1 a

$$\vec{F}(x,y,z) = \begin{pmatrix} x^2\cos z \\ \ln(xy) \\ \tan(yz) \end{pmatrix}$$

Jacobimatrisen til $\vec{F}$:

$$\vec{F}'(x,y,z) = \begin{pmatrix} \frac{\partial}{\partial x}(x^2\cos z) & \frac{\partial}{\partial y}(x^2\cos z) & \frac{\partial}{\partial z}(x^2\cos z) \\ \frac{\partial}{\partial x}(\ln xy) & \frac{\partial}{\partial y}(\ln xy) & \frac{\partial}{\partial z}(\ln xy) \\ \frac{\partial}{\partial x}(\tan yz) & \frac{\partial}{\partial y}(\tan yz) & \frac{\partial}{\partial z}(\tan yz) \end{pmatrix}$$

$$= \begin{pmatrix} 2x\cos z & 0 & -x^2\sin z \\ \frac{1}{xy}\cdot y & \frac{1}{xy}\cdot x & 0 \\ 0 & \frac{1}{\cos^2(yz)}\cdot z & \frac{1}{\cos^2 yz}\cdot y \end{pmatrix}$$

$$\frac{\partial}{\partial x}\ln(xy) = \frac{1}{x}$$

$$= \frac{\partial}{\partial x}\ln(x)$$

$$\underset{\ln(xy)}{\frac{\partial}{\partial x}(\ln x + \ln y)}$$

$$= \begin{pmatrix} 2x\cos z & 0 & -x^2\sin z \\ \frac{1}{x} & \frac{1}{y} & 0 \\ 0 & \frac{z}{\cos^2(yz)} & \frac{y}{\cos^2(yz)} \end{pmatrix}$$

b)

$$F'(1,2,0) = \begin{pmatrix} 2\cdot 1\cos 0 & 0 & -1\cdot\sin 0 \\ \frac{1}{1} & \frac{1}{2} & 0 \\ 0 & \frac{0}{\cos^2 0} & \frac{2}{\cos^2(0)} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$(F'(1,2,0))^2 = \begin{pmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 5/2 & 1/4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$(F'(1,2,0))^3 = \begin{pmatrix} 4 & 0 & 0 \\ 5/2 & 1/4 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 & 0 \\ \frac{21}{4} & \frac{1}{8} & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

2 $f(x,y) = y\sin(xy)$ $\quad \vec{a} = \left(\frac{\pi}{6}, -1\right)$ $\quad \vec{r} = (2,3)$

Vi skal regne ut:

$$\underline{f'(\vec{a},\vec{r}) = \nabla f(\vec{a}) \cdot \vec{r}}$$

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right) = \left(y^2\cos(xy), \sin(xy) + yx\cos xy\right)$$

$$\nabla f(\vec{a}) = \nabla f\left(\frac{\pi}{6}, -1\right) = \left(\cos\left(-\frac{\pi}{6}\right), \sin\left(-\frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right)\cdot\cos\left(-\frac{\pi}{6}\right)\right)$$

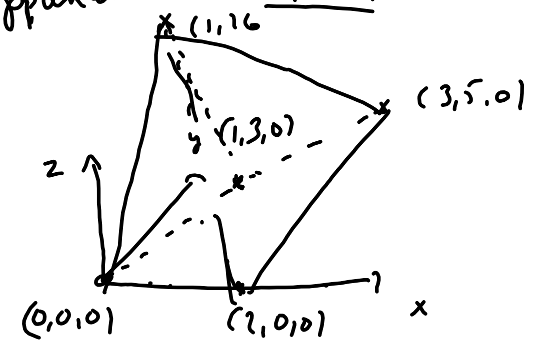
$$= \left(\frac{1}{2}\sqrt{3}, -\frac{1}{2} - \frac{\pi}{6}\cdot\frac{1}{2}\cdot\sqrt{3}\right)$$

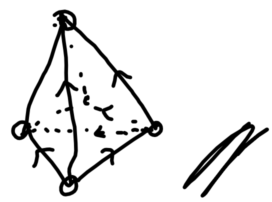
$$= \left(\frac{1}{2}\sqrt{3}, -\frac{1}{2} - \frac{\pi}{12}\cdot\sqrt{3}\right)$$

$$\underline{\underline{f'(\vec{a},\vec{r})}} = \nabla f(\vec{a})\cdot\vec{r} = \left(\frac{1}{2}\sqrt{3}, -\frac{1}{2} - \frac{\pi}{12}\sqrt{3}\right)\cdot(2, 3)$$

