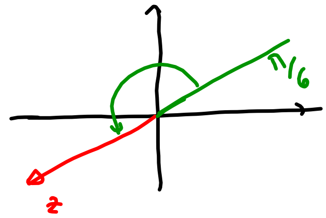
"Livet etter studiene" torsdag 14/10 kl 16^{15} i aud 1 VB
Pizza fra 16

Oppgave 1: $z = -2\sqrt{3} - 2i$

Polarkoordinater:



$$r = \sqrt{a^2+b^2} = \sqrt{(-2\sqrt{3})^2 + (-2)^2}$$

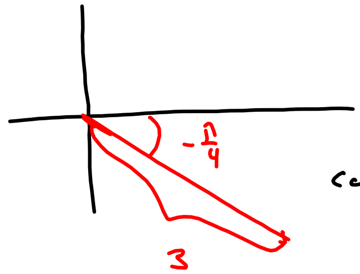
$$= \sqrt{12 + 4} = \underline{\underline{4}}$$

$$\sin\vartheta = \frac{b}{r} = \frac{-2}{4} = \underline{\underline{-\frac{1}{2}}} \quad , \vartheta = \frac{\pi}{6} + \pi = \underline{\underline{\frac{7\pi}{6}}}$$

Oppgave 2: $z = 3e^{-i\frac{\pi}{4}}$

$\vartheta = -\frac{\pi}{4}$

$r = 3$



$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$z = r(\cos\vartheta + i\sin\vartheta)$$

$$= 3\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$= 3\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = \underline{\underline{\frac{3\sqrt{2}}{2} - i\frac{3\sqrt{2}}{2}}}$$

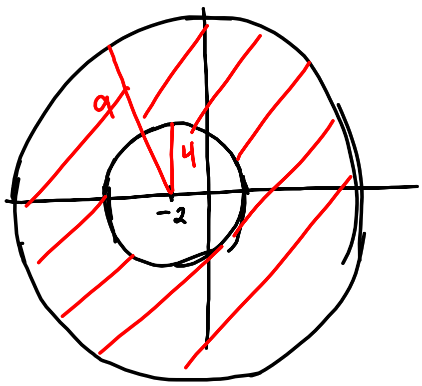
Oppgave 3: $2z - i = 4 - iz$

$$(2+i)z = 4 + i$$

$$z = \frac{4+i}{2+i} = \frac{(4+i)(2-i)}{(2+i)(2-i)} = \frac{4\cdot 2 - 4i + 2i \overbrace{-i^2}^{+1}}{4 \underbrace{-i^2}_{+1}}$$

$$= \frac{9-2i}{5} = \underline{\underline{\frac{9}{5} - \frac{2}{5}i}}$$

Oppgave 4: $A = \{z \in \mathbb{C} : \underline{4} < \underbrace{|z+2|}_{\underbrace{|z-(-2)|}_{\text{avstand mellom } z \text{ og } -2}} < \underline{9}\}$



Oppgave 5: $\lim\limits_{n\to\infty} \dfrac{n+3n^3-5n^4}{3+2n+n^3+2n^4}$

$= \lim\limits_{n\to\infty} \dfrac{\cancel{n^4}\left(\frac{1}{n^3}+\frac{3}{n}-5\right)}{\cancel{n^4}\left(\frac{3}{n^4}+\frac{2}{n^3}+\frac{1}{n}+2\right)} = \dfrac{-5}{2} = -\dfrac{5}{2}$

Oppgave 6: $\lim\limits_{x\to 0} \dfrac{1+3x-e^{3x}}{x^2} \overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{3-e^{3x}\cdot 3}{2x}$

$= \lim\limits_{x\to 0} \dfrac{3-3e^{3x}}{2x} \overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{-3\cdot e^{3x}\cdot 3}{2} = \dfrac{-9}{2} = -\dfrac{9}{2}$

Oppgave 7: $f(x) = \cot(\ln x)$

$f'(x) = -\dfrac{1}{\sin^2(\ln x)} \cdot \dfrac{1}{x} = -\dfrac{1}{x\sin^2(\ln x)}$

Oppgave 8: $f(x) = \arctan(e^x)$

$f'(x) = \dfrac{1}{1+(e^x)^2} \cdot e^x = \dfrac{e^x}{1+e^{2x}}$

Oppgave 9: $\lim\limits_{x\to 0} \dfrac{\arcsin x}{\sin 2x} \overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1-x^2}} \to 1}{\underbrace{\cos(2x)}_{\downarrow\, 1}\cdot 2} = \dfrac{1}{1\cdot 2} = \dfrac{1}{2}$

Oppgave 10: $f(x) = e^{\sqrt{x}-3}$, finn omvendt funksjon g

$y = e^{\sqrt{x}-3}$ løser for x:

