Matriseregning og vektorfunksjoner.

2014	12
2017	4
2018	4
2017	1, 2

2014 12:

$$A = \begin{pmatrix} 1 & 0.2 & 0.1 \\ 0.1 & 1.02 & 0.01 \\ -0.2 & -0.04 & 0.98 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -0.2 & -0.1 \\ -0.1 & 1 & 0 \\ 0.2 & 0 & 1 \end{pmatrix}$$

a) Vil vise at B er inversmatrisen til A, det vil si at $AB = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$A \cdot B = \begin{pmatrix} 1 & 0.2 & 0.1 \\ 0.1 & 1.02 & 0.01 \\ -0.2 & -0.04 & 0.98 \end{pmatrix} \begin{pmatrix} 1 & -0.2 & -0.1 \\ -0.1 & 1 & 0 \\ 0.2 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \underline{1 \cdot 1 - 0.2 \cdot 0.1 + 0.1 \cdot (0.2)} & \underline{-0.2} + \underline{0.2} + \underline{0.1 \cdot 0} & \underline{-0.1} + 0 + \underline{0.1} \\ \underline{0.1} - \underline{0.1 \cdot 1.02} + 0.01 \cdot 0.2 & -0.2 \cdot 0.1 + 1.02 & \underline{-0.1 \cdot 0.1} + \underline{0.01} \\ -0.2 + 0.04 \cdot 0.1 + 0.98 \cdot 0.2 & 0.2 \cdot 0.2 - 0.04 & 0.1 \cdot 0.2 + 0.98 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0.1 - 0.102 + 0.002 & 1 & 0 \\ -0.2 + 0.004 + 0.196 & 0 & 0.02 + 0.98 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

b) $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $x = \# X$

$y = \# Y$

$z = \# Z$

Dersom $\vec{r}$ er bestanden ett år så er

$A\vec{r}$ bestanden neste år.

$\vec{r}_0 = \begin{pmatrix} 4000 \\ 2000 \\ 1000 \end{pmatrix}$ er bestanden ett år,

vil finne bestanden $\vec{r}_{-1}$ året før.

Vet $A\,\vec{r}_{-1} = \vec{r}_0$

Da er $B \cdot A\,\vec{r}_{-1} = B\,\vec{r}_0$

Men $B \cdot A = A \cdot B = I$ så $I \cdot \vec{r}_{-1} = B\,\vec{r}_0$

så $\vec{r}_{-1} = B\,\vec{r}_0 = \begin{pmatrix} 1 & -0.2 & -0.1 \\ -0.1 & 1 & 0 \\ 0.2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4000 \\ 2000 \\ 1000 \end{pmatrix}$

$= \begin{pmatrix} 4000 - 400 - 100 \\ -400 + 2000 \\ 800 + 1000 \end{pmatrix} = \begin{pmatrix} 3500 \\ 1600 \\ 1800 \end{pmatrix}$

så bestanden året før var

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3500 \\ 1600 \\ 1800 \end{pmatrix}$

2017
4

x_1 sier at de vil stemme på P
x_2 — " — Q
x_3 — " — R

y_1 som stemmer på P
y_2 — " — Q
y_3 — " — R.

i) + ii) + iii):
$$y_1 = \frac{7}{10}x_1 + \frac{2}{10}x_2 + \frac{2}{10}x_3$$
$$y_2 = \frac{2}{10}x_1 + \frac{7}{10}x_2 + \frac{2}{10}x_3$$
$$y_3 = \frac{1}{10}x_1 + \frac{1}{10}x_2 + \frac{6}{10}x_3$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0.7 & 0.2 & 0.2 \\ 0.2 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.6 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

b)

$$B = \begin{pmatrix} 1.6 & -0.4 & -0.4 \\ -0.4 & 1.6 & -0.4 \\ -0.2 & -0.2 & 1.8 \end{pmatrix}$$

B er invers til A.

Hvis $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 800 \\ 700 \\ 500 \end{pmatrix}$ hva er $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$?

