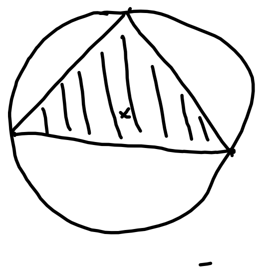
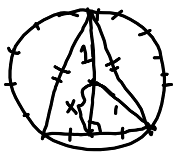
Anvendelser av kurvedrøfting.

- maksimumsproblemer
- koblede hastigheter.



Hvilken av de innskrevne trekantene har størst areal?

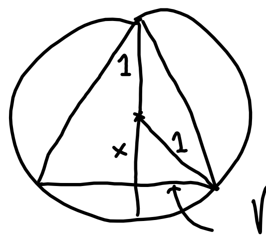
- Den likesidede 
- En trekant med en side gjennom sentrum. 



$A = gh \cdot \frac{1}{2}$

Kan anta at trekanten er likebeint.

Anta at radius = 1.



$g = 2\sqrt{1-x^2}$ $\quad h = 1+x$

$$\underline{A} = \frac{1}{2} g \cdot h$$
$$= \underline{\frac{1}{2} 2\sqrt{1-x^2} \cdot (x+1)} \qquad \underline{0 \leq x \leq 1}$$

$$A' = \left(\sqrt{1-x^2}\,(x+1)\right)'$$
$$= \frac{-x}{\sqrt{1-x^2}}(x+1) + \sqrt{1-x^2} \cdot 1$$
$$= \frac{(-x^2-x) + 1-x^2}{\sqrt{1-x^2}}$$
$$= \frac{-2x^2-x+1}{\sqrt{1-x^2}} = 0$$

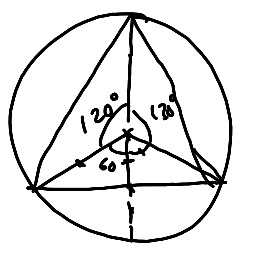
$$\Rightarrow \quad 2x^2 + x - 1 = 0$$
$$x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$

Av løsningene er bare $x = \frac{1}{2}$ relevant.

$$x = \frac{1}{2} \Rightarrow A = \sqrt{1-x^2} \cdot (1+x) = \sqrt{\frac{3}{4}} \cdot \frac{3}{2} = \frac{3\sqrt{3}}{4} \approx \frac{5}{4} > 1$$

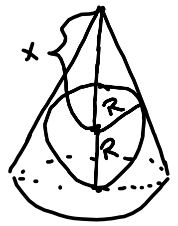
$$A_{maks} = \frac{3\sqrt{3}}{4} \quad \text{når} \quad x = \frac{1}{2}$$

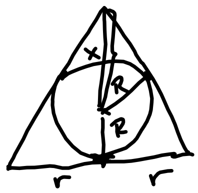
og trekanten er likesidet!



(2)

Hva er den (største) rette kjeglen som er omskrevet om ei kule?
minste

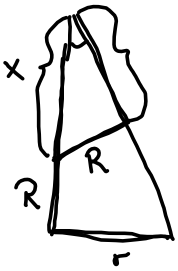




Finn x slik at volumet til kjeglen er størst.

høyden h.

$$V(x) = \frac{1}{3} G \cdot h = \frac{1}{3} \pi r^2 \cdot (R + x)$$



$$\frac{r}{R} = \frac{x+R}{\sqrt{x^2 - R^2}} \quad \text{(formlikhet)}$$

$$r = \frac{x+R}{\sqrt{x^2-R^2}} \cdot R$$

$\longrightarrow$ så

$x^2 - R^2 = (x-R)(x+R)$

$$V(x) = \frac{1}{3}\pi \frac{(x+R)^2}{x^2 - R^2} \cdot R^2 \cdot (R + x) = \frac{\pi R^2}{3} \cdot \frac{(x+R)^2}{x-R}$$

$$V'(x) = \frac{\pi R^2}{3} \frac{2(x+R)(x-R) - (x+R)^2}{(x-R)^2}$$

$$= \frac{\pi R^2}{3} \frac{2x^2 - 2R^2 - x^2 - 2Rx - R^2}{(x-R)^2}$$

$$= \frac{\pi R^2}{3} \frac{x^2 - 2Rx - 3R^2}{(x-R)^2} = 0$$

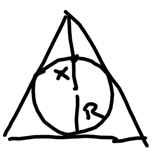
$$x^2 - 2Rx - 3R^2 = 0$$

$$x = \frac{2R \pm \sqrt{4R^2 + 12R^2}}{2}$$

$$= R \pm 2R = \begin{cases} 3R \\ -R \end{cases}$$

Av disse er bare $x = 3R$ relevant.

$$V(x) = \frac{\pi R^2}{3} \frac{(x+R)^2}{x-R} \qquad R < x$$

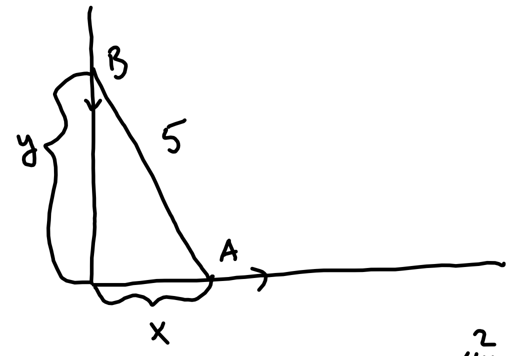


Vi får ekstremverdi når $x = 3R$

$V(x)$ har et minimum når $x = 3R$.

