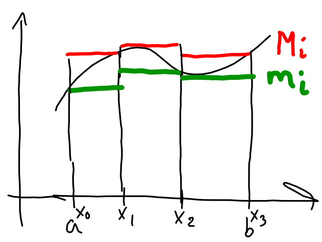
INTEGRASJON

$f : [a,b] \longrightarrow \mathbb{R}$

begrenset

Partisjon $\Pi = \{x_0, \dots, x_n\}$

8.2 DARBOUX



$A = \sum_{i=1}^{n} m_i (x_i - x_{i-1})$

$N(\pi)$

$\int_a^b f(x)\,dx = \sup \{N(\pi)\}$

$A = \sum_{i=1}^{n} M_i (x_i - x_{i-1})$

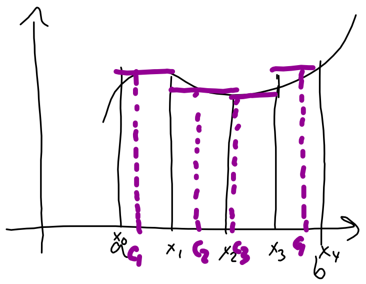
$\varnothing(\pi)$

$\int_a^{b} f(x)\,dx = \inf \{\varnothing(\pi)\}$

Hvis de er like er f integrerbar på $[a,b]$

$= \int_a^b f(x)\,dx$

8.5 RIEMANN



Utvalg $U = \{c_1, \dots, c_n\}$

$A = \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1})$

$R(\Pi, U)$

f er Riemannintegrerbar hvis det eksisterer en α slik at

$\lim_{n \to \infty} R(\Pi_n, U_n) = \alpha$

for alle følger $\{\Pi_n, U_n\}$ slik at $|\Pi_n| \to 0$.

DISSE ER EKVIVALENTE

8.4. DET UBESTEMTE INTEGRALET

$\int f(x)\,dx$ for den generelle antiderivete til f

$= F(x) + C \quad , \quad C \in \mathbb{R}$

$D[F(x) + C] = f(x)$

ELEMENTÆRE: $a \in \mathbb{R}$

- $\int a\,dx = ax + C \qquad \int dx = \int 1\,dx = x + C$

$a \neq -1$ • $\int x^a\,dx = \frac{1}{a+1} x^{a+1} + C$

$a = -1 \quad \int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$

$\int \frac{1}{\sin^2 x}\,dx = -\cot x + C$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

- $\int \frac{1}{1+x^2}\,dx = \arctan x + C$

SETTE SAMMEN

- $\int a f(x)\,dx = a \int f(x)\,dx$
- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- ENKEL SUBSTITUSJON (kjerneregel baklengs)
  - $\int f[g(x)] \cdot g'(x)\,dx = F[g(x)] + C$
  - $\int f(ax)\,dx = \frac{F(ax)}{a} + C$

MORAL: ALLTID MULIG Å SJEKKE

8.4.1. a) (med twist)

- $\int x^2 dx = \frac{1}{2+1} x^{2+1} + C = \underline{\underline{\frac{1}{3} x^3 + C}}$
- $\int x^{\pi} dx = \underline{\underline{\frac{1}{\pi+1} x^{\pi+1} + C}}$
- $\int x^{-1} dx = \ln|x| + C \quad = \int \frac{1}{x} dx$
- $\int \frac{1}{x+3} dx = \ln|x+3| + C$ FLAKS!

  SUBST: $u = x+3$, $\frac{du}{dx} = 1$, $\underline{du = dx}$

  $= \int \frac{1}{u} du = \ln|u| + C = \underline{\underline{\ln|x+3| + C}}$

- $\int \frac{1}{2x+3} dx$

  $u = 2x+3$, $du = 2\,dx$, $\frac{1}{2} du = dx$

  $= \frac{1}{2} \int \frac{1}{u} du$

  $= \frac{1}{2} \ln|u| + C$

  $= \underline{\underline{\frac{1}{2} \ln|2x+3| + C}}$

HUSK TIL NESTE UKE!

