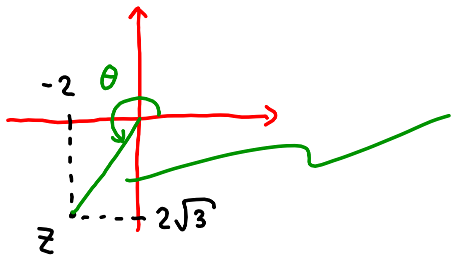
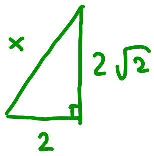
Løsningsforslag midtveis prøveeksamen MAT 1100

Lørdag 1. oktober 2022

Oppgave 1





$$2^2 + (2\sqrt{3})^2 = x^2$$
$$4 + 4\cdot 3 = x^2$$
$$x^2 = 16$$
$$x = 4$$

Så trekanten er 30/60/90

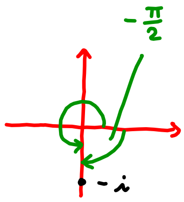
Altså: $\theta = 180° + 60°$
$$= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Dermed $z = 4e^{i\frac{4\pi}{3}}$

$\boxed{B}$

Oppgave 2

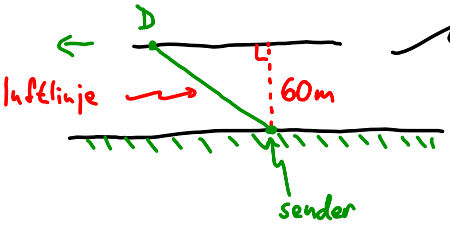
Vil ha $e^{ix} = -i$

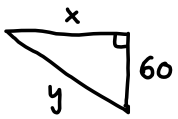


$$-i = re^{i\theta}$$
$$= 1e^{i\left(\frac{3\pi}{2}\right)}$$
$$= e^{i\left(-\frac{\pi}{2}\right)}$$

Så $x = -\frac{\pi}{2}$ $\boxed{E}$

Oppgave 3





$$x^2 + 60^2 = y^2$$
$$\boxed{2x\cdot x' + 0 = 2y\cdot y'} \quad (*)$$

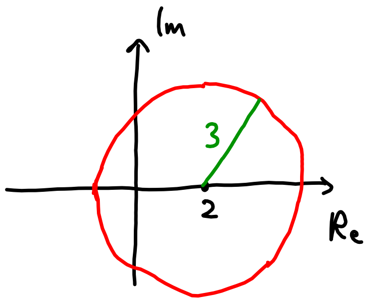
Vårt øyeblikk: $x = 80$ gir
$$y^2 = 80^2 + 60^2 = 10\,000$$
$$y = 100$$

Innsatt i $(*)$ fås
$$2\cdot 80\cdot 5 = 2\cdot 100\cdot y'$$
$$y' = \frac{400}{100} = 4 \quad (m/s)$$

$\boxed{D}$

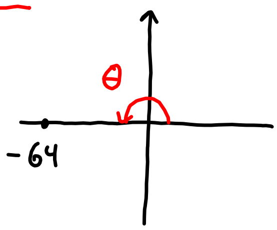
Oppgave 4

Kravet $|z-2| > 3$ betyr at avstanden mellom z og $2 = 2 + 0i$ skal være større enn 3



$\boxed{A}$

Oppgave 5



Må ha vinkel $\frac{\pi}{6}$ (prinsipal rot)

$r = \sqrt[6]{64} = 2$

Får $2e^{i(\pi/6)}$

$\boxed{D}$

Oppgave 6

$\boxed{C}$

Oppgave 7

$$f(x) = \arctan\left(e^{2x}\right) + e^{2\arctan x}$$

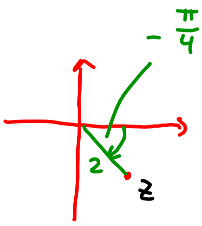
$$f'(x) = \frac{1}{1 + \left(e^{2x}\right)^2} \cdot e^{2x} \cdot 2 + e^{2\arctan x} \cdot 2 \cdot \frac{1}{1+x^2}$$

