1.

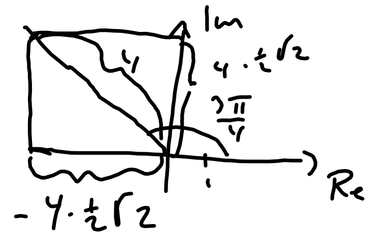
$$z = 4e^{i\frac{3\pi}{4}} \qquad \underline{r = 4} \quad \underline{\theta = \tfrac{3\pi}{4}}$$

$$= 4\left(\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right)$$

$$= 4\left(-\tfrac{1}{2}\sqrt{2} + i\,\tfrac{1}{2}\sqrt{2}\right)$$

$$= -2\sqrt{2} + i\,2\sqrt{2}.$$

e



2

$$z = (2 - 2i\sqrt{3})^2$$

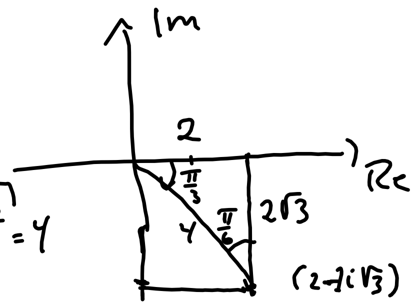
$$= \left(4e^{-i\frac{\pi}{3}}\right)^2$$

$$= 16e^{-i\frac{2\pi}{3}} \qquad \sqrt{2^2 + (2\sqrt{3})^2} = 4$$

$$\Rightarrow \underline{|z| = 16} \qquad \operatorname{Arg} z = \theta = -\tfrac{2\pi}{3} + 2k\pi$$

$$= \underline{\tfrac{4\pi}{3}}$$

e



3

$$z = \tfrac{1}{2}e^{i\frac{\pi}{4}} \qquad w = 2e^{i\frac{7\pi}{12}}$$

$$\underline{z\cdot w} = \tfrac{1}{2}e^{i\frac{\pi}{4}}\cdot 2e^{i\frac{7\pi}{12}} = e^{i\left(\frac{\pi}{4}+\frac{7\pi}{12}\right)} = e^{i\left(\frac{3\pi}{12}+\frac{7\pi}{12}\right)}$$

$$= e^{i\frac{10\pi}{12}} = \underline{e^{i\frac{5\pi}{6}}}$$

b

$$= \cos\tfrac{5\pi}{6} + i\sin\tfrac{5\pi}{6}$$

$$= \underline{-\tfrac{1}{2}\sqrt{3} + i\,\tfrac{1}{2}}$$

4

(d)

$$\lim_{n\to\infty} \frac{3n^3 + n\sqrt{n^4+1}}{\sqrt{n^4+4n^3}} \sim \frac{n^3}{n^2} = n \qquad (i^2 = -1) \qquad \sqrt{-1} = \pm i$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}\left(3n^3 + n\sqrt{n^4+1}\right)}{\frac{1}{n^2}\sqrt{n^4+4n^3}} = \lim_{n\to\infty} \frac{3n + n\sqrt{1+\frac{1}{n^4}}}{\underbrace{\sqrt{1+\frac{4}{n}}}_{\to 1}} = \underline{\infty}$$

5

(a)

$$\lim_{x\to 0} \frac{\overset{\to 0}{\sin x}}{\underbrace{\ln x^2 + 1}_{\to -\infty}} = 0. \qquad \lim_{x\to 0^+} \ln x = -\infty$$

$$\left(\frac{\sin x}{\ln(x^2+1)} \text{"="} \text{"}\frac{0}{0}\text{"} \right)$$

6

(c)

$$\lim_{x\to\frac{\pi}{2}} \frac{2\ln(\sin x)}{\cos x} \qquad \ln(\sin x) \xrightarrow[x\to\frac{\pi}{2}]{} 0 \qquad \cos x \xrightarrow[x\to\frac{\pi}{2}]{} 0$$

så $\frac{2\ln(\sin x)}{\cos x}$ er et "$\frac{0}{0}$" uttrykk når $x \to \frac{\pi}{2}$.

L'H:

$$\frac{(2\ln(\sin x))'}{(\cos x)'} = \frac{2\cdot\frac{1}{\sin x}\cdot\cos x}{-\sin x} = -2\,\frac{\cos x}{\sin^2 x} \xrightarrow[x\to\frac{\pi}{2}]{} 0$$

så $\lim_{x\to\frac{\pi}{2}} \frac{2\ln(\sin x)}{\cos x} = \underline{0}$.