- $\int \frac{1}{ax+b} dx$ $\quad a,b \in \mathbb{R}$, $a \neq 0$

  $u = ax+b$, $du = a\,dx$, $\frac{1}{a} du = dx$

  $= \frac{1}{a} \int \frac{1}{u} du$

  $= \underline{\underline{\frac{1}{a} \ln|ax+b| + C}}$

8.4.1. b) $\int 7x + 3x^{\frac{1}{2}} - \cos x \, dx$ $\qquad x = x^1$

$= 7 \cdot \frac{1}{2} x^2 + 3 \cdot \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} - \sin x + C$

$= \underline{\frac{7}{2} x^2 + 2x^{\frac{3}{2}} - \sin x + C}$ $\qquad \cancel{u = 1 + 2x^2}$

c) $\int \frac{1}{1+2x^2} dx = \underline{\int \frac{1}{1+(\sqrt{2}x)^2} dx}$ $\qquad u = \sqrt{2}x$, $du = \sqrt{2} dx$, $\frac{1}{\sqrt{2}} du = dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{1+u^2} du$

$= \frac{1}{\sqrt{2}} \underline{\arctan(\sqrt{2}x) + C}$

d) $\int 8e^{7x} + \frac{1}{\sqrt{x}} dx = \int 8e^{7x} dx + \int x^{-\frac{1}{2}} dx$ $\qquad u = 7x$, $\frac{1}{7} du = dx$

$\boxed{f(ax)} = \frac{8}{7} e^{7x} + \frac{1}{-\frac{1}{2}+1} x^{\frac{1}{2}} + C$

$\underline{\frac{8}{7} e^{7x} + 2\sqrt{x} + C}$ $\qquad = \underline{\frac{8}{7} e^{7x} + 2x^{\frac{1}{2}} + C}$

$$\int e^{7x}\,dx$$

$$= \frac{1}{7}\int e^{u}\,du$$

$$u = 7x$$

$$\frac{du}{dx} = u'_{x} = 7 \quad \text{"}\,|\cdot dx\,\text{"}$$

$$du = 7\,dx$$

$$\frac{1}{7}\,du = dx$$

$$u(x,y)$$

$$\frac{du}{dx}$$

$$\frac{du}{dy}$$

8.4.2.

a) $\int \frac{42}{\sin^2(7x)}\,dx$

$= \frac{42}{7}\int \frac{1}{\sin^2 u}\,du$

$= 6\,(-\cot u) + C$

$= -6\cot(7x) + C$

$u = 7x$
$du = 7\,dx$
$\frac{1}{7}du = dx$

$\boxed{\begin{array}{l} \cancel{u = \sin(7x)} \\ \cancel{du = \cos(7x)\cdot 7\,dx} \end{array}}$

$D\left[-6\cot(7x) + C\right]$

$= -6\cdot\left(-\frac{1}{\sin^2(7x)}\cdot 7\right)$

$= \frac{42}{\sin^2(7x)}$

b) $\int x e^{-x^2}\,dx$

$= -\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u + C$

$= -\frac{1}{2}e^{-x^2} + C$

$u = -x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

c) $\int e^x\cos(e^x)\,dx$

$= \int \cos u\,du$

$= \sin(e^x) + C$

$u = e^x$
$du = e^x\,dx$

d) $\int \frac{dx}{\sqrt{x}\cos^2(\sqrt{x})}$

$u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$

$2du = x^{-\frac{1}{2}} dx$

$= \int \frac{dx}{x^{\frac{1}{2}} \cos^2(x^{\frac{1}{2}})}$

$= 2\int \frac{du}{\cos^2 u}$

$= 2\tan(x^{\frac{1}{2}}) + C$

$= 2\tan\sqrt{x} + C$

e) $\int \frac{1+x}{1+x^2} dx$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x dx$