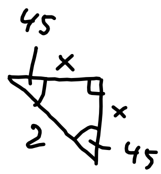
$\to e^{4x}$

$\boxed{C}$

Oppgave 8

$$z = 2e^{-i\left(\frac{\pi}{4}\right)} = 2e^{i\left(-\frac{\pi}{4}\right)}$$





$2x^2 = 4$, $x^2 = 2$, $x = \sqrt{2}$

Så $z = \sqrt{2} - i\sqrt{2}$

$\boxed{A}$

Oppgave 9

$$\lim_{x\to 0^+} (4x)^{2\sin x} \overset{[0^0]}{=} \lim_{x\to 0^+} \left(e^{\ln 4x}\right)^{2\sin x} = \lim_{x\to 0^+} e^{(\ln 4x)\cdot 2\sin x}$$

Eksponenten:

$$\lim_{x\to 0^+} (\ln 4x)\cdot 2\sin x \overset{[\infty\cdot 0]}{=} \lim_{x\to 0^+} \frac{\ln 4x}{\frac{1}{2\sin x}}$$

$$\overset{[\frac{\infty}{\infty}]}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cancel{4}x}\cdot\cancel{4}}{\frac{0-1\cdot 2\cos x}{4\sin^2 x}} \begin{matrix}\cdot 4\cdot x\cdot \sin^2 x\\ \cdot 4\cdot x\cdot \sin^2 x\end{matrix}$$

$$= \lim_{x\to 0^+} \frac{4\sin^2 x}{-2x\cos x} = -2\lim_{x\to 0^+} \frac{\sin^2 x}{x\cos x}$$

$$\overset{[\frac{0}{0}]}{=} -2\lim_{x\to 0^+} \frac{2\sin x\cos x \to 0}{\underset{\to 1}{\cos x} - \underset{\to 0}{x\sin x}} = 0$$

$$\text{Så } \lim_{x\to 0^+} (4x)^{2\sin x} = e^0 = 1 \qquad \boxed{A}$$

Oppgave 10

$$G(x) = \int_1^{\sqrt{x+1}} \ln t \, dt$$

$$F(x) = \int_1^{x} \ln t \, dt \qquad u(x) = \sqrt{x+1} \qquad \text{gir}$$

$$G(x) = F(u(x))$$

Kjerneregelen:

$$F'(x) = \ln x$$

$$G'(x) = F'(u(x))\cdot u'(x)$$

$$= \ln\left(\sqrt{x+1}\right)\cdot\frac{1}{2\sqrt{x+1}}, \quad \text{så } G'(1) = \frac{\ln\sqrt{2}}{2\sqrt{2}} = \frac{\ln 2^{1/2}}{2\sqrt{2}}$$

$$= \frac{\frac{1}{2}\ln 2}{2\sqrt{2}} = \frac{\ln 2}{4\sqrt{2}} \qquad \boxed{B}$$

Oppgave 11

$$\lim_{n\to\infty}\left[(-1)^n + \underbrace{\frac{n^2+8n+9}{n^3+7n}}_{0}\right]$$

Divergerer

$\boxed{E}$

Oppgave 12

$$V = \int_a^6 \pi\cdot[f(x)]^2\,dx = \int_0^1 \pi\left[x^7+14\right]dx$$

$$= \pi\left[\frac{1}{8}x^8+14x\right]_0^1 = \pi\left(\frac{1}{8}+14\right) = \frac{113\pi}{8}$$

$\boxed{D}$

Oppgave 13

$$\lim_{x\to 0}\frac{\sin^3 x}{2x^3+x^4} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0}\frac{3\sin^2 x\cdot\cos x}{6x^2+4x^3}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0}\frac{6\sin x\cos x\cdot\cos x + 3\sin^2 x\cdot(-\sin x)}{12x+12x^2}$$

$$= \lim_{x\to 0}\frac{6\sin x\cdot\cos^2 x - 3\sin^3 x}{12x+12x^2}$$

$$\overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0}\frac{6\cos^3 x + \overbrace{6\sin x\cdot 2\cos x(-\sin x) - 9\sin^2 x\cdot\cos x}^{0}}{12+\underbrace{24x}_{0}}$$

$$= \frac{6}{12} = \frac{1}{2}$$

$\boxed{B}$

Alternativt:

