Repetisjon de neste to ukene:

Forelesninger: 18., 20., 22., 25. og 29. november

Gruppearbeid: 18. – 22. november

Felles orakel 18., 20., 22., 25., 27., 29. november
(12 – 14)

"panikkhjelp / orakel" 29. november 10 – 16 Aud 1.

→ mest gjennomgang av tidligere eksamensoppgaver.

Derivasjon av vektorfunksjoner.

$$\vec{F} : \mathbb{R}^n \to \mathbb{R}^m$$

$$\vec{F} = \begin{pmatrix} F_1(x_1, \dots, x_n) \\ F_2(x_1, \dots, x_n) \\ \vdots \\ F_m(x_1, \dots, x_n) \end{pmatrix}$$

Hver F_i er et skalarfelt : $F_i : \mathbb{R}^n \to \mathbb{R}$

Hva er $\vec{F}'$? Jacobimatrisen

$$\vec{F}' = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \cdots & \cdots & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial F_m}{\partial x_1} & \cdots & \cdots & \frac{\partial F_m}{\partial x_n} \end{pmatrix}^{m,n}$$

$$F_i' = \left(\frac{\partial F_i}{\partial x_1}, \dots, \frac{\partial F_i}{\partial x_n}\right)^{1,n} = \nabla F_i$$

gradienten til F_i

$f : \mathbb{R} \to \mathbb{R}$ $\quad f'(x) = \frac{d}{dx} f \quad$ "Jacobimatrisen til f."

<u>eksempel</u>

$$\vec{F} : \mathbb{R}^2 \to \mathbb{R}^3$$

$$\vec{F}(x,y) = \begin{pmatrix} xy^3 \\ e^{x+y^2} \\ 3x^2y \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

$$\vec{F}\,' = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} \end{pmatrix}^{3,2} = \begin{pmatrix} y^3 & 3xy^2 \\ e^{x+y^2} & 2ye^{x+y^2} \\ 6xy & 3x^2 \end{pmatrix}$$

Jacobi

eller "den deriverte" til $\vec{F}$.

f er deriverbar i a, dersom: $\left\{ \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ eksisterer} \right\}$ $= f'(a)$

F, skalarfelt.
er derivertbart i $\vec{a}$ dersom: $\boxed{\left\{ \lim_{\vec{r} \to \vec{0}} \frac{F(\vec{a} + \vec{r}) - F(\vec{a}) - \nabla F \cdot \vec{r}}{|\vec{r}|} = 0 \right\}}$

$$\lim_{\vec{r} \to \vec{0}} \frac{F(\vec{a} + \vec{r}) - F(\vec{a})}{|\vec{r}|} = \lim_{\vec{r} \to \vec{0}} \frac{\nabla F \cdot \vec{r}}{|\vec{r}|}$$

$\vec{F}$ vektorfelt $\quad \vec{F} : \mathbb{R}^n \to \mathbb{R}^m$

$$\left. \begin{array}{l} \vec{F}' = \left(\frac{\partial F_i}{\partial x_j} \right)^{m,n} \\ \vec{r} \in \mathbb{R}^n \qquad \vec{r} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} \end{array} \right\}.$$

$$\vec{F}' \cdot \vec{r} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial F_m}{\partial x_1} & \cdots & \vdots \end{pmatrix}^{m,n} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}^{n,1} = \begin{pmatrix} r_1 \cdot \frac{\partial F_1}{\partial x_1} + r_2 \frac{\partial F_1}{\partial x_2} + \cdots + r_n \frac{\partial F_1}{\partial x_n} \\ \vdots \\ \vdots \end{pmatrix}$$

$$= \begin{pmatrix} \nabla F_1 \cdot \vec{r} \\ \vdots \\ \nabla F_m \cdot \vec{r} \end{pmatrix}$$

$\vec{F} : \mathbb{R}^n \to \mathbb{R}^m$

$\vec{F}$ er deriverbar i $\vec{a}$

dersom
$$\lim_{\vec{r} \to (0,0,\ldots,0)} \frac{\vec{F}(\vec{a} + \vec{r}) - \vec{F}(\vec{a}) - \vec{F}' \cdot \vec{r}}{|\vec{r}|} = \vec{0} \quad !$$

Jacobimatrisen.

elementene i teller er $(m \times 1)$-matriser eller vektorer med m koordinater.
i nevner er et reelt tall!