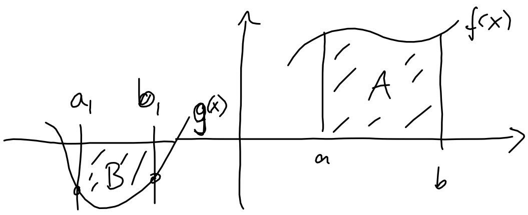
$= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$

$= \arctan x + \frac{1}{2}\int \frac{du}{u}$

$= \arctan x + \frac{1}{2}\ln|1+x^2| + C$

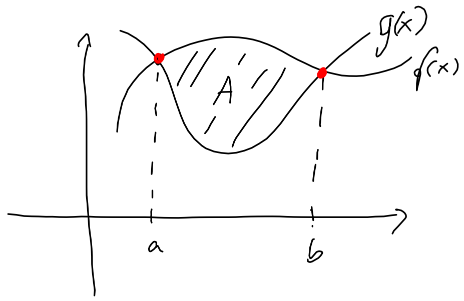
$= \arctan x + \frac{1}{2}\ln(1+x^2) + C$

8.6. ANVENDELSER AV INTEGRALET



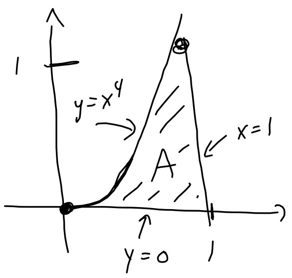
$A = \int_a^b f(x)\,dx \quad f(x) \geq 0$ på $[a, b]$

$B = -\int_{a_1}^{b_1} g(x)\,dx \quad g(x) \leq 0$ på $(a_1, b_1]$



$A = \int_a^b f(x) - g(x)\,dx$

8.6.1. a)



$y = x^4$

x - aksen $(y=0) \Rightarrow x=0$

$x = 1$

$$A = \int_0^1 x^4\,dx$$

$$= \left[\frac{1}{5}x^5\right]_0^1$$

$$= \frac{1}{5}\cdot 1^5 - \frac{1}{5}\cdot 0^5$$

$$= \underline{\underline{\frac{1}{5}}}$$

8.6.1. g)

I $y = 2x^2$

II $y = x^2$

III $y = \frac{1}{x}$

I & II: $2x^2 = x^2$

$x = 0$

I & III $2x^2 = \frac{1}{x}$

$2x^3 = 1$

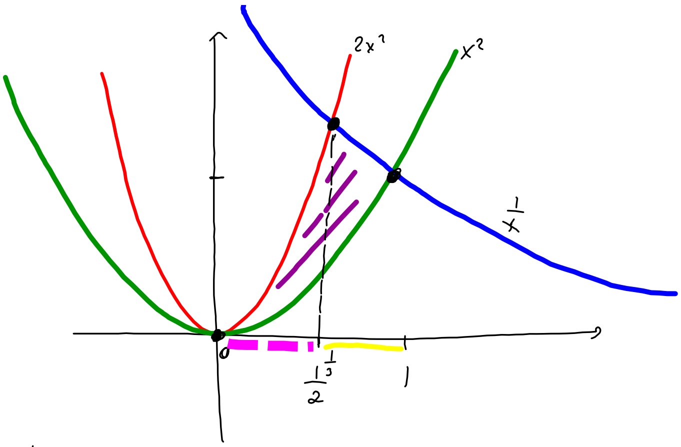
$x^3 = \frac{1}{2}$

$x = \frac{1}{2}^{\frac{1}{3}}$

II & III $x^2 = \frac{1}{x}$

$x^3 = 1$

$x = 1$



$$A = \int_0^{\frac{1}{2}^{\frac{1}{3}}} 2x^2 - x^2 \, dx + \int_{\frac{1}{2}^{\frac{1}{3}}}^{1} \frac{1}{x} - x^2 \, dx$$

$$= \left[\frac{1}{3}x^3\right]_0^{\frac{1}{2}^{\frac{1}{3}}} + \left[\ln|x| - \frac{1}{3}x^3\right]_{\frac{1}{2}^{\frac{1}{3}}}^{1}$$

