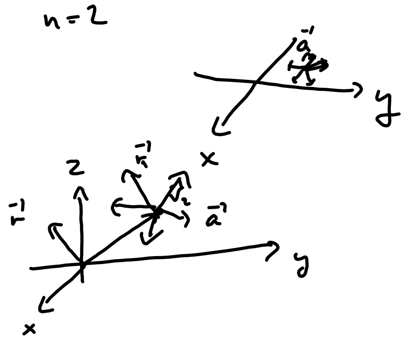
2.4 Derivasjon av skalarfelt

$f: \mathbb{R}^n \to \mathbb{R}$ $\qquad (\vec{F}: \mathbb{R}^n \to \mathbb{R}^m)$

$\vec{a} \in \mathbb{R}^n$ punkt $\qquad n=2$

$\vec{r} \in \mathbb{R}^n$ retning



~~Retningsderivert~~

til f i $\vec{a}$ i retningen $\vec{r}$:

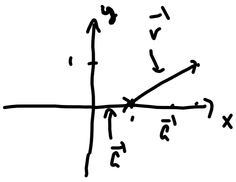
$$f'(\vec{a};\vec{r}) = \lim_{h\to 0} \frac{f(\vec{a}+h\vec{r}) - f(\vec{a})}{h}$$

retningsderiverte til f i retning $\vec{r}$ i $\vec{a}$.

$\vec{a} + h\vec{r} = (1+2h, h)$

$f: \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = x^2 + xy \qquad \vec{a} = (1,0) \qquad \vec{r} = (2,1)$



$$\to f'\left(\vec{a}; \frac{\vec{r}}{|\vec{r}|}\right) = \lim_{h\to 0} \frac{f\left(\vec{a} + h\frac{\vec{r}}{\sqrt{5}}\right) - f(\vec{a})}{h}$$

$$= \lim_{h\to 0} \frac{f\left(\left(1+\frac{2h}{\sqrt{5}}, 0+\frac{1}{\sqrt{5}}h\right)\right) - f(1,0)}{h}$$

$|\vec{r}| = \sqrt{2^2+1^2} = \sqrt{5}$

$$= \lim_{h\to 0} \frac{\left(1+\frac{2h}{\sqrt{5}}\right)^2 + \left(1+\frac{2h}{\sqrt{5}}\right)h - 1}{h}$$

$\frac{\vec{r}}{|\vec{r}|}$

$$= \lim_{h\to 0} \frac{1 + \frac{4}{\sqrt{5}}h + \frac{4}{5}h^2 + \frac{h}{\sqrt{5}} + 2\frac{h^2}{5} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{5}}5h + 6h^2\frac{1}{5}}{h}$$

r

$$= \lim_{h\to 0} \frac{5}{\sqrt{5}} + \frac{6h}{5} = \sqrt{5}$$

$f'(\vec{a};\vec{r})\cdot h = 5\cdot h$

$f : \mathbb{R}^n \to \mathbb{R}$

$\vec{a} \in \mathbb{R}^n$ $\vec{a} = (a_1, a_2, \quad, a_n)$

$\vec{e_i} = (0, 0, \ldots, 1, 0, \quad 0)$

$\uparrow$ i-t plass

Def: $f'(\vec{a}; \vec{e_i}) = \frac{\partial f}{\partial x_i}(\vec{a}) = \lim_{h \to 0} \frac{f(\vec{a} + h\vec{e_i}) - f(\vec{a})}{h}$

$$= \lim_{h \to 0} \frac{f(a_1 + h\cdot 0, a_2 + h\cdot 0, \ldots a_i + h\cdot 1, \ldots a_n + h\cdot 0) - f(\vec{a})}{h}$$

Den retningsderiverte i retning $\vec{e_i}$:

$$\frac{\partial f}{\partial x_i}(\vec{a}) = f'(\vec{a}, \vec{e_i}) = \lim_{h \to 0} \frac{f(\vec{a} + h\vec{e_i}) - f(\vec{a})}{h}$$

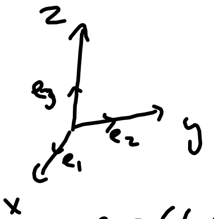
retningsderivert i retning $\vec{r} = (r_1, r_2, r_3, \ldots, r_n)$

$$\lim_{t \to 0} \frac{f(\vec{a} + t\boxed{\vec{r}}) - f(\vec{a})}{t} = \frac{1}{|\vec{r}|}\left(r_1 \frac{\partial f}{\partial x_1}(\vec{a}), + \quad + , r_n \frac{\partial f}{\partial x_n}(\vec{a})\right)$$