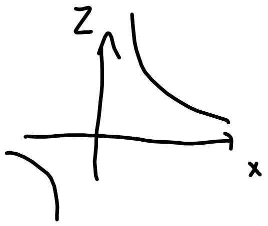
Setning: $\vec{F}$ er deriverbar i $\vec{a}$ dersom $\vec{F}$ er definert i en omegn om $\vec{a}$, en liten ball med sentrum i $\vec{a}$, og alle partiellderiverte i Jacobimatrisen er definert i en omegn om $\vec{a}$ og er kontinuerlige i $\vec{a}$.

eksempel

$$\vec{F}(x,y,z) = \begin{pmatrix} y \sin z \\ x \ln(y^2-1) \\ \tan(xz) \end{pmatrix} \qquad \vec{F}: \mathbb{R}^3 \to \mathbb{R}^3$$

$$y^2 - 1 > 0$$

$$xz \neq \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}.$$



$$xz = \frac{\pi}{2}$$

$$x = \frac{\pi}{2z}$$

$$\vec{F}: A \to \mathbb{R}^3$$

$$A = \left\{ (a_1, a_2, a_3) \in \mathbb{R}^3 \;\middle|\; a_2^2 - 1 > 0,\ a_1 a_3 \neq \frac{\pi}{2} + k\pi \right\}$$

$$\vec{F} = \begin{pmatrix} y \sin z \\ x \ln(y^2-1) \\ \tan(xz) \end{pmatrix} \qquad \vec{F}' = \begin{pmatrix} 0 & \sin z & y \cos z \\ \ln(y^2-1) & \frac{2xy}{y^2-1} & 0 \\ \frac{z}{\cos^2(xz)} & 0 & \frac{x}{\cos^2(xz)} \end{pmatrix}$$

$y^2 - 1 > 0$

$xz \neq \frac{\pi}{2} + k\pi$

$\vec{F}$ er deriverbar i alle $\vec{a}$ der $\vec{F}$ er definert.

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$\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

$$\vec{F}(x,y) = \begin{pmatrix} e^{x^2+y} - 1 \\ x^2+y \end{pmatrix}$$

$$\vec{F}'(x,y) = \begin{pmatrix} 2xe^{x^2+y} & e^{x^2+y} \\ 2x & 1 \end{pmatrix}$$

$$\vec{F}' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\vec{F}(x,y) = \begin{pmatrix} e^{x^2+y} - 1 \\ x^2+y \end{pmatrix}$$

Finn en 2×2 matrise M s.a.

$$\lim_{(x,y)\to(0,0)} \frac{\vec{F}(x,y) - M\cdot\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} = \vec{0}.$$

$$\frac{\vec{F}(x,y) - \vec{F}(0,0) - ?\cdot\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{\vec{F}((0,0)+(x,y)) - \vec{F}(0,0) - \vec{F}'(0,0)\begin{pmatrix} x \\ y \end{pmatrix}}{|(x,y)|} \overset{?}{=} \vec{0}$$

Likheten gjelder dersom $\vec{F}$ er deriverbar i $(0,0)$.

Siden alle leddene i Jacobimatrisen til $\vec{F}$ er kontinuerlige i $(0,0)$, så er $\vec{F}$ deriverbar i $(0,0)$ og $\vec{F}'(0,0)$ oppfyller kravet i oppgaven.

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \xleftarrow{(x,y)\to(0,0)} \frac{\vec{F}(x,y) - \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} = \frac{\begin{pmatrix} e^{x^2+y} - 1 \\ x^2+y \end{pmatrix} - \begin{pmatrix} y \\ y \end{pmatrix}}{\sqrt{x^2+y^2}}$$

$$= \begin{pmatrix} e^{x^2+y} - 1 - y \\ x^2 \end{pmatrix}\frac{1}{\sqrt{x^2+y^2}}$$

$$\boxed{M = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} \dfrac{e^{x^2+y} - 1 - y}{\sqrt{x^2+y^2}} \\ \dfrac{x^2}{\sqrt{x^2+y^2}} \end{pmatrix}$$