$\frac{1}{3}\left(\frac{1}{2}^{\frac{1}{3}}\right)^3 = \frac{1}{3} \cdot \frac{1}{2}$

$$= \frac{1}{3} \cdot \frac{1}{2} + (\overset{0}{\overset{\shortparallel}{\ln}}) - \frac{1}{3} - \left(\ln \frac{1}{2}^{\frac{1}{3}} - \frac{1}{3} \cdot \frac{1}{2}\right)$$

$$= \frac{1}{3} \cdot \frac{1}{2} + \left(-\frac{1}{3}\right) - \ln \frac{1}{2}^{\frac{1}{3}} + \frac{1}{3} \cdot \frac{1}{2}$$

③

$$= -\frac{1}{3} \ln \frac{1}{2}$$

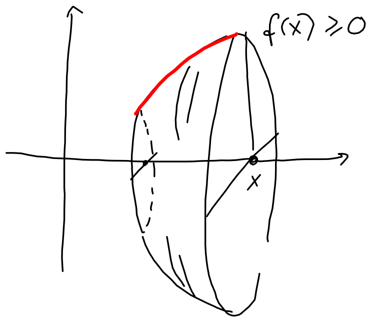
②

$$= -\frac{1}{3}(\ln 1 - \ln 2)$$

$$= \frac{1}{3} \ln 2$$

OMDREININGSLEGMER

X-aksen



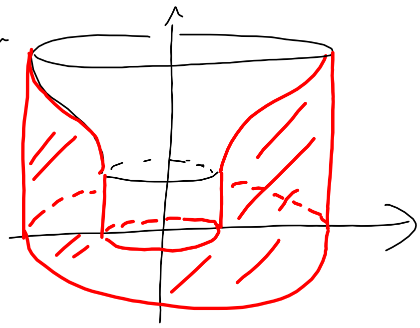
Mange små sirkler med areal

$A = \pi \cdot f(x)^2$

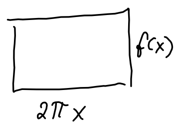
Legge sammen

$$V = \int_a^b \pi f(x)^2 \, dx$$

Y-aksen



Mange små sylindre



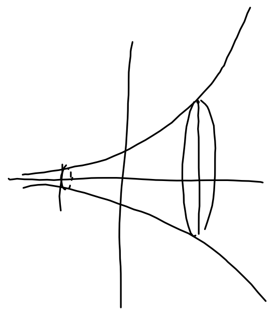
$A = 2\pi x f(x)$

Legge sammen

$$V = \int_a^b 2\pi x f(x) \, dx \qquad a \geq 0$$

8.6.5. f) $y = e^x$ skal roteres om x-aksen

$x = -1$ og $x = 1$



$$V = \int_{-1}^{1} \pi (e^x)^2 \, dx$$

$$= \int_{-1}^{1} \pi e^{2x} \, dx$$

$$= \pi \left[\frac{1}{2} e^{2x} \right]_{-1}^{1}$$

$$= \underline{\frac{\pi}{2} \left(e^2 - e^{-2} \right)}$$

8.6.7. b) $y = \sqrt{x}$ roteres om y-aksen

$x = 1$

$x = 4$

$$V = \int_{1}^{4} 2\pi x \cdot \sqrt{x} \, dx$$

$$= 2\pi \int_{1}^{4} x^{\frac{3}{2}} \, dx$$

$$= 2\pi \left[\frac{2}{5} x^{\frac{5}{2}} \right]_{1}^{4}$$

$$\rightsquigarrow \underline{\frac{124\pi}{5}}$$