$\ln y = \ln(e^{\sqrt{x}-3}) = \sqrt{x}-3 \Rightarrow \ln y + 3 = \sqrt{x}$

$\Rightarrow (\ln y+3)^2 = x \Rightarrow x = (\ln y+3)^2 = g(y)$

$g(x) = (\ln x+3)^2$

Oppgave 11: Deriver $f(x) = (x^2+1)^x$

To metoder:

(i) Logaritmisk derivasjon: $f'(x) = (\ln|f(x)|)' f(x)$

$$f'(x) = \left(\ln\left[(x^2+1)^x\right]\right)' \cdot f(x) = \left(x \ln(x^2+1)\right)' \cdot f(x)$$

$$= \left(1 \cdot \ln(x^2+1) + x \cdot \frac{1}{x^2+1} \cdot 2x\right) (x^2+1)^x$$

$$= (x^2+1)^x \left[\ln(x^2+1) + \frac{2x^2}{x^2+1}\right]$$

(ii) Flytter x-avhengigheten opp i eksponenten:

$$f'(x) = \left[(x^2+1)^x\right]' = \left[\left(e^{\ln(x^2+1)}\right)^x\right]'$$

$$= \left(e^{x\ln(x^2+1)}\right)' = e^{x\ln(x^2+1)} \left(x\ln(x^2+1)\right)'$$

$$= (x^2+1)^x \left[1 \cdot \ln(x^2+1) + x \frac{1}{x^2+1} \cdot 2x\right]$$

$$= (x^2+1)^x \left[\ln(x^2+1) + \frac{2x^2}{x^2+1}\right]$$

Oppgave 12 $P(z) = z^4 - 2z^3 + 4z - 4$

$1+i$ er en rot
$1-i$ er også en rot siden $P(z)$ er reell.

$(z-(1+i))(z-(1-i))$ deler $P(z)$

$$= \left[(z-1)-i\right]\left[(z-1)+i\right] = (z-1)^2 - i^2$$

$$= z^2 - 2z + 1 + 1 = z^2 - 2z + 2$$

Polynomdivisjon:

$$z^4 - 2z^3 + 4z - 4 : z^2 - 2z + 2 = z^2 - 2$$

$$-(z^4 - 2z^3 + 2z^2)$$

$$-2z^2 + 4z - 4$$

$$-(-2z^2 + 4z - 4)$$

$$0$$

Altså

$$z^4 - 2z^3 + 4z - 4 = (z^2 - 2z + 2)(z^2 - 2) = (z-(1+i))(z-(1-i))(z-\sqrt{2})(z+\sqrt{2})$$

Røtter $(1+i)$, $(1-i)$, $\sqrt{2}$, $-\sqrt{2}$.

Oppgave 13:

$$\lim_{n\to\infty}\sqrt{n^4-n^2}-n^2 = \lim_{n\to\infty}\frac{\left(\sqrt{n^4-n^2}-n^2\right)\left(\sqrt{n^4-n^2}+n^2\right)}{\sqrt{n^4-n^2}+n^2}$$

$$= \lim_{n\to\infty}\frac{\left(\sqrt{n^4-n^2}\right)^2-n^4}{\sqrt{n^4-n^2}+n^2} = \lim \frac{\cancel{n^4}-n^2-\cancel{n^4}}{\sqrt{n^4-n^2}+n^2}$$

$$\longrightarrow = -\lim_{n\to\infty}\frac{\cancel{n^2}}{\cancel{n^2}\underbrace{\left(\sqrt{1-\frac{1}{n^2}}+1\right)}_{2}} = -\frac{1}{2}$$

Alternativ metode:

$$\lim_{n\to\infty}\left(\sqrt{n^4-n^2}-n^2\right) = \lim_{n\to\infty} n^2\left(\sqrt{1-\frac{1}{n^2}}-1\right)$$

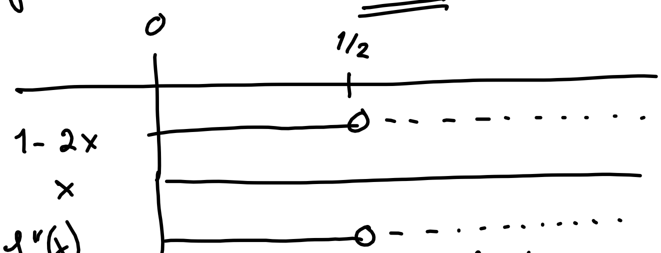
$$= \lim_{n\to\infty}\frac{\sqrt{1-\frac{1}{n^2}}-1}{\frac{1}{n^2}} \overset{L'H}{=} \lim \frac{\frac{1}{2\sqrt{1-\frac{1}{n^2}}}\left(-\cancel{\frac{1}{n^2}}\right)'}{\cancel{\left(\frac{1}{n^2}\right)'}} = \lim_{n\to\infty}\frac{-1}{2\sqrt{1-\frac{1}{n^2}}} = -\frac{1}{2}$$