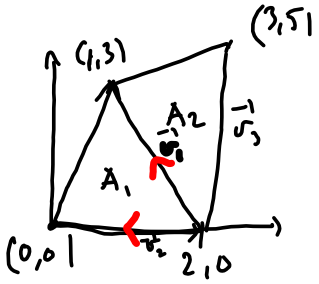
$$= \sqrt{3} - \frac{3}{2} - \frac{3\pi}{12}\sqrt{3} = \underline{\underline{\sqrt{3} - \frac{3}{2} - \frac{\pi}{4}\sqrt{3}}}.$$

3 Skal finne volumet V til pyramiden med grunnflate med hjørner $(0,0,0)$, $(2,0,0)$, $(1,3,0)$, $(3,5,0)$ og toppunkt i $(1,2,6)$.





$$V = \frac{1}{3} G \cdot h$$



$$A_1 = \frac{1}{2} |\vec{v}_1 \times \vec{v}_2| = \frac{1}{2} \left| \begin{vmatrix} \vec{v}_1 \\ \vec{v}_2 \end{vmatrix} \right|$$

$$\vec{v}_1 = (1,3) - (2,0) = (-1, 3)$$

$$\vec{v}_2 = (0,0) - (2,0) = (-2, 0)$$

$$A_1 = \frac{1}{2} \left| \begin{vmatrix} -1 & 3 \\ -2 & 0 \end{vmatrix} \right| = \frac{1}{2} |6| = \underline{\underline{3}}$$

$$\vec{v}_3 = (3,5) - (2,0) = (1, 5)$$

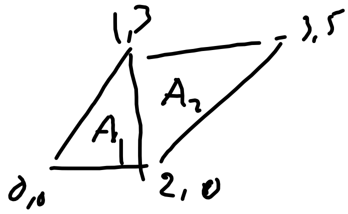
$$A_2 = \frac{1}{2} \left| \begin{vmatrix} \vec{v}_1 \\ \vec{v}_3 \end{vmatrix} \right| = \frac{1}{2} \left| \begin{vmatrix} -1 & 3 \\ 1 & 5 \end{vmatrix} \right| = \frac{1}{2} |-8| = \underline{4}$$

$$G = A_1 + A_2 = \underline{7}, \quad h = \underline{6}.$$

$$\underline{V} = \frac{1}{3} \cdot G \cdot h = \frac{1}{3}\, 7 \cdot 6 = \underline{\underline{14}}$$

$(0,0,0)$ $(2,0,0)$ $(1,3,0)$ $(3,5,0)$

$(1,2,6)$



V_1 = volum av pyramiden utspent av $(0,0,0)$ $(2,0,0)$ $(1,3,0)$ og $(1,2,6)$.

$$V_1 = \frac{1}{6}\begin{vmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 2 & 6 \end{vmatrix} = \frac{1}{3} A_1 \cdot h$$

$$= \frac{1}{3}\left(\frac{1}{2}\begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix}\right) \cdot 6$$

4a

$$\int_1^{\sqrt{3}} \frac{1+\tan^2 x}{\tan^2 x + 2\tan x + 2}\, dx = \int_{x=1}^{x=\sqrt{3}} \frac{du}{u^2+2u+2}$$

$u = \tan x \qquad du = (1+\tan^2 x)\, dx$

$$= \int_{x=1}^{x=\sqrt{3}} \frac{du}{u^2+2u+1\ +1} = \int_{x=1}^{x=\sqrt{3}} \frac{du}{(u+1)^2+1} = \int_{x=1}^{x=\sqrt{3}} \frac{dv}{1+v^2}$$

$v = u+1 \qquad dv = du$

$$= \Big[\arctan v\Big]_{x=1}^{x=\sqrt{3}} = \Big[\arctan(u+1)\Big]_{x=1}^{x=\sqrt{3}}$$

$$= \Big[\arctan(\tan x + 1)\Big]_{x=1}^{x=\sqrt{3}}$$

$$= \underline{\arctan(\tan\sqrt{3}+1) - \arctan(\tan 1 + 1)}$$

b) $I = \int_1^{\infty} \frac{\sin x}{2x^{\frac{3}{2}} - 1}\, dx$

Avgjør om I konvergerer eller divergerer.

teller er begrenset, nevner går mot ∞.

$(*)$ $\underline{\int_1^{\infty} \frac{1}{x^{3/2}}\, dx}$ konvergerer.

