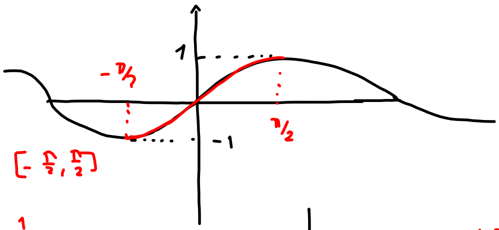
Arcusfunksjoner

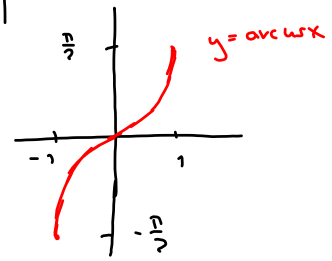
Minner om:

arcussinus:

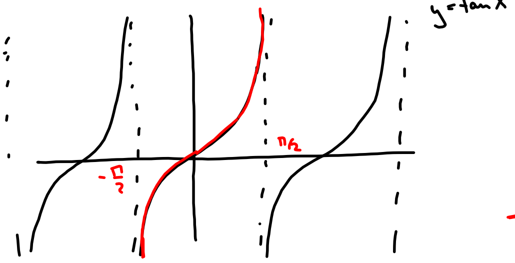
omvendt funksjon til sinus restriktert til $[-\frac{\pi}{2}, \frac{\pi}{2}]$

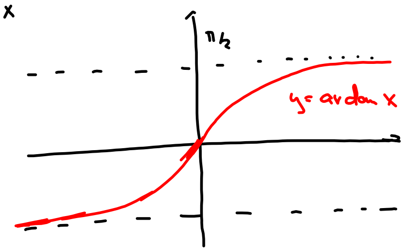


$$D(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$



arcustangens

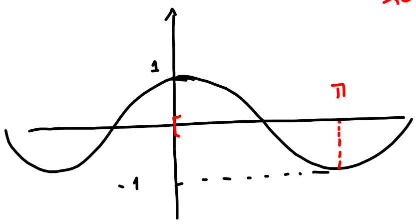


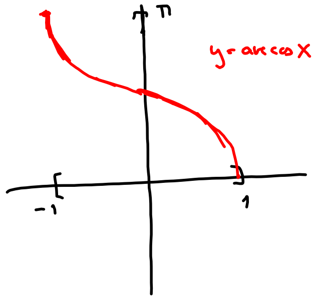


$$D(\arctan x) = \frac{1}{1+x^2}$$

arcuscosinus.

arccos er den omvendte funksjonen til cosinus restriktert til $[0,\pi]$





$$D(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

Eksakte verdier:

$\arcsin(0) = 0$ fordi $\sin 0 = 0$

$\arcsin(\frac{1}{2}) = \frac{\pi}{6}$ fordi $\sin\frac{\pi}{6} = \frac{1}{2}$

$\arcsin(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$ —"— $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$ —"— $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\arcsin(1) = \frac{\pi}{2}$ —"— $\sin\frac{\pi}{2} = 1$

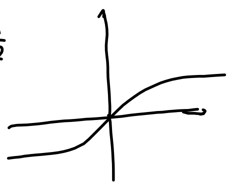
$\arctan(0) = 0$ fordi $\tan 0 = 0$

$\arctan(\frac{\sqrt{3}}{3}) = \frac{\pi}{6}$ —"— $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\arctan(1) = \frac{\pi}{4}$ —"— $\tan\frac{\pi}{4} = 1$

$\arctan\sqrt{3} = \frac{\pi}{3}$ —"— $\tan\frac{\pi}{3} = \sqrt{3}$

$\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$, $\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$



Eksempel: $\lim_{x\to 0} \frac{\arctan x}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{1} = \underline{\underline{1}}$

Eksempel: Finn asymptoten til $f(x) = x \arctan x$ når $x \to \infty$

(hvis den finnes).

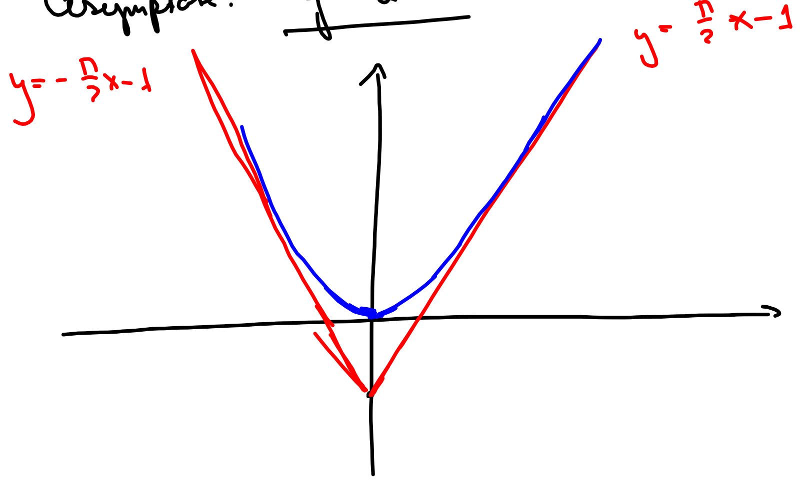
$\underline{\underline{1}}$: $a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\cancel{x} \arctan x}{\cancel{x}} = \underline{\underline{\frac{\pi}{2}}}$