Husk

Oppgave 14: $f(x) = x\ln x - x^2,\quad x>0$

$$f'(x) = 1\cdot\ln x - x\cdot\frac{1}{x} - 2x = \ln x - 1 - 2x$$

$$f''(x) = \frac{1}{x} - 2 = \frac{1-2x}{x}$$



konveks $(0, 1/2]$ konkav $[1/2, \infty)$

Oppgave 15

$$\lim_{x\to\frac{\pi}{4}}(\tan x)^{\frac{1}{x-\pi/4}} \quad \dots\to \text{"}1^{\infty}\text{"}$$

$$= \lim_{x\to\frac{\pi}{4}}\left(e^{\ln(\tan x)}\right)^{\frac{1}{x-\frac{\pi}{4}}} = \lim_{x\to\frac{\pi}{4}} e^{\frac{\ln(\tan x)}{x-\frac{\pi}{4}}} = e^2$$

Mellomregning:

$$\lim_{x\to\frac{\pi}{4}}\frac{\ln(\tan x)}{x-\frac{\pi}{4}} \overset{L'H}{=} \lim_{x\to\frac{\pi}{4}}\frac{\frac{1}{\tan x}\cdot\frac{1}{\cos^2 x}}{1}$$

1 $\quad \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

$$= \frac{\left(\frac{1}{1}\cdot\frac{1}{\frac{1}{2}}\right)}{(1)} = 2$$

Oppgave 16: $f(x) = x \arctan x$ g omvendt funksjon

$f(1) = 1 \cdot \arctan 1 = \frac{\pi}{4}$.

$$g'\left(\frac{\pi}{4}\right) = \frac{1}{f'(1)} = \frac{1}{\arctan 1 + \frac{1}{1+1^2}} = \frac{1}{\frac{\pi}{4} + \frac{1}{2}} = \underline{\frac{4}{\pi + 2}}$$

$$g'(y) = \frac{1}{f'(x)}, \quad y = f(x)$$

$$f'(x) = 1 \cdot \arctan x + x \, \frac{1}{1+x^2} = \underline{\arctan x + \frac{x}{1+x^2}}$$

Oppgave 17: $x_1 = 4$, $x_{n+1} = \frac{x_n^2 + 5}{6}$

Anta at $\lim_{n\to\infty} x_n = x$

$$\downarrow$$

$$x = \frac{x^2+5}{6} \Rightarrow \underline{x^2 - 6x + 5 = 0}$$

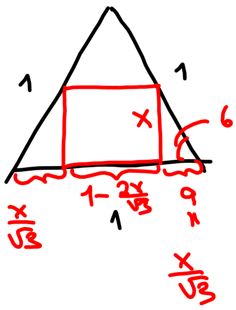
$$\underline{\underline{x = 1, x = 5}}$$

$$x_2 = \frac{x_1^2 + 5}{6} = \frac{16+5}{6} = \frac{21}{6} < \frac{24}{6} = 4 = x_1$$

Vis ved induksjon at (x_n) er avtagende og $x_n > 1$

$$\lim_{n\to\infty} x_n = \underline{\underline{1}}$$

Oppgave 19



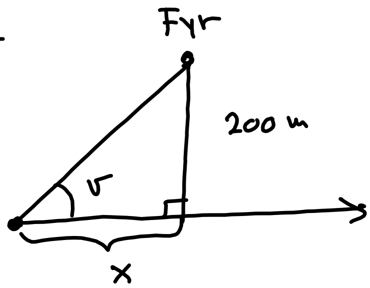
$$\sqrt{3} = \tan 60 = \frac{x}{a} \Rightarrow a = \frac{x}{\sqrt{3}}$$

$$A(x) = x\left(1 - \frac{2x}{\sqrt{3}}\right)$$

$A'(x) = 0$ når $x = \frac{\sqrt{3}}{4}$

$$A\left(\frac{\sqrt{3}}{4}\right) = \underline{\underline{\frac{\sqrt{3}}{8}}}$$

Oppgave 20:



$$\cot v = \frac{x}{200}$$

$$-\frac{1}{\sin^2 v} v'(t) = \frac{x'}{200}$$

$$x' = -\frac{200}{\sin^2 v} \underset{\substack{\| \\ 0{,}02 \text{ rad/s}}}{v'} \qquad v = \frac{\pi}{4}$$

$$= -\frac{200}{\frac{1}{2}} \cdot 0{,}02 = \underline{\underline{8 \text{ m/s}}}$$