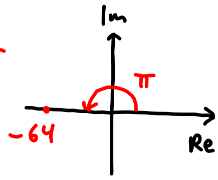
Løsningsforslag midtveis eksamen MAT 1100

10. oktober 2022

Oppgave 1



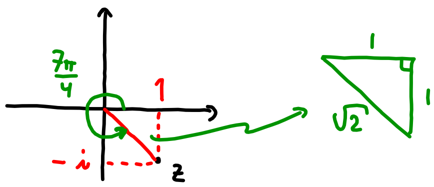
$$-64 = 64e^{i\pi} \quad \text{så}$$

$$w_1 = \sqrt[3]{64}\, e^{i(\pi/3)} = 4e^{i(\pi/3)}$$

er en tredjerot av -64

$\boxed{B}$

Oppgave 2



$$z = \sqrt{2}\, e^{i\left(\frac{7\pi}{4}\right)}$$

$\boxed{C}$

Oppgave 3

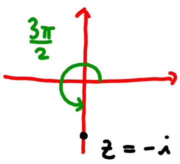
$\boxed{E}$

Oppgave 4

$$\begin{aligned}(z-i)\cdot(z-(-i)) &= (z-i)\cdot(z-1+i)\\ &= z^2 - z + iz - iz + i + 1\\ &= z^2 - z + 1 + i\end{aligned}$$

$\boxed{B}$

Oppgave 5



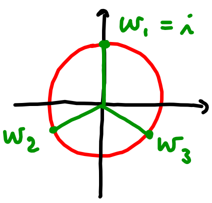
$$z = -i = 1\cdot e^{i\left(\frac{3\pi}{2}\right)}$$

Prinsipal tredjerot:

$$w_1 = \sqrt[3]{1}\, e^{i\left(\frac{\pi}{2}\right)} = e^{i\left(\frac{\pi}{2}\right)} = i$$

Vi har $w_+ = e^{i\left(\frac{2\pi}{3}\right)} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

Så røttene ligger slik:



$$w_2 = w_1 w_+ = i\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2}i - \frac{\sqrt{3}}{2}$$

$$w_3 = w_2 w_+ = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

figur

$\boxed{D}$

Oppgave 6

Kravet er $|z-(1+i)| = 2$, dvs. avstanden fra z til $1+i$ skal være 2.
Altså sirkel med sentrum $1+i$ og radius 2.

$\boxed{D}$

Oppgave 7

Deler på dominerende ledd

$$\lim_{n\to\infty} \frac{(-3)^n + e^n}{4^n - 5n} = \lim_{n\to\infty} \frac{\frac{(-3)^n}{4^n} + \frac{e^n}{4^n}}{1 - \frac{5n}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\left(-\frac{3}{4}\right)^n + \left(\frac{e}{4}\right)^n}{1 - \frac{5n}{4^n}} = 0$$

0 0 0

$\boxed{B}$

Oppgave 8

$\boxed{C}$

Oppgave 9

$$\lim_{x\to 0} \frac{x}{\sin 2x} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0} \frac{1}{\cos 2x \cdot 2} = \frac{1}{2}$$

$\boxed{B}$

Oppgave 10

$f(x) = \arctan(\arctan x)$ gir

$$f'(x) = \frac{1}{1+(\arctan x)^2} \cdot \frac{1}{1+x^2}$$

$\boxed{A}$

Oppgave 11

$$F(x) = \int_0^{x^2} \sin(1+t^4)\,dt$$

La $G(x) = \int_0^x \sin(1+t^4)\,dt$ og $u(x) = x^2$

Da er $F(x) = G(u(x))$, så kjerneregelen gir

$$F'(x) = G'(u(x)) \cdot u'(x) = \sin\left(1+(x^2)^4\right) \cdot 2x$$

$$= \sin(1+x^8) \cdot 2x$$

$G'(x) = \sin(1+x^4)$ ved fundamentalteoremet
Vi har $u'(x) = 2x$

$\boxed{C}$

Oppgave 12

$$V = 2\pi \int_1^2 x\, f(x)\, dx = 2\pi \int_1^2 x \cdot \frac{1}{x}\, dx = 2\pi \int_1^2 1\, dx$$

$$= 2\pi \left[x\right]_1^2 = 2\pi(2-1) = 2\pi \qquad \boxed{C}$$

Oppgave 13

$$\lim_{x\to 0^+} x^{\arctan x} \overset{[0^0]}{=} \lim_{x\to 0^+} \left(e^{\ln x}\right)^{\arctan x} = \lim_{x\to 0^+} e^{(\ln x)\cdot \arctan x}$$