$\underline{\underline{2}}$ $b = \lim_{x\to\infty} [f(x) - ax] = \lim_{x\to\infty} [x \arctan x - \frac{\pi}{2} x]$

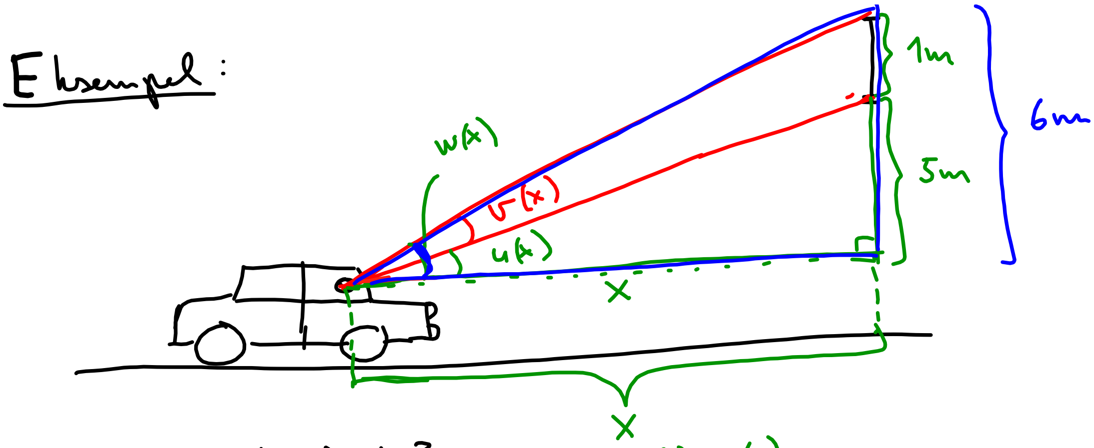
$= \lim_{x\to\infty} \overset{\infty \uparrow}{x} [\underbrace{\arctan x - \frac{\pi}{2}}_{\downarrow 0}] = \lim_{x\to\infty} \frac{\arctan x - \frac{\pi}{2}}{\frac{1}{x}}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = -\lim_{x\to\infty} \frac{x^2}{1+x^2} \overset{L'H}{=} -\lim_{x\to\infty} \frac{2x}{2x} = \underline{-1}$

Asymptote: $\underline{y = \frac{\pi}{2} x - 1}$ når $x \to \infty$



Eksempel:



Når er v(x) størst? $v(x) = w(x) - u(x)$

$\tan u(x) = \frac{5}{x} \Rightarrow u(x) = \arctan \frac{5}{x}$

$\tan w(x) = \frac{6}{x} \Rightarrow w(x) = \arctan \frac{6}{x}$

Altså $v(x) = \arctan \frac{6}{x} - \arctan \frac{5}{x}$ ← maksimere.

Deriverer:

$$v'(x) = \frac{1}{1+\left(\frac{6}{x}\right)^2}\left(-\frac{6}{x^2}\right) - \frac{1}{1+\left(\frac{5}{x}\right)^2}\left(-\frac{5}{x^2}\right)$$

$$= -\frac{6}{x^2+6^2} + \frac{5}{x^2+5^2} = -\frac{6}{x^2+36} + \frac{5}{x^2+25}.$$

Setter $v'(x) = 0$:

$$\frac{6}{x^2+36} = \frac{5}{x^2+25} \quad \Big| \, (x^2+36)(x^2+25)$$

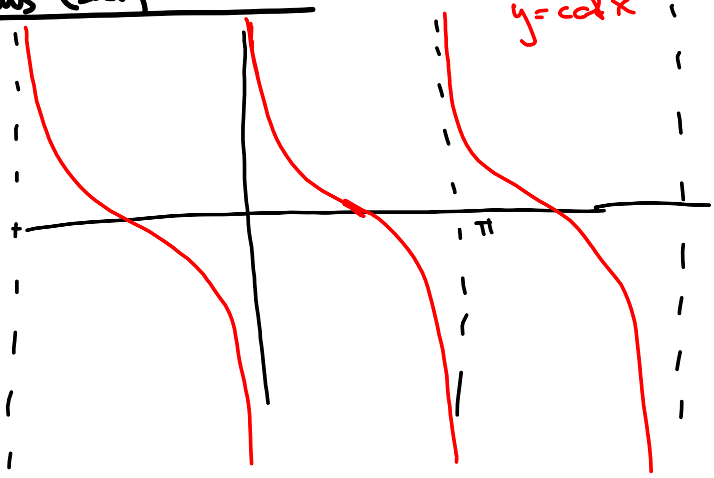
$$6(x^2+25) = 5(x^2+36)$$

$$6x^2 + 150 = 5x^2 + 180$$

$$x^2 = 30$$

så $x = \sqrt{30}$

Cotangens (Seksjon 7.5)



$$\cot(x) = \frac{\cos x}{\sin x}$$

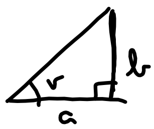
$$\cot(x) = \frac{1}{\tan x}$$

$$(\cot(x))' = \left(\frac{\cos x}{\sin x}\right)'$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cos x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$\int \frac{1}{\sin^2 x}\,dx = -\cot x + C$$