Gradienten:

$$\nabla f(\vec{a}) = \left(\frac{\partial f}{\partial x_1}(\vec{a}), \frac{\partial f}{\partial x_2}(\vec{a}), \quad , \frac{\partial f}{\partial x_n}(\vec{a}) \right)$$

eks $\quad f(x,y,z) = x^2 y e^{yz}$



$\vec{a} = (1, -2, 0)$ $\qquad e_1 = (1,0,0)$

$$\frac{\partial f}{\partial x}(\vec{a}) = f'(\vec{a}, \vec{e}_1) = 2x \cdot ye^{yz}\Big|_{\vec{a}} = 2 \cdot 1 \cdot (-2) e^{-2\cdot 0} = -4e^0 = -4$$

$$\frac{\partial f}{\partial y}(\vec{a}) = f'(\vec{a}, \vec{e}_2) = x^2 e^{yz} + x^2 y \cdot e^{yz} \cdot z\Big|_{\vec{a}} = 1 \cdot e^{-2\cdot 0} + 1^2 \cdot (-2) \cdot e^{-2\cdot 0} \cdot 0 = 1$$

$$\frac{\partial f}{\partial z}(\vec{a}) = f'(\vec{a}, \vec{e}_3) = x^2 y \; y \cdot e^{yz}\Big|_{\vec{a}} = 1^2 (-2) \cdot (-2) \cdot e^{-2\cdot 0} = 4$$

$$\nabla f(1,-2,0) = (-4, 1, 4)$$

$\frac{\partial f}{\partial x}(\vec{a}) = f'(\vec{a}, \vec{e}_1) = -4 \quad \frac{\partial f}{\partial y}(\vec{a}) = f'(\vec{a}, \vec{e}_2) = 1 \quad \frac{\partial f}{\partial z}(\vec{a}) = f'(\vec{a}, \vec{e}_3) = 4$

$\boxed{\vec{v} = (1, 2, 3)}$

$$f'(\vec{a}, \vec{v}) = f'(\vec{a}, 1\vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3) = 1 \cdot \frac{\partial f}{\partial x}(\vec{a}) + 2\frac{\partial f}{\partial y}(\vec{a}) + 3\frac{\partial f}{\partial z}(\vec{a})$$

$$= 1 \cdot -4 + 2 \cdot 1 + 3 \cdot 4 = 10$$

$$f'\left(\vec{a}, \frac{\vec{v}}{|\vec{v}|}\right) = \frac{1}{|\vec{v}|}\left(\quad 10 \quad \right) = \frac{10}{\sqrt{14}}$$

$|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$$\vec{v} = v_1 \vec{e}_1 + v_2 \vec{e}_2 + v_3 \vec{e}_3$$

$$f'(\vec{a}, \vec{v}) = v_1 f'(\vec{a}, \vec{e}_1) + v_2 f'(\vec{a}, \vec{e}_2) + v_3 f'(\vec{a}, \vec{e}_3)$$

$$= v_1 \frac{\partial f}{\partial x}(\vec{a}) + v_2 \frac{\partial f}{\partial y}(\vec{a}) + v_3 \frac{\partial f}{\partial z}(\vec{a}).$$

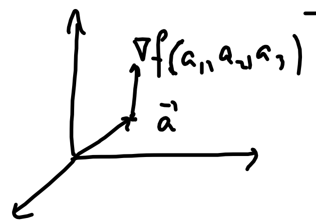
$$f'\left(\vec{a}, \frac{\vec{v}}{|\vec{v}|}\right) = \frac{1}{|\vec{v}|}\left(v_1 \frac{\partial f}{\partial x}(\vec{a}) + \quad + v_3 \frac{\partial f}{\partial z}(\vec{a}) \right)$$

$$f(x,y,z) \; : \; \mathbb{R}^3 \to \mathbb{R}$$

$$\underbrace{\nabla f(x,y,z)} = \left(\frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z) \right)$$

$$\vec{a} \in \mathbb{R}^3 \to \underline{\nabla f(a_1, a_2, a_3)} = \left(\frac{\partial f}{\partial x}(\vec{a}), \frac{\partial f}{\partial y}(\vec{a}), \frac{\partial f}{\partial z}(\vec{a}) \right) \in \mathbb{R}^3$$

$$\vec{a} = (a_1, a_2, a_3)$$



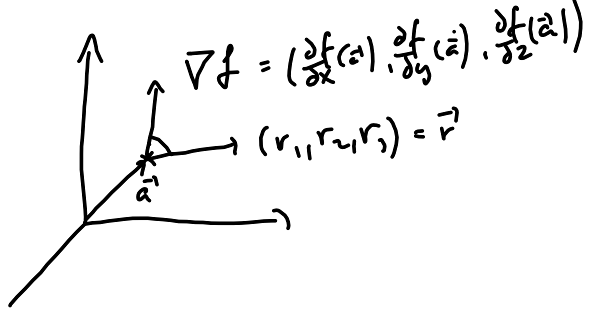
$$|\vec{r}| = 1 \qquad f'(\vec{a}; \vec{r}) = f'(\vec{a}; r_1\vec{e}_1 + r_2\vec{e}_2 + r_3\vec{e}_3)$$

$$= r_1 f'(\vec{a}, \vec{e}_1) + r_2 f'(\vec{a}, \vec{e}_2) + r_3 f'(\vec{a}, e_3)$$

$$= r_1 \frac{\partial f}{\partial x}(\vec{a}) + r_2 \frac{\partial f}{\partial y}(\vec{a}) + r_3 \frac{\partial f}{\partial z}(\vec{a})$$

$$= (r_1, r_2, r_3) \cdot \left(\frac{\partial f}{\partial x}(\vec{a}), \frac{\partial f}{\partial y}(\vec{a}), \frac{\partial f}{\partial z}\vec{a} \right)$$

$$= (r_1, r_2, r_3) \cdot \nabla f = \vec{r} \cdot \nabla f$$



retningsderiverte

til f i retning $\vec{r}$

i $\vec{a}$