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$B \cdot A = I$

$$B\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = B \cdot A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = B\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1.6 & -0.4 & -0.4 \\ -0.4 & 1.6 & -0.4 \\ -0.2 & -0.2 & 1.8 \end{pmatrix}\begin{pmatrix} 800 \\ 700 \\ 500 \end{pmatrix}$$

$$\begin{pmatrix} 1.6 \cdot 800 + (-0.4) \cdot 700 + (-0.4) \cdot 500 \\ -0.4 \cdot 800 + 1.6 \cdot 700 + (-0.4) \cdot 500 \\ -0.2 \cdot 800 + (-0.2) \cdot 700 + 1.8 \cdot 500 \end{pmatrix} = \begin{pmatrix} 1280 - 280 - 200 \\ -320 + 1.120 - 200 \\ -160 - 140 + 900 \end{pmatrix}$$

$$= \begin{pmatrix} 800 \\ 600 \\ 600 \end{pmatrix}$$

så: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 800 \\ 600 \\ 600 \end{pmatrix}$

2018

4. Ved år (måned) n e det

x_n unge dyr,

y_n voksne dyr,

z_n gamle dyr.

Hva med år $n+1$?

$$x_{n+1} = 3x_n + 10y_n$$
$$y_{n+1} = 0.8x_n$$
$$z_{n+1} = 0.5y_n$$

Finn M s.a

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = M \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

Så
$$M = \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}$$

Er M invertarbar, dvs. finis det en N s.a

$$N \cdot M = M \cdot N = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I} \ ?$$

$$\begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{pmatrix} \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} = \begin{pmatrix} & & 0 \\ & & 0 \\ & & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = M \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

Finn A s.a $\begin{pmatrix} x_{n+3} \\ y_{n+3} \\ z_{n+3} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$

$$\begin{pmatrix} x_{n+3} \\ y_{n+3} \\ z_{n+3} \end{pmatrix} = M \begin{pmatrix} x_{n+2} \\ y_{n+2} \\ z_{n+2} \end{pmatrix} = M \cdot M \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = M \cdot M \cdot M \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

så $A = M \cdot M \cdot M = (M)^3 = M^3$

$$\underbrace{\begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}} \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 17 & 30 & 0 \\ 2.4 & 8 & 0 \\ 0.4 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 & 10 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 75 & 170 & 0 \\ 13.6 & 24 & 0 \\ 1.2 & 4 & 0 \end{pmatrix} = \underline{\underline{A}}$$

Fins det en B s.a

$$\begin{pmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \end{pmatrix} = B \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

da må $B = M^{-1}$

$$M \begin{pmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = M \cdot B \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

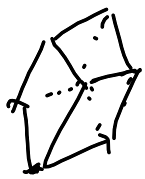
så $M \cdot B = I$ men M^{-1}

eksisterer ikke (av a) så B

fins ikke!

2017 2 Volumet til parallellepipedet P utspent av vektorene $\vec{a} = (1,-2,1)$ $\vec{b} = (2,1,-2)$

$\vec{c} = (-1,0,1)$



$$\underline{V(P)} = \left\| \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{vmatrix} \right\|$$

$$= \left| 1\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} \right|$$

$$= \left| \underline{1} + 2 \cdot 0 + 1 \right| = \underline{\underline{2}}$$

2017 1.

$$f(x,y) = x\,e^{xy}$$

a) $$\frac{\partial f}{\partial x} = e^{xy} + x\,e^{xy}\cdot y = \underline{(1+xy)e^{xy}}$$

$$\frac{\partial f}{\partial y} = x\,e^{xy}\cdot x = \underline{x^2 e^{xy}}$$

$$\nabla f = \left((1+xy)e^{xy},\ x^2 e^{xy}\right)$$

b) $$\vec{a} = (2,1):\quad \nabla f(\vec{a}) = \left((1+2\cdot 1)e^2,\ 2^2\cdot e^2\right) = (3e^2, 4e^2)$$

f vokser raskest i retning $\underline{(3e^2, 4e^2)}$ i punktet $(2,1)$.

$$\vec{u} = \frac{(3e^2, 4e^2)}{|(3e^2,4e^2)|} = \frac{(3e^2, 4e^2)}{\sqrt{9e^4 + 16e^4}} = \frac{(3e^2, 4e^2)}{5e^2} = \underline{\left(\frac{3}{5}, \frac{4}{5}\right)}$$

$$\underline{\underline{f'(\vec{a}, \vec{u})}} = \nabla f(\vec{a})\cdot\vec{u} = (3e^2, 4e^2)\cdot\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= \left(\frac{9}{5} + \frac{16}{5}\right)e^2$$

$$= \underline{\underline{5e^2}}$$

$5e^2$ er også stigningstallet til f i den retningen der den vokser raskest.