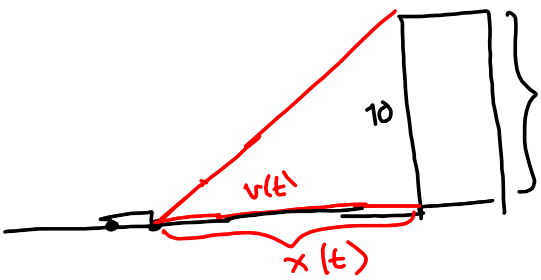


$$\cot v = \frac{a}{b}$$

Omvendt funksjon:

$\operatorname{arccot}(x)$.

Eksempel:



v øker med $2^{\text{rad}}/\text{sek}$.

Hvor fort nærmer vi oss veggen når $v(t) = \frac{\pi}{4}$?

Vi ser at

$$\cot v(t) = \frac{x(t)}{10}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

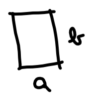
Deriverer:

$$-\frac{1}{\sin^2 v(t)} \cdot v'(t) = \frac{x'(t)}{10}$$

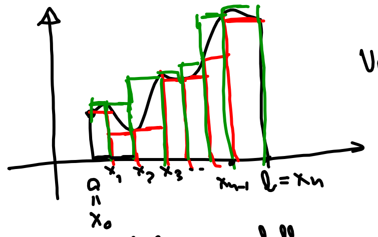
$$x'(t) = -\frac{10}{\sin^2 v(t)} \cdot v'(t) = -\frac{10}{\frac{1}{2}} \cdot 2 = -40\ \text{m/s}$$

Kapittel 8: Integrasjon

Enkle arealberegninger:



$A = ab$ Vel.

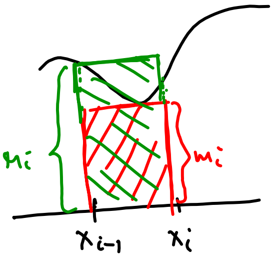


Vil regne ut!

Partisjon av $[a,b]$: En endelig følge av tall

$$\underbrace{a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b}_{\Pi}$$

i-te intervall: $[x_{i-1}, x_i]$



$m_i = \inf\{f(x) : x \in [x_{i-1}, x_i]\}$

$M_i = \sup\{f(x) : x \in [x_{i-1}, x_i]\}$

Arealet til i-te "underboks": $m_i(x_i - x_{i-1})$

Arealet til i-te "overboks": $M_i(x_i - x_{i-1})$

Nedre trappesum = arealet av alle rødebokser:

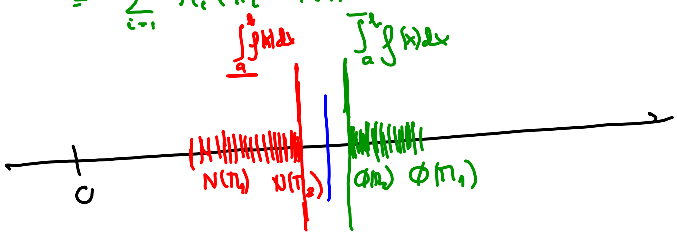
$$N(\Pi) = m_1(x_1 - x_0) + m_2(x_2 - x_1) + \dots + m_i(x_i - x_{i-1}) + \dots + m_n(x_n - x_{n-1})$$

$$= \sum_{i=1}^{n} m_i(x_i - x_{i-1})$$

Øvre trappesum = arealet av alle grønne bokser:

$$\Phi(\Pi) = M_1(x_1 - x_0) + M_2(x_2 - x_1) + \dots + M_i(x_i - x_{i-1}) + \dots + M_n(x_n - x_{n-1})$$

$$= \sum_{i=1}^{n} M_i(x_i - x_{i-1})$$



Nedre integral: $\underline{\int_a^b} f(x)\,dx = \sup\{N(\Pi) : \Pi \text{ en partisjon av } [a,b]\}$

Øvre integral: $\overline{\int_a^b} f(x)\,dx = \inf\{\Phi(\Pi) : \Pi \text{ —"— }\}$

Definisjon: En begrenset funksjon $f : [a,b] \to \mathbb{R}$ kalles integrerbar dersom $\underline{\int_a^b} f(x)dx = \overline{\int_a^b} f(x)dx$. I så fall definerer vi integralet til f over $[a,b]$ til å være

$$\int_a^b f(x)dx = \underline{\int_a^b} f(x)dx = \overline{\int_a^b} f(x)dx.$$