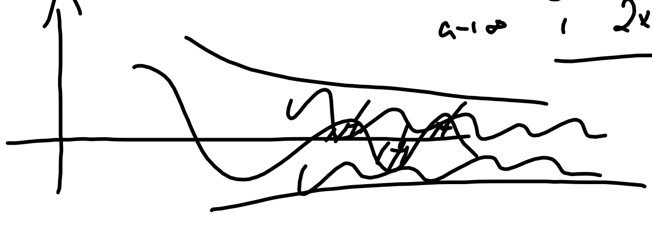
$$\underline{-\frac{1}{x^{3/2}} < \frac{\sin x}{2x^{3/2}-1} < \frac{1}{x^{3/2}}} \qquad x > 100000000$$

Siden $\int_1^{\infty} -\frac{1}{x^{3/2}}\, dx$ og $(*)$ konvergerer så vil

også $\int_1^{\infty} \frac{\sin x}{2x^{3/2}-1}\, dx$ konvergere. $\square$.

$$\parallel$$

$$\underline{\lim_{a \to \infty} \int_1^{a} \frac{\sin x}{2x^{3/2}-1}\, dx} \quad \text{eksisterer}$$



$$\Updownarrow$$

$\lim_{a \to \infty} \int_{10}^{a} \frac{\sin x}{2x^{3/2}-1}\, dx$ eksisterer.

5 a) $G(x) = \int_a^{g(x)} f(t)\,dt \qquad a \in \mathbb{R}$

$G(g(x)) = G(u) \qquad u = g(x) \qquad G(u) = \int_a^{u} f(t)\,dt.$

$\frac{dG(x)}{dx} = G'(x) = \frac{dG(u)}{du} \cdot \frac{du}{dx}$

Fundamentalsetningen: $= f(u) \cdot g'(x) = f(g(x)) \cdot g'(x)$

b) $F_1(x) = \int_1^{e^x} (\ln t - 1)\,dt$

$F_1'(x) = (\ln e^x - 1) \cdot e^x = (x-1)e^x$

$F_2(x) = \int_1^{x^2} (t^2 + t^4)\,dt$

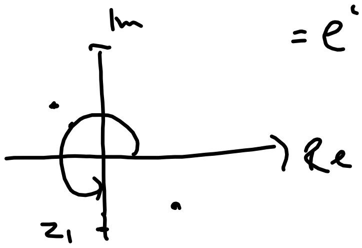
$F_2'(x) = (x^4 + x^8) \cdot 2x = 2x^5 + 2x^9$

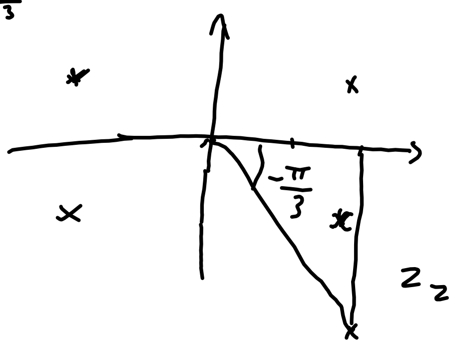
c) F_1 og F_2 er definert for alle $x \in \mathbb{R}$ og er kontinuerlige med kontinuerlige deriverte, da har F_1 og F_2 invers bare om den deriverte ikke skifter fortegn.

Både F_1' og F_2' skifter fortegn, så hverken F_1 eller F_2 har en invers.

6

$z_1 = -i = e^{i\frac{3\pi}{2}}$ $\qquad$ $z_2 = 2 - 2i\sqrt{3} = 4\cdot e^{i\frac{5\pi}{3}}$





$w^2 = z_1$

$w = e^{i\frac{3\pi}{4}}$ ellr $w = e^{i\frac{7\pi}{4}}$

$= -\frac{1}{2}\sqrt{2} + \frac{i}{2}\sqrt{2}$ $\qquad$ $= \frac{1}{2}\sqrt{2} - \frac{i}{2}\sqrt{2}$

$w^2 = z_2$

$w = 2e^{i\frac{5\pi}{6}}$ ellr $w = 2e^{i\frac{11\pi}{6}}$

$= -2\cdot\frac{1}{2}\sqrt{3} + 2\,\frac{1}{2}i$ $\qquad$ $= \sqrt{3} - i$

$= -\sqrt{3} + i$

6)

$$\left(z^2 - (2-2i\sqrt{3})\right)\left(z^2 - (2-2i\sqrt{3})\right)$$

$$= (z^2 - 2 + 2i\sqrt{3})\,(z^2 - 2 - 2i\sqrt{3})$$

$$= (z^2-2)^2 - (2i\sqrt{3})^2$$

$$= z^4 - 4z^2 + 4 + 12 = z^4 - 4z^2 + 16$$

7) $\quad f : [0,1] \to \mathbb{R}$ monoton i $(0,1)$

men ikke integrerbar på $[0,1]$.

$$f(x) = \begin{cases} \frac{1}{x} & x \in (0,1] \\ 0 & x = 0 \end{cases} \qquad \int_0^1 f(x)\,dx \text{ divergerer.}$$