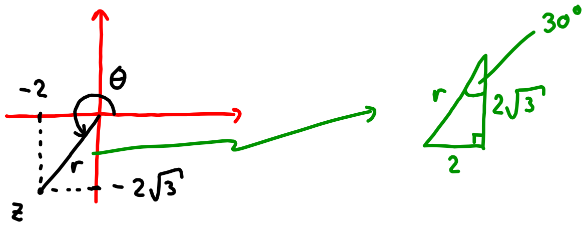
Løsningsforslag midtveis prøveeksamen Mat 1100
Lørdag 3. oktober 2020

Oppgave 1



Pytagoras:

$2^2 + (2\sqrt{3})^2 = r^2$

$4 + 4 \cdot 3 = r^2$

$r^2 = 16$

$r = 4$

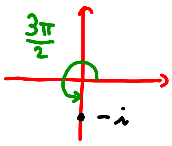
Så trekanten er 30/60/90

Så $\theta = 180° + 60° = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$. Dermed $z = 4e^{i(4\pi/3)}$

$\boxed{B}$

Oppgave 2

Vil ha $e^{ix} = -i$

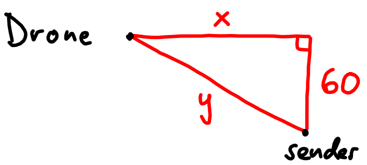


Så $-i = e^{i(3\pi/2)}$

$= e^{i(-\frac{\pi}{2})}$

$\boxed{E}$

Oppgave 3



$x^2 + 60^2 = y^2$

gir $2x \cdot x' + 0 = 2y \cdot y'$

$\boxed{x \cdot x' = y \cdot y'}$ (*)

Vårt øyeblikk: $x = 80$ gir $80^2 + 60^2 = y^2$

$y^2 = 10\,000$

$y = 100$

Innsatt i (*) får vi, med $x = 80$, $y = 100$ og $x' = 5$

$80 \cdot 5 = 100 \cdot y'$

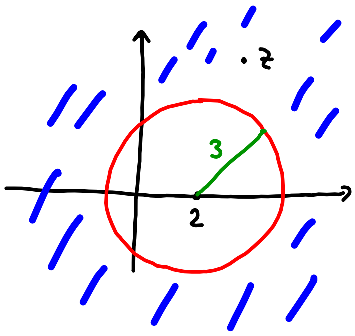
$y' = \frac{400}{100} = 4 \quad (m/s)$

D

Oppgave 4

$\{z : |z-2| > 3\}$

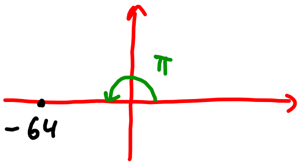
Kravet $|z-2| > 3$ betyr at avstanden mellom z og 2 skal være større enn 3.



$\boxed{A}$

Oppgave 5

Sjetterot til $z = -64 = 64e^{i\pi}$



Har $\sqrt[6]{64} = 2$, så $w_0 = 2e^{i(\pi/6)}$ er en rot.

$\boxed{D}$

Oppgave 6

$\boxed{C}$

Oppgave 7

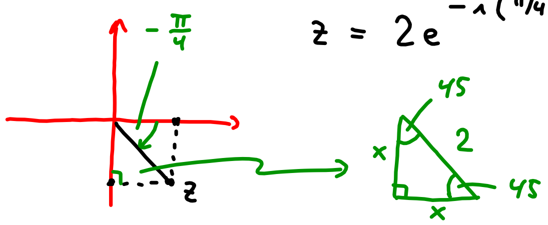
$$f(x) = \arctan(e^{2x}) + e^{2\arctan x}$$

$$f'(x) = \frac{1}{1+(e^{2x})^2} \cdot e^{2x} \cdot 2 + e^{2\arctan x} \cdot 2 \cdot \frac{1}{1+x^2}$$

$\boxed{C}$

Oppgave 8

$$z = 2e^{-i(\pi/4)} = 2e^{i(-\frac{\pi}{4})}$$



$$\left.\begin{aligned} x^2 + x^2 &= 2^2 \\ 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \sqrt{2} \end{aligned}\right\}$$

Altså $z = \sqrt{2} - i\sqrt{2}$

$\boxed{A}$

Oppgave 9

$$\lim_{x\to 0^+} (4x)^{2\sin x} \overset{[0^0]}{=} \lim_{x\to 0^+} \left[e^{\ln(4x)}\right]^{2\sin x}$$

$$= \lim_{x\to 0^+} e^{(\ln 4x)\cdot 2\sin x}$$

Eksponenten:

$$\lim_{x\to 0^+} (\ln 4x)\cdot 2\sin x \overset{[\infty\cdot 0]}{=} \lim_{x\to 0^+} \frac{\ln 4x}{\frac{1}{2\sin x}}$$

$$\overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cancel{4}x}\cdot \cancel{4}}{\frac{0 - 1\cdot 2\cos x}{4\sin^2 x}} = \lim_{x\to 0^+} \frac{4\sin^2 x}{-2x\cos x}$$

$$= -2 \cdot \lim_{x\to 0^+} \frac{\sin^2 x}{x\cos x}$$

$$\overset{\left[\frac{0}{0}\right]}{=} -2 \cdot \lim_{x\to 0^+} \frac{2\sin x\cos x}{1\cdot\cos x - x\sin x} = -2\cdot 0 = 0$$