$$\lim_{x\to 0}\frac{\sin^3 x}{2x^3+x^4}$$

$$= \lim_{x\to 0}\frac{\left(\overbrace{\frac{\sin x}{x}}^{1}\right)^3}{2+x}$$

$$= \frac{1}{2}$$

Oppgave 14

g er den omvendte funksjonen til $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to\mathbb{R}$

gitt ved $f(x) = \sin^3 x + 1$

$$D_g = V_f = [0,2]$$

$\boxed{C}$

Oppgave 15

$$\lim_{x\to\infty}\left(e^{x} - x\ln x\right) \overset{[\infty-\infty]}{=} \lim_{x\to\infty} \underbrace{e^{x}}_{\to +\infty}\Big(1 - \underbrace{\frac{x\ln x}{e^{x}}}_{\to 0}\Big) = +\infty$$

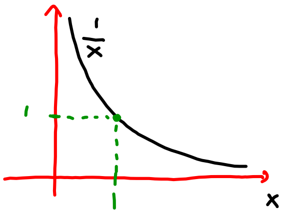
pga dette

Her: $$\lim_{x\to\infty}\frac{x\ln x}{e^{x}} \overset{[\frac{\infty}{\infty}]}{=} \lim_{x\to\infty}\frac{1\cdot\ln x + x\cdot\frac{1}{x}}{e^{x}}$$

$$= \lim_{x\to\infty}\frac{\ln x + 1}{e^{x}} \overset{[\frac{\infty}{\infty}]}{=} \lim_{x\to\infty}\frac{\overbrace{\frac{1}{x}}^{\to 0}}{\underbrace{e^{x}}_{\to +\infty}} = 0$$

$\boxed{A}$

Oppgave 16



$f(1) = 1$

$f(2) = \frac{1}{2}$

Siden f er kontinuerlig i $x = 1$, vet vi at det for alle $\varepsilon > 0$ fins $\delta > 0$ slik at

$$|x-1| < \delta \Rightarrow |f(x) - f(1)| < \varepsilon$$

Innsetting av $f(1) = 1$ og navnebytte $\delta \to t$ gir $\boxed{C}$

Oppgave 17

Sjekker skråasymptoter for $f(x) = 13x\underbrace{e^{\overbrace{-\frac{2}{x}}^{\to 0 \text{ når } x\to+\infty}}}_{\to 1}$

Skråasymptote: $y = ax + b$

$$a = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty} 13\underbrace{e^{-\frac{2}{x}}}_{\to 1} = 13$$

$$b = \lim_{x\to\infty}\left[f(x) - ax\right] = \lim_{x\to\infty}\left[13x\,e^{-\frac{2}{x}} - 13x\right]$$

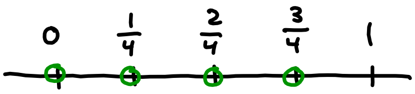
$$\overset{[\infty-\infty]}{=} \lim_{x\to\infty} 13x\left[e^{-\frac{2}{x}} - 1\right] \overset{[\infty\cdot 0]}{=} \lim_{x\to\infty} 13\cdot\frac{e^{-\frac{2}{x}} - 1}{\frac{1}{x}}$$

$$\overset{[\frac{0}{0}]}{=} 13\cdot\lim_{x\to\infty}\frac{e^{-\frac{2}{x}}\left(\frac{2}{\cancel{x^2}}\right)}{-\frac{1}{\cancel{x^2}}} = -26\cdot\lim_{x\to\infty}\underbrace{e^{-\frac{2}{x}}}_{\to 1} = -26$$

Altså er $y = 13x - 26$ en skråasymptote for f $\boxed{D}$

Oppgave 18

Intervall : $[0, 1]$



$$R(\Pi, u) = f(0)\cdot\frac{1}{4} + f\left(\frac{1}{4}\right)\cdot\frac{1}{4} + f\left(\frac{2}{4}\right)\cdot\frac{1}{4} + f\left(\frac{3}{4}\right)\cdot\frac{1}{4}$$

$$= \cancel{0^3\cdot\frac{1}{4}} + \left(\frac{1}{4}\right)^3\cdot\frac{1}{4} + \left(\frac{2}{4}\right)^3\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^3\cdot\frac{1}{4}$$

$$= \frac{1}{4^4}