Eksponenten:

$$\lim_{x\to 0^+} (\ln x)\cdot \arctan x \overset{[\infty\cdot 0]}{=} \lim_{x\to 0^+} \frac{\arctan x}{\frac{1}{\ln x}}$$

$$\overset{[\frac{0}{0}]}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+x^2}}{\frac{-1}{(\ln x)^2}\cdot \frac{1}{x}} = -\lim_{x\to 0^+} \frac{x\cdot(\ln x)^2}{1+x^2}$$

Her har vi

$$\lim_{x\to 0^+} x\cdot(\ln x)^2 \overset{[0\cdot\infty]}{=} \lim_{x\to 0^+} \frac{(\ln x)^2}{\frac{1}{x}} \overset{[\frac{\infty}{\infty}]}{=} \lim_{x\to 0^+} \frac{2(\ln x)\cdot\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= -2\cdot \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \overset{[\frac{\infty}{\infty}]}{=} -2\cdot \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= 2\cdot \lim_{x\to 0^+} x = 0$$

Altså går eksponenten mot 0, så svaret blir $e^0 = 1$. $\boxed{B}$

Oppgave 14

Skråasymptote : $y = ax + b$

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} 8e^{\overset{\to 0}{-\frac{1}{x}}} = 8$$

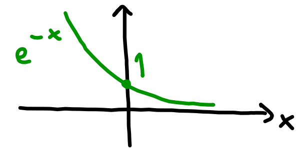
$$b = \lim_{x\to\infty} \left(f(x) - ax\right) = \lim_{x\to\infty} \left(8x e^{-\frac{1}{x}} - 8x\right)$$

$$\overset{[\infty-\infty]}{=} \lim_{x\to\infty} 8x\left(e^{-\frac{1}{x}} - 1\right) \overset{[\infty\cdot 0]}{=} 8\cdot \lim_{x\to\infty} \frac{\overset{\to 1}{e^{-\frac{1}{x}}} - 1}{\frac{1}{x}}$$

$$\overset{[\frac{0}{0}]}{=} 8\cdot \lim_{x\to\infty} \frac{e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = -8\cdot \lim_{x\to\infty} e^{-\frac{1}{x}} = -8$$

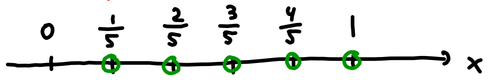
Så skråasymptoten er $y = 8x - 8$ $\boxed{E}$

Oppgave 15



$\boxed{A}$

Oppgave 16



$$R = f\left(\tfrac{1}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{2}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{3}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{4}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{5}{5}\right)\cdot\tfrac{1}{5}$$

$$= \frac{1}{25}\cdot\frac{1}{5} + \frac{4}{25}\cdot\frac{1}{5} + \frac{9}{25}\cdot\frac{1}{5} + \frac{16}{25}\cdot\frac{1}{5} + \frac{25}{25}\cdot\frac{1}{5}$$

$$= \frac{1}{125}\left(1+4+9+16+25\right) = \frac{55}{125} = \frac{11}{25}$$

$\boxed{D}$

Oppgave 17

$f : [0,\infty) \to \mathbb{R}$ ved $f(x) = \arcsin \frac{1}{1+x}$

Vi har $f(0) = \arcsin 1 = \frac{\pi}{2}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \arcsin \frac{1}{1+x} = 0$ (med $\frac{1}{1+x} \to 0$)

Så f er strengt avtakende på hele D_f

Altså $V_f = \left(0, \frac{\pi}{2}\right]$. Så f har en omvendt funksjon med definisjonsområde $\left(0, \frac{\pi}{2}\right]$.

$\boxed{C}$

Oppgave 18

$f(x) = 5x + 8$

$f(4) = 20 + 8 = 28$

Ser at $\delta = \frac{\varepsilon}{5}$ holder

$\boxed{A}$