7

(c)

$$f(x) = \ln\left(\sqrt{x^2+1} - x\right)$$

$$f'(x) = \frac{1}{\sqrt{x^2+1} - x} \cdot \left(\frac{x}{\sqrt{x^2+1}} - 1 \right) = \frac{1}{\sqrt{x^2+1} - x}\left(x - \sqrt{x^2+1}\right)\frac{1}{\sqrt{x^2+1}}$$

$$= -\frac{1}{\sqrt{x^2+1}}.$$

8

(d)

$y = f(x) = \ln(x^3 - 1)$

$x = g(y)$, så løser med hensyn på x

$$y = \ln(x^3-1)$$
$$e^y = x^3 - 1$$
$$e^y + 1 = x^3$$
$$x = (e^y+1)^{\frac{1}{3}} = g(y)$$
$$\underline{g(x) = (e^x+1)^{\frac{1}{3}}}$$

9 $\quad y = f(x) = e^{(2x+1)} \qquad g'(e) \stackrel{?}{=}$

$x = g(y)$

ⓐ $\quad g'(y) = \dfrac{1}{f'(x)} \qquad \underline{y = e} \Rightarrow e = e^{(2x+1)}$

$\Rightarrow \underline{x = 0}$

$g'(e) = \dfrac{1}{f'(0)} = \underline{\dfrac{1}{2e}} \qquad f'(x) = 2 \cdot e^{(2x+1)}$

$f'(0) = 2 \cdot e^1 = 2e$

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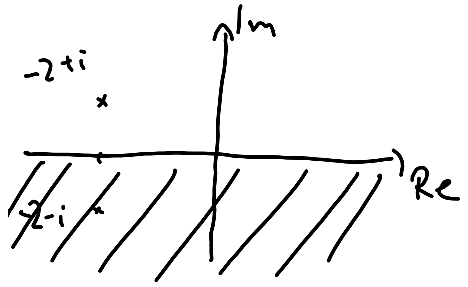
10 $\quad \{ z \mid \underline{|z + 2 + i| \leq |z + 2 - i|} \} \subseteq \mathbb{C}$



$|z + 2 + i|$

$\|$

$|z - (-2 - i)|$

$|z + 2 - i| = |z - (-2 + i)|$

ⓔ. $\quad$ z ligger nærmere $-2-i$ enn $-2+i$

z ligger ikke over den reelle aksen.

11. $i+1$ er ei fjerderot av z, da er ei anna fjerderot til z lik:

$\sqrt{2}$

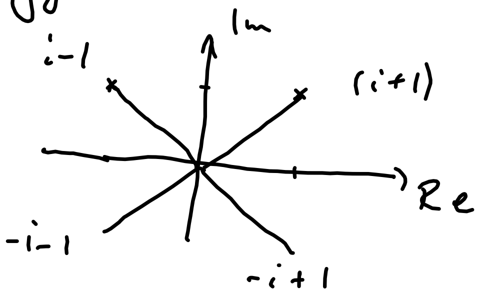
$-\sqrt{2}$

$i\sqrt{2}$

$\boxed{1-i}$

$-i\sqrt{2}$

d



$\underline{w^4 = z} = re^{i\theta}$

$\Rightarrow w = i+1$

eller ...

$w = r^{\frac{1}{4}} e^{i\frac{\theta}{4} + \frac{2k\pi}{4}}$

$k = 0,1,2,3$

$$w = \underline{r^{\frac{1}{4}} e^{i\frac{\theta}{4}}} \cdot e^{i\frac{2k\pi}{4}}, \qquad e^{i\frac{2k\pi}{4}} = e^{ik\frac{\pi}{2}}$$

$k = 0,1,2,3$

12.

$$f(x) = \begin{cases} \dfrac{\sin ax}{x} & x > 0 \\ \cos \pi x - b & -1 \le x \le 0 \\ e^{(x^2-1)} & x < -1 \end{cases}$$

Finn a, b s.a. f er kontinuerlig.

Vi ha

a

$\underline{x=0}:$ $\longrightarrow$ $\displaystyle\lim_{x\to 0^+} \frac{\sin ax}{x} = \lim_{x\to 0^-} (\cos \pi x - b)$

$a = 1 - b \Rightarrow a = 3.$

$\underline{x=-1}:$ $\longrightarrow$ $\displaystyle\lim_{x\to -1^+} \cos \pi x - b = \lim_{x\to -1^-} e^{x^2-1}$

$-1 - b = 1 \Rightarrow \underline{b = -2}$

13.

