$$\int \frac{5x^{3/2}}{\ln x + 4x^{5/2}}\,dx \qquad x \to \infty \Rightarrow x^{5/2} > \ln x$$

$$\underline{\frac{1}{x}} = \frac{5x^{3/2}}{x^{5/2} + 4x^{5/2}} < \frac{5x^{3/2}}{\ln x + 4x^{5/2}}$$

siden $\int_1^\infty \frac{1}{x}\,dx$ divergerer så

divergerer også $\int_1^\infty \frac{5x^{3/2}}{\ln x + 4x^{5/2}}\,dx$!

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$$\int \frac{\cos x}{\sin^2 x + 6\sin x + 25}\,dx$$

$u = \sin x \qquad du = \cos x\,dx$

$$= \int \frac{du}{u^2+6u+25} = \int \frac{du}{u^2+6u+9+16}$$

$$= \int \frac{du}{(u+3)^2+16}$$

$$\left(\arctan v = \int \frac{1}{1+v^2}\,dv \right)$$

$$= \int \frac{\frac{1}{16}\,du}{\frac{(u+3)^2}{16}+1}$$

$$= \frac{1}{16}\int \frac{du}{\left(\frac{u+3}{4}\right)^2+1} \qquad v = \frac{u+3}{4} \qquad dv = \frac{1}{4}\,du$$

$$du = 4\cdot dv$$

$$= \frac{1}{16}\int \frac{4\cdot dv}{v^2+1} = \frac{1}{4}\int \frac{dv}{v^2+1} = \frac{1}{4}\arctan v + C$$

$$= \frac{1}{4}\arctan\left(\frac{u+3}{4}\right) + C = \frac{1}{4}\arctan\left(\frac{\sin x+3}{4}\right) + C$$

2016	12
2017	5
2018	5

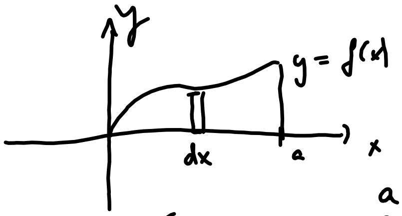
12: $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{vmatrix} x^3 & 3x & 1 \\ e^{x^3} & 0 & 0 \\ x^2 & x & -1 \end{vmatrix}$$

$$= x^3 \begin{vmatrix} 0 & 0 \\ x & -1 \end{vmatrix} - 3x \begin{vmatrix} e^{x^3} & 0 \\ x^2 & -1 \end{vmatrix} + 1 \cdot \begin{vmatrix} e^{x^3} & 0 \\ x^2 & x \end{vmatrix}$$

$$= -3x\left(-e^{x^3}\right) + xe^{x^3} = \underline{\underline{4xe^{x^3}}}$$

a)



Volum av omdreining om x-aksen:

$$\underline{V} = \int dV = \int_0^a \pi f(x)^2 \cdot dx = \pi \int_0^a 16x^2 e^{2x^3} dx = \underline{16\pi \int_0^a x^2 e^{2x^3} dx}$$

b) $$V = 16\pi \int_0^a x^2 e^{2x^3} dx = 16\pi \int_{x=0}^{x=a} \frac{1}{6} e^u \, du = \frac{16}{6}\pi \int_{x=0}^{x=a} e^u \, du$$

$u = 2x^3 \qquad du = 6x^2 dx$

$x^2 dx = \frac{1}{6} du$

$$= \frac{8\pi}{3} \left[e^u\right]_{x=0}^{x=a}$$

$$= \frac{8\pi}{3} \left[e^{2x^3}\right]_{x=0}^{x=a}$$

$$= \underline{\underline{\frac{8\pi}{3}\left(e^{2a^3} - 1\right)}}.$$

2017 nr 5:

skal regne ut

a) $\int \cos\sqrt{x}\, dx$ og b) $\int \frac{1}{x^2+6x+18}\, dx$

a) $\int \cos\sqrt{x}\, dx = \int 2u\cos u\, du$

$u = \sqrt{x}$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2u} \Rightarrow dx = 2u\, du$

$= 2\int u\cos u\, du = 2u\sin u - 2\int 1\cdot \sin u\, du$

$u = u \quad v' = \cos u$

$u' = 1 \quad v = \sin u$

$(x^a)' = ax^{a-1}$

$\int x^a dx = \frac{1}{a+1}x^{a+1}$

$= 2u\sin u + 2\cos u + C$

$= \underline{2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C}$

(b) $\displaystyle\int \frac{1}{x^2+6x+18}\,dx = \int \frac{1}{\underline{x^2+6x+9}+9}\,dx$

arctan v $= \displaystyle\int \frac{1}{1+v^2}\,dv$

$\displaystyle = \int \frac{1}{(x+3)^2+9}\,dx = \int \frac{\frac{1}{9}}{\frac{(x+3)^2}{9}+1}\,dx$

$\left((x+3)^2+9 = (x+3+3i)(x+3-3i) \right)$
$-(-9)$

$v = \frac{x+3}{3}$ $\quad dv = \frac{1}{3}\,dx$
$dx = 3\,dv$

$\displaystyle = \frac{1}{9}\int \frac{1}{\left(\frac{x+3}{3}\right)^2+1}\,dx = \frac{1}{9}\int \frac{3\,dv}{v^2+1}$

$\displaystyle = \frac{1}{3}\int \frac{1}{1+v^2}\,dv = \frac{1}{3}\arctan v + C$

$\displaystyle = \frac{1}{3}\arctan\left(\frac{x+3}{3}\right) + C$

2018 5.

a) $\int \frac{x}{\sqrt{1-x^4}}\,dx$ $\qquad$ $\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin u + C$

$u = x^2 \qquad du = 2x\,dx$

$x\,dx = \frac{1}{2}\,du$

$$= \int \frac{\frac{1}{2}\,du}{\sqrt{1-u^2}} = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin u + C$$

$$= \frac{1}{2}\arcsin x^2 + C$$

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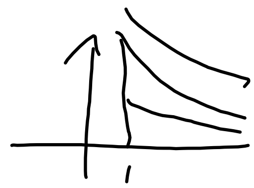
b) $\int_1^\infty \frac{1}{x^2 - \sin x}\,dx$



$0 < x^2 - \sin x$ når $x > 1$ $\qquad$ $\int_1^\infty \frac{1}{x^2}\,dx$ konvergerer

$\int_1^a \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^a = 1 - \frac{1}{a}$

$x^2 - \sin x > \frac{1}{2}x^2$ $\qquad$ $x > 100000$

så $\frac{1}{x^2 - \sin x} < 2\frac{1}{x^2}$.

Siden $\int_1^\infty \frac{2}{x^2}\,dx = 2\int_1^\infty \frac{1}{x^2}\,dx$ konvergerer $(=2)$

så konvergerer også $\int_1^\infty \frac{1}{x^2 - \sin x}\,dx$.

$\int_1^\infty \frac{1}{x^2 - \sin x}\,dx$ $\qquad$ $\int_1^\infty \frac{1}{x^2}\,dx$ konverger.

$$\frac{\frac{1}{x^2}}{\frac{1}{x^2 - \sin x}} = \frac{x^2 - \sin x}{x^2} = \frac{1 - \frac{\sin x}{x^2}}{1} \xrightarrow[x\to\infty]{} 1$$

grensesammenligningssetning sier at

$\int_1^\infty \frac{1}{x^2 - \sin x}\,dx$ konvergerer siden $\int_1^\infty \frac{1}{x^2}\,dx$ konvergerer!