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- forelesninger (passiv)
- gruppeundervisning (aktiv)

  smågruppe / gruble gruppe / felles orakel
- obligatoriske oppgaver
  - skriftlig
  - muntlig
- eksamen
  - midtveis
  - slutt

Komplekse tall

Tredjegradslikning:

$$x^3 = px + q \qquad p, q > 0$$

har en positiv rot x_0 :

Cardano $\quad x_0 = \sqrt[3]{D + \frac{q}{2}} - \sqrt[3]{D - \frac{q}{2}}$

der $D = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}$.

Eksempel. 1. $\quad x^3 = 3x + 2 \qquad x_{-1} = -1, \quad \underline{x_0 = 2}$

$$D = \sqrt{\left(\frac{2}{2}\right)^2 - \left(\frac{3}{3}\right)^3} = \sqrt{1-1} = 0$$

$$x_0 = \sqrt[3]{0 + \frac{2}{2}} - \sqrt[3]{0 - \frac{2}{2}} = \sqrt[3]{1} - \sqrt[3]{-1}$$

$$= 1 - (-1) = \underline{2}$$

2. $\quad x^3 = 15x + 4 \qquad p = 15 \quad q = 4$

$$D = \sqrt{\left(\frac{4}{2}\right)^2 - \left(\frac{15}{3}\right)^3} = \sqrt{4 - 125} = \sqrt{-121}$$

$$x_0 = \sqrt[3]{\sqrt{-121} + 2} - \sqrt[3]{\sqrt{-121} - 2}$$

$$\sqrt{-121} = \sqrt{-1 \cdot 11^2} = \sqrt{-1} \cdot 11 = 11i \qquad i = \sqrt{-1}, \quad \underline{i^2 = -1}$$

$$x_0 = \sqrt[3]{11i + 2} - \sqrt[3]{11i - 2}$$

tips: $\sqrt[3]{11i+2} = i + 2 \qquad \sqrt[3]{11i-2} = i - 2$

$$\underline{(i+2)^3} = i^3 + 3i^2 \cdot 2 + 3i \cdot 2^2 + 2^3$$

$$= i^2 \cdot i + 3(-1) \cdot 2 + 3i \cdot 4 + 8$$

$$= -i - 6 + 12i + 8 = \underline{11i + 2}$$

$$\underline{(i-2)^3} = (i-2)(i-2)^2 = (i-2)(i^2 - 4i + 4)$$

$$= (i-2)(3-4i) = (-2 \cdot 3 + i \cdot (-4i) + 3i + 2 \cdot 4i)$$

$$= (-6 + 4 + 11i) = \underline{11i - 2}$$

$$\underline{x_0} = \sqrt[3]{11i+2} - \sqrt[3]{11i-2} = i + 2 - (i-2) = \underline{\underline{4}}$$

Regneregler:

$z = a + ib$ a, b reelle tall

kalles et komplekst tall.

$Re(z) = a$ kalles realdelen

$Im(z) = b$ kalles imaginærdelen

$z = a + ib \qquad w = c + id$

$(w+z =)\ z + w = a + ib + c + id$
$= (a+c) + i(b+d)$

$(w \cdot z =)\ z \cdot w = (a+ib)\cdot(c+id)$
$= a\cdot c + a\cdot i\cdot d + i\cdot b\cdot c + \underbrace{i\cdot i}_{i^2=-1}\cdot b\cdot d$
$= (a\cdot c - bd) + i(ad + bc)$

$w \neq 0 \qquad \frac{z}{w} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \frac{ac - ibd + i(bc - iad)}{c^2 - (id)^2}$

$= \frac{ac + bd + i(bc - ad)}{c^2 + d^2} \qquad i^2 d^2 = -d^2$

$= \frac{ac+bd}{c^2+d^2} + i\,\frac{bc - ad}{b^2+d^2}$

$z = a + ib$

$z \neq 0 \quad \frac{1}{z} = \frac{1}{a+ib} = \frac{(a-ib)}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2}$

$= \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$

<u>men</u>!

$\underline{1} = \sqrt{1 \cdot 1} = \sqrt{(-1)\cdot(-1)} = \sqrt{-1}\cdot\sqrt{-1} = i \cdot i = -\underline{1}$

Konjugert:

$z = a + ib$

$\bar{z} = a - ib$ $\bar{z}$ er konjugert til z.

