$\underline{(4,3) = (2,4) + (2,-1)}$