$\to 0$, $\to 1$, $\to 0$

Så $\lim_{x\to 0^+} (4x)^{2\sin x} = e^0 = 1$

$\boxed{A}$

Oppgave 10

$s(x) = f(g(h(x)))$ gir

$s'(x) = f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$

$\boxed{E}$

Oppgave 11

$$\lim_{n\to\infty}\left[(-1)^n + \frac{n^2+8n+9}{n^3+7n}\right] = \lim_{n\to\infty}\left[(-1)^n + \underbrace{\frac{\frac{n^2}{n^3}+\frac{8n}{n^3}+\frac{9}{n^3}}{1+\frac{7n}{n^3}}}_{\to 0}\right]$$

$\to 0$, $\to 0$, $\to 0$, $\to 0$

Fins ikke, divergerer

$\boxed{E}$

Oppgave 12

$g(f(x)) = x$ gir $g'(f(x)) \cdot f'(x) = 1$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$g'(\underbrace{f(1)}_{=2}) = \frac{1}{f'(1)}$ dvs. $g'(2) = \frac{1}{\frac{1}{3}} = 3$

$\boxed{A}$

Oppgave 13

$$\lim_{x \to 0} \frac{\sin^3 x}{2x^3 + x^4} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x \to 0} \frac{3\sin^2 x \cdot \cos x}{6x^2 + 4x^3}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{x \to 0} \frac{6 \sin x \cos x \cdot \cos x + 3 \sin^2 x (-\sin x)}{12x + 12x^2}$$

$$= \lim_{x \to 0} \frac{6 \sin x \cos^2 x - 3 \sin^3 x}{12x + 12x^2}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{x \to 0} \frac{6 \cos^3 x + 6 \sin x \cdot 2 \cos x (-\sin x) - 9 \sin^2 x \cdot \cos x}{12 + 24x}$$

$$= \frac{6}{12} = \frac{1}{2}$$

$\boxed{B}$

Alternativt:

$$\frac{\sin^3 x}{2x^3 + x^4} = \frac{\left(\frac{\sin x}{x}\right)^3}{2 + x}$$

Vet at

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Så svaret blir $\frac{1}{2+0}$

Oppgave 14

$f(x) = \sin^3 x + 1$ har verdimengde $V_f = [0, 2]$

Altså $D_g = V_f = [0, 2]$

$\boxed{C}$

Oppgave 15

$$\lim_{x \to \infty} (e^x - x \ln x) \overset{[\infty - \infty]}{=} \lim_{x \to \infty} e^x \left(1 - \frac{x \ln x}{e^x}\right)$$

Ser på denne separat

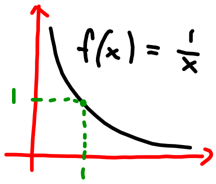
Her har vi $\lim_{x \to \infty} \frac{x \ln x}{e^x} \overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x \to \infty} \frac{1 \cdot \ln x + x \cdot \frac{1}{x}}{e^x} = \lim_{x \to \infty} \frac{\ln x + 1}{e^x}$

$$\overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{e^x} = \lim_{x \to \infty} \frac{1}{x e^x} = 0$$

Altså er grensen vår $+\infty$.

$\boxed{A}$

Oppgave 16



$D_f = (0, \infty)$

$f(1) = 1$

Siden f er kontinuerlig i $x = 1$, vet vi at for alle $\varepsilon > 0$ fins $\delta > 0$ slik at

$$|x - 1| < \delta \quad \text{medfører} \quad |f(x) - f(1)| < \varepsilon$$

Innsetting av $f(1) = 1$ og navnebytte $\delta \to t$ gir $\boxed{C}$

Oppgave 17

Sjekker skråasymptoter for $f(x) = 13x\,e^{-2/x}$:

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} 13\,e^{-2/x} = 13$$

$$b = \lim_{x\to\infty} [f(x) - ax] = \lim_{x\to\infty} [13x\,e^{-2/x} - 13x]$$

$$\overset{[\infty-\infty]}{=} \lim_{x\to\infty} 13x\left[e^{-2/x} - 1\right] \overset{[\infty\cdot 0]}{=} 13 \cdot \lim_{x\to\infty} \frac{e^{-2/x} - 1}{\frac{1}{x}}$$

$$\overset{[\frac{0}{0}]}{=} 13 \cdot \lim_{x\to\infty} \frac{e^{-2/x} \cdot \left(\frac{2}{x^2}\right)}{-\frac{1}{x^2}} = -26 \lim_{x\to\infty} e^{-2/x}$$

$$= -26 \cdot e^0 = -26$$

Alltså er $y = 13x - 26$ en skråasymptote for f. $\boxed{D}$

Oppgave 18

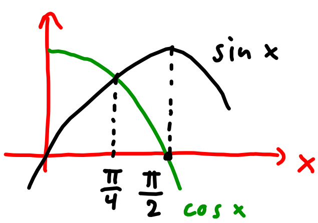
$f(x) = \sin(e^x)$ gir

$$f'(x) = \cos(e^x) \cdot e^x$$

$$f''(x) = -\sin(e^x) \cdot e^x \cdot e^x + \cos(e^x) \cdot e^x$$

$$f''(0) = -\sin(e^0) \cdot e^0 \cdot e^0 + \cos(e^0) \cdot e^0$$

$$= -\sin 1 + \cos 1$$



$\frac{\pi}{4}$ er mindre enn 1, så $\sin 1 > \cos 1$

Alltså $f''(0) < 0$. $\boxed{E}$