$h(x) = f(x)\ln(f(x))$ $\qquad f(x) > 0.$

b

$$h'(x) = f'(x)\cdot\ln(f(x)) + f(x)\cdot\ln(f(x))'$$
$$= f'(x)\ln(f(x)) + f(x)\frac{1}{f(x)}\cdot f'(x)$$
$$= f'(x)\left(\ln(f(x)) + 1\right)$$

14

$f:[a,b]\to\mathbb{R}$ kont. på $[a,b]$ og deriverbar på (a,b)

$f(a) = -2f(b)$

det fins et punkt $c\in(a,b)$ s.a.

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(b)+2f(b)}{b-a} = \frac{3f(b)}{b-a}$$

d

$$3f(b) = f'(c)(b-a)$$

det fins $c\in[a,b]$ $\quad f(c)=0 \qquad f(a) = -2f(b)$

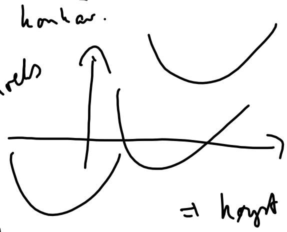
15 P er et fjerdegradspolynom

P'' har ei dobbel rot. P'' er et andregradspolynom.

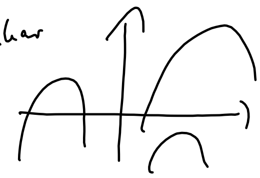
Hvor mange røtter har P?

P konveks
$\Leftarrow$ så $P'' \geq 0$ for alle x
eller $P'' \leq 0$ for alle x
$\Leftarrow$
P konkav.

konveks

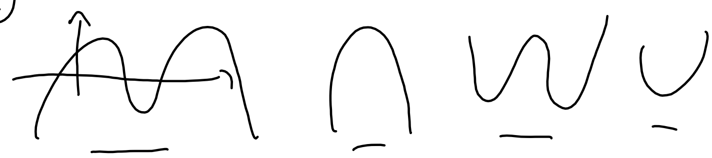


konkav



$\Rightarrow$ høyst 2 røtter

(c)



16 $f(x) = x e^{-x^2}$
$x \in [-1,1]$

$f'(x) = x\cdot(-2x)\cdot e^{-x^2} + 1\, e^{-x^2}$
$= (-2x^2+1)e^{-x^2}$
$= 0$ når $x = \pm\frac{1}{\sqrt{2}}$

(b) maximum når $x > 0$:

$x = \frac{1}{\sqrt{2}}$. $f(\frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}} \cdot \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{2e}}$

$x = 1$ $f(1) = e^{-1} = \frac{1}{e}$

globalt max i $\frac{1}{\sqrt{2}}$. $2 < e \Rightarrow \frac{1}{\sqrt{2e}} > \frac{1}{e}$.

17 $\lim_{x \to \frac{\pi}{2}^+} \cos x \ln(2x - \pi)$ "$0 \cdot -\infty$"

l'H X $\frac{(\ln(2x-\pi))'}{(\frac{1}{\cos x})'} = \frac{\frac{2}{2x-\pi}}{\frac{-\sin x}{\cos^2 x}} = -\frac{2\cos^2 x}{(2x-\pi)\sin x}$ $x \to \frac{\pi}{2}$

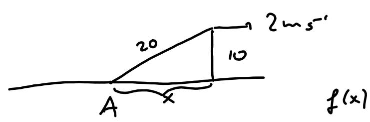
l'H $\frac{(\cos^2 x)'}{(2x-\pi)'} = \frac{-2\cos x \sin x}{2} \to 0$ $x \to \frac{\pi}{2}$

(b)

$\Rightarrow X \to 0$ $x \to \frac{\pi}{2}$

$\Rightarrow \cos x \ln(2\pi - x) \to 0$ $x \to \frac{\pi}{2}$

18



$f(x)$ = lengden til mora

(d) $f(x(t)) = \sqrt{10^2 + x(t)^2}$

$\frac{d}{dt} f(x) = f'(x(t)) = \frac{x(t)}{\sqrt{10^2 + x^2(t)}} \cdot x'(t)$ $x'(t) = 2$

$f(x) = 20$
$\sqrt{x^2+10^2} = 400$
$x = 10\sqrt{3}$

$f'(t_0) = \frac{10\sqrt{3}}{\sqrt{10^2 + (10\sqrt{3})^2}} \cdot 2 = \frac{10\sqrt{3}}{20} \cdot 2 = \sqrt{3}$