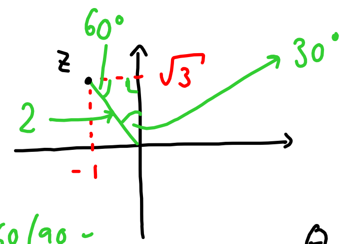
Løsningsforslag midtveis eksamen 09.10.2015 Mat 1100

Oppgave 1 $\boxed{D}$

Oppgave 2

$$z = -1 + i\sqrt{3}$$



30/60/90 - trekant med korteste katet 1

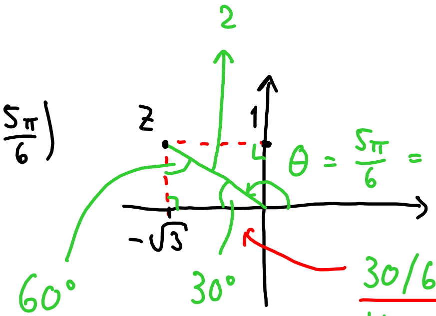
Så hvis $z = re^{i\theta}$, har vi $r = 2$

$$\theta = 30^\circ + 90^\circ = \frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$\boxed{C}$

Oppgave 3

$$z = 2e^{i\left(\frac{5\pi}{6}\right)}$$



$$\theta = \frac{5\pi}{6} = \frac{5}{6}\cdot 180^\circ = 150^\circ$$

30/60/90-trekant med hypotenus 2
Korteste katet er halvparten av hypotenusen, dvs. 1.
Siste side: $x^2 + 1^2 = 2^2$ gir $x = \sqrt{3}$

Ergo $z = -\sqrt{3} + i$. $\boxed{A}$

Oppgave 4

$$(1-i)^2 - 2(1-i) + (1-2i) = (1-i)(1-i) - 2 + 2i + 1 - 2i$$
$$= 1 - 2i - 1 - 1 \neq 0 \quad \text{nix}$$

$$(-i)^2 + 2i + (1-2i) = -1 + 2i + 1 - 2i = 0 \quad \text{ok}$$

$$(2+i)^2 - 2(2+i) + (1-2i) = 4 + 4i - 1 - 4 - 2i + 1 - 2i = 0 \quad \text{ok}$$

Dvs. begge løsningene fra B) passer. $\boxed{B}$

Oppgave 5

$\boxed{B}$

Oppgave 6

$$\lim_{x\to\infty} \frac{\ln(4x^2+1)}{\ln(x+1)} \overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x\to\infty} \frac{\frac{1}{4x^2+1}\cdot 8x}{\frac{1}{x+1}\cdot 1}$$

$$= \lim_{x\to\infty} \frac{8x(x+1)}{4x^2+1} = \lim_{x\to\infty} \frac{8x^2+8x}{4x^2+1}$$

$$= \lim_{x\to\infty} \frac{8+\frac{8}{x} \to 0}{4+\frac{1}{x^2} \to 0} = 2$$

$\boxed{E}$

Oppgave 7

$$\lim_{x\to 0} \frac{(\sin x)^2}{5x+x^2} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0} \frac{2\sin x\cos x}{5+2x} = 0$$

$\boxed{C}$

Oppgave 8

$\boxed{D}$

Oppgave 9

Anta at følgen konvergerer mot et tall L. Lar vi $n\to\infty$ på begge sider av likningen

$$a_{n+1} = \sqrt[3]{(a_n^3+1)/2}$$

får vi da

$$L = \sqrt[3]{(L^3+1)/2}$$

$$L^3 = (L^3+1)/2$$

$$2L^3 = L^3+1$$

$$L^3 = 1, \quad \text{dvs. } L = 1$$

$\boxed{B}$

Oppgave 10

$$\lim_{n\to\infty} \frac{\cos n + (-2)^n}{e^n} = \lim_{n\to\infty} \frac{\frac{\cos n}{e^n} + \left(\frac{-2}{e}\right)^n}{1} = \frac{0+0}{1} = 0$$

deler på dominerende ledd, nemlig e^n

$\boxed{B}$

Oppgave 11

$\boxed{D}$

Oppgave 12

$$f'(x) = \cos(\sin x)\cdot\cos x + e^x \ln(2x+1) + e^x \cdot \frac{1}{2x+1}\cdot 2$$

$\boxed{B}$

Oppgave 13

$$x^2 : (x-1) = x+1$$
$$\frac{x^2 - x}{\quad x}$$
$$\frac{x-1}{\quad 1}$$

Ergo:

$$\frac{x^2}{x-1} = x+1+\frac{1}{x-1}$$

$\boxed{C}$

Oppgave 14

$$\lim_{x\to 0}(1-x)^{1/x} \overset{[1^\infty]}{=} \lim_{x\to 0}\left[e^{\ln(1-x)}\right]^{\frac{1}{x}} = \lim_{x\to 0} e^{\frac{\ln(1-x)}{x}}$$

Eksponenten:

$$\lim_{x\to 0}\frac{\ln(1-x)}{x} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0}\frac{\frac{1}{1-x}(-1)}{1} = -1$$

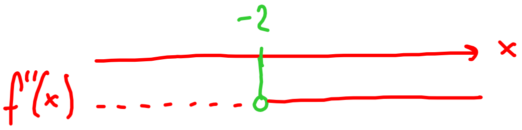
Ergo $\lim_{x\to 0}(1-x)^{1/x} = e^{-1}$.

$\boxed{D}$

Oppgave 15

$$f'(x) = e^x + xe^x$$

$$f''(x) = e^x + e^x + xe^x = e^x(2+x)$$



$\boxed{D}$

Oppgave 16

$$f'(x) = \frac{1}{a+\ln(b+\ln(c+x))} \cdot \frac{1}{b+\ln(c+x)} \cdot \frac{1}{c+x}$$

$\boxed{E}$

Oppgave 17

Middelverdisetningen: $\dfrac{f(b)-f(a)}{b-a} = f'(c)$

Dette gir $f(b) - f(a) = f'(c)\cdot(b-a)$

$$f(b) = f(a) + f'(c)\cdot(b-a)$$

$\boxed{C}$

Oppgave 18

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} e^{7/x} = e^0 = 1$$

$$b = \lim_{x\to\infty}\left[f(x) - ax\right] = \lim_{x\to\infty} xe^{7/x} - x \overset{[\infty-\infty]}{=} \lim_{x\to\infty} x\left(e^{7/x} - 1\right)$$

$$\overset{[\infty\cdot 0]}{=} \lim_{x\to\infty} \frac{e^{7/x} - 1}{\frac{1}{x}} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to\infty} \frac{e^{7/x}\cdot\left(\frac{-7}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = e^0 \cdot 7 = 7$$

Skråasymptote: $y = ax + b = x + 7$.

$\boxed{A}$

Oppgave 19

$z = 8e^{i\left(\frac{3\pi}{2}\right)}$ har prinsipal tredjerot

$$w_0 = \sqrt[3]{8}\, e^{i(\pi/2)} = 2e^{i(\pi/2)}$$

Denne er ikke blant alternativene. Vi finner neste rot ved å multiplisere w_0 med

$$w_+ = e^{i(2\pi/3)}$$

Da får vi

$$w_1 = 2e^{i(\pi/2)} e^{i(2\pi/3)} = 2e^{i\left(\frac{\pi}{2}+\frac{2\pi}{3}\right)}$$

$$= 2e^{i\left(\frac{3\pi}{6}+\frac{4\pi}{6}\right)} = 2e^{i(7\pi/6)}$$

$\boxed{A}$

Oppgave 20

$\boxed{A}$