Løsningsforslag midtveis MAT 1100 H19

(1) $z = 2e^{i\frac{5\pi}{6}} = -2\cdot\frac{1}{2}\sqrt{3} + 2\cdot\frac{1}{2}i = \underline{-\sqrt{3}+i}$

(2) $z = -3-3i \Rightarrow \underline{r} = \sqrt{3^2+3^2} = \underline{3\sqrt{2}}$, $\cos\theta = \sin\theta = -\frac{1}{2}\sqrt{2}$

$\Rightarrow \underline{\theta = \frac{5\pi}{4}}$

(3) $z = 2e^{i\frac{3\pi}{4}}$ $w = 4e^{i\frac{7\pi}{12}}$ $\underline{zw} = 8e^{i\frac{16\pi}{12}}$

$= 8\cos\frac{4\pi}{3} + 8i\sin\frac{4\pi}{3}$

$= \underline{-4-4i\sqrt{3}}$

(4) $\lim_{x\to\infty}\left(\sqrt{x^2-2x}-x\right) = \lim_{x\to\infty}\frac{(x^2-2x)-x^2}{\sqrt{x^2-2x}+x} = \lim_{x\to\infty}\frac{-2}{\sqrt{1-\frac{2}{x}}+1} = \underline{\underline{-1}}$

(5) $\lim_{x\to 0}\frac{\tan 3x}{e^x-1} \overset{l'H}{=} \lim_{x\to 0}\frac{\frac{3}{\cos^2 3x}}{e^x} = \underline{\underline{3}}$

(6) $\lim_{x\to\frac{\pi}{2}}\frac{2\ln(\sin x)}{\cos x} \overset{l'H}{=} \lim_{x\to\frac{\pi}{2}}\frac{2\frac{1}{\sin x}\cos x}{-\sin x} = \underline{\underline{0}}$

(7) $f(x) = y = e^{x^3-1} \Rightarrow \ln y = x^3-1 \Rightarrow x = (\ln y+1)^{\frac{1}{3}}$

$g(x) = f^{-1}(y) = \underline{(\ln x+1)^{\frac{1}{3}}}$

(8) $f(x) = \ln(2x+1) = 0 \Rightarrow x = 0$ $g = f^{-1}$ $f' = \frac{2}{2x+1}$

så $\underline{g'(0)} = \frac{1}{f'(0)} = \underline{\frac{1}{2}}$

(9) $\lim_{n\to\infty}\frac{7n^2+n\sqrt{n}}{\sqrt{3n^4+4n^3}} = \lim_{n\to\infty}\frac{7+\sqrt{\frac{1}{n}}}{\sqrt{3+\frac{4}{n}}} = \underline{\underline{\frac{7}{\sqrt{3}}}}$

(10) $f(x) = xe^x$ $f'(x) = (x+1)e^x$ $f''(x) = (x+2)e^x$

$f''(x) \leq 0$ når $x \leq -2$, så $\underline{\text{konkav på } [-\infty, -2]}$

(11) Middelverdisetningen: $\frac{(f+g)(b) - (f+g)(a)}{b-a} = (f+g)'(c)$ for en $c \in (a,b)$

$$\Rightarrow \frac{f(b) + g(b) - f(a) - g(a)}{b-a} = 0 = f'(c) + g'(c)$$

$$\Rightarrow \underline{f'(c) = -g'(c)} \quad \text{for en } c \in (a,b)$$

(12) $h(x) = f(x)^{f(x)} \Rightarrow \ln(h(x)) = f(x) \cdot \ln f(x)$

$$\Rightarrow \frac{h'(x)}{h(x)} = f'(x) \ln f(x) + f(x) \cdot \frac{f'(x)}{f(x)}$$

$$= f'(x)(\ln f(x) + 1)$$

så $\underline{h'(x) = f'(x) \cdot f(x)^{f(x)} (\ln f(x) + 1)}$

(13) $(-2i)^3 = -8 \cdot i^3 = 8i$

Andre 3'dje røtter av $8i$: $-2i \cdot e^{\frac{2\pi i}{3}}$, $-2i e^{\frac{4\pi i}{3}}$

$$-2i e^{\frac{2\pi i}{3}} = -2i\left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right) = \underline{\sqrt{3} + i}$$

(14) Røttene til $P(z)$ er $1, \pm i, \pm 3i$

Så $P(z) = z^5 - z^4 + 10z^3 - 10z^2 + 9z - 9$.

(15) $f(x) = (1-2x)e^{-x^2} \quad x \in [-2, 2]$

$f'(x) = ((1-2x)(-2x) - 2)e^{-x^2} = (4x^2 - 2x - 2)e^{-x^2}$

globalt minimum når $(1-2x) < 0$, $x > \frac{1}{2}$

$x > \frac{1}{2}$, $f'(x) = 0 \Rightarrow x = 1 \quad f(1) = -e^{-1} = -\frac{1}{e}$

$$f(2) = -3e^{-4} = -\frac{3}{e^4} > -\frac{1}{e}$$

så globalt minimum i $\underline{x = 1}$

(16)

$$\lim_{x\to 0^+} \underline{\sin x \ \ln(2x)} \overset{l'H}{=} \lim_{x\to 0^+} \frac{(\ln 2x)'}{\left(\frac{1}{\sin x}\right)'}$$

$$= \lim_{x\to 0^+} \frac{\frac{2}{2x}}{-\frac{1}{\sin^2 x}\cdot \cos x} = \lim_{x\to 0^+} -\frac{\sin^2 x}{x \cos x} = \underline{\underline{0}}$$

(17)

$T(x)$ er tiden (i minutt) det tar om båten når stranda ved x meter fra B.

S = strekning, V = hastighet

$$T(x) = \frac{S(x)}{V(x)} = \frac{\sqrt{100^2 + x^2}}{100} + \frac{300 - x}{200}$$

$$T(x)' = \frac{1}{100} \cdot \frac{x}{\sqrt{100^2 + x^2}} - \frac{1}{200} = 0$$

$$\Rightarrow \frac{x}{\sqrt{100^2 + x^2}} = \frac{1}{2} \Rightarrow 2x = \sqrt{100^2 + x^2}$$

$$\Rightarrow 3x^2 = 100^2 \Rightarrow \underline{x = \frac{100}{\sqrt{3}}}$$

Raskest om $\underline{x = \frac{100}{\sqrt{3}}}$ meter

(18)

x = lengden på kanten til rektangelet langs hypotenusen

Da er høyden $\frac{1}{2}(1 - x)$, så arealet

er $A(x) = x \cdot \frac{1}{2}(1 - x) = \frac{1}{2}x - \frac{1}{2}x^2$

$A'(x) = \frac{1}{2} - x = 0$ når $x = \frac{1}{2}$

Størst areal: $\underline{A\left(\frac{1}{2}\right)}$ =

$$= \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{8}}}$$