$V(x)$ har ikke noe maksimum.

koblede hastigheter:



A og B beveger seg samtidig, men uavhengig med forskjellig hastighet.

x, y er funksjoner av tiden t.

$$x^2(t) + y^2(t) = 25 \qquad (*)$$

deriverer på begge sider m.h.p t.

$$\frac{d}{dt} x^2(t) + \frac{d}{dt} y^2(t) = \frac{d}{dt} 25$$

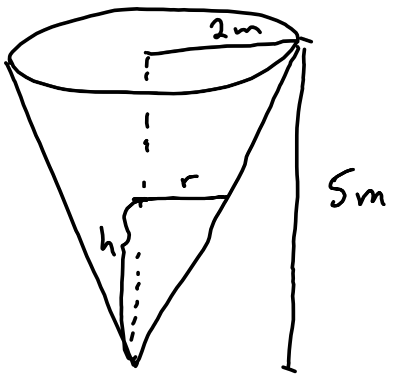
$$2x(t) \cdot x'(t) + 2y(t) \cdot y'(t) = 0$$

$$x'(t) = -\frac{y(t)\, y'(t)}{x(t)}$$

$$y'(t) = -\frac{x(t)\, x'(t)}{y(t)} \leftarrow$$

Hva er $y'(t)$ når $x = 4$ og $x'(t) = \underline{2}$

$x = 4 \Rightarrow \underline{y} = \sqrt{25 - x^2} = \underline{3}$. $\quad y'(t) = -\dfrac{4 \cdot 2}{3} = \underline{\underline{-\dfrac{8}{3}}}$



$0.1 \ m^3/min$ vann.

$$\frac{r}{h} = \frac{2}{5}$$

$$r = \frac{2}{5} \cdot h$$

$$V(h) = \frac{1}{3} G \cdot h = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \frac{4}{25} \cdot h^2 \cdot h$$

$$= \frac{4\pi}{75} h^3$$

Vil finne $h'(t)$!

$$V(h(t)) = \frac{4\pi}{75} h^3(t)$$

$$V(h(t)) = \frac{4\pi}{75} h^3(t)$$

$$\frac{d}{dt} V(h(t)) = \frac{d}{dt}\left(\frac{4\pi}{75} h^3(t)\right)$$

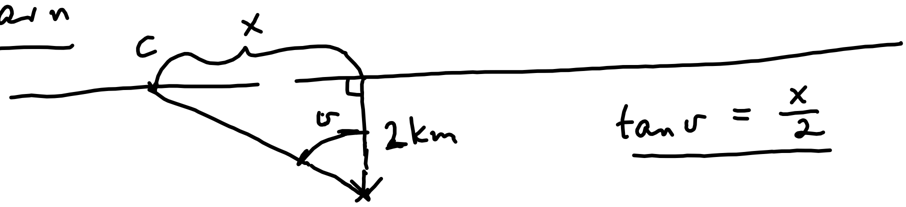
$$0.1 \ m^3/min = V'(h) \cdot h'(t) = \frac{4\pi}{75} \cdot 3 \cdot h(t)^2 \cdot h'(t).$$

$$= \frac{4\pi}{25} \cdot h^2 \cdot h'$$

$$h' = \frac{25}{4\pi \cdot h^2} \cdot 0.1 \ m^3/min = \frac{25}{36 \cdot \pi} \cdot 0.1 \ m/min$$

$$h = 3m \quad = 0.022 \ m/min.$$

Fyrtårn



$\underline{\tan v = \frac{x}{2}}$

Lyset fra fyrtårnet roterer med en hastighet på 2 min pr omdreining : $\underline{v'(t)} = \frac{2\pi}{2} = \underline{\pi}$

Vil finne hastigheten til c på bergveggen.
c = der lysstrålen treffer.

Til dette trenger en relasjon mellom x og v.
x, v er funksjoner i t!

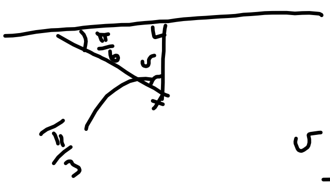
$$\boxed{\tan v(t) = \frac{x(t)}{2} \qquad (*)}$$

$$\Rightarrow \frac{d}{dt} \tan v(t) = \frac{d}{dt} \frac{x(t)}{2} \qquad \underline{v'(t) = \pi}$$

$$(1 + \tan^2 v) \cdot v'(t) = \frac{1}{2} \cdot x'(t)$$

$$\underline{\underline{x'(t)}} = 2(1 + \tan^2 v)\, v'(t)$$

$$= 2(1 + \tan^2 v) \cdot \pi$$



$\underline{v = \frac{\pi}{3}}$

$$= 2\left(1 + \tan^2 \tfrac{\pi}{3}\right) \cdot \pi = 2(1 + \sqrt{3}^2) \cdot \pi$$

$$= \underline{\underline{8 \cdot \pi}} \text{ km/min.}$$