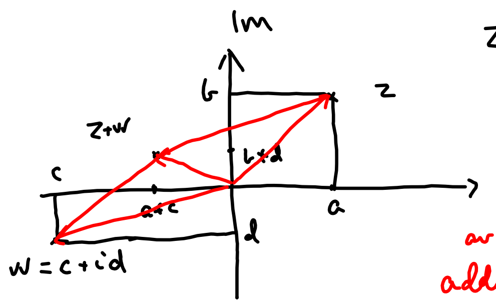
$\overline{z+w} = \bar{z} + \bar{w}$

$\underline{z\bar{z}} = (a+ib)(a-ib) = \underline{a^2+b^2}$

geometrisk tolkning på ny side)

Geometrisk tolkning av komplekse tall.

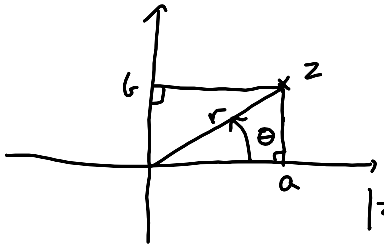
$z = a + ib$ Wessel



$z + w = (a+c) + i(b+d)$

addisjon av komplekse tall er vektor addisjon i planet.

For å tolke multiplikasjon bruker vi polarkoordinater.



z har polarkoordinater (r, θ)

$0 \le \theta < 2\pi \qquad r \ge 0$

$a = r\cos\theta \qquad b = r\sin\theta$

$|z| = r = \sqrt{a^2+b^2}$ kalles modulus.

θ er bestemt av $\underset{\frac{a}{r}}{\cos\theta}$ og $\underset{\frac{b}{r}}{\sin\theta}$.

(r_1, θ_1)

- $z = a + ib = r_1\cos\theta_1 + i\, r_1 \sin\theta_1$

$w = c + id = r_2 \cos\theta_2 + i\, r_2 \sin\theta_2$

(r_2, θ_2)

HUSK: $\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$

$\sin(\theta_1+\theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$

$$z\cdot w = r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1\cdot r_2\,(\cos\theta_1\cdot\cos\theta_2 + \overset{=-1}{i^2}\sin\theta_1\sin\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2)$$

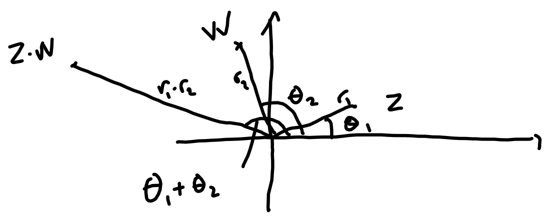
$$\longrightarrow \quad = r_1 r_2\left[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\right]$$

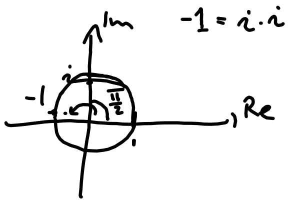
$$\longrightarrow \quad = r_1 r_2\,(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

$Re(zw) = \underline{r_1 r_2}\cos(\underline{\theta_1+\theta_2})$

$z\cdot w$ har polarkoordinater $(\underline{r_1 r_2}, \underline{\theta_1+\theta_2})$

Så ved multiplikasjon adderes argumentene og multipliseres modulusene.



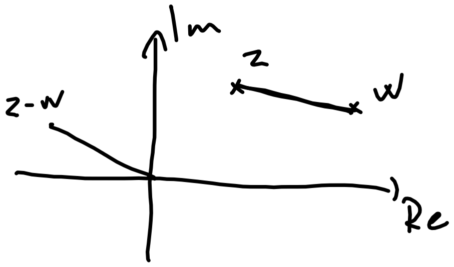


$i = 0 + i\cdot 1 \qquad (1, \frac{\pi}{2})$

$-1 = i\cdot i \qquad (1\cdot 1, \frac{\pi}{2}+\frac{\pi}{2})$

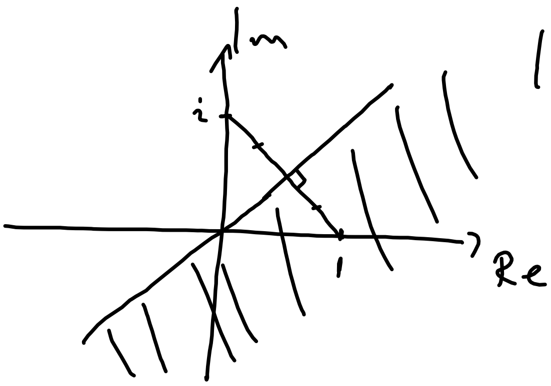
$= (1, \pi)$

Dersom z, w er komplekse tall
gir $z < w$ ikke mening



$|z| < |w|$ gir mening

$|z-w|$



$|z-1| \leq |z-i|$

z skal være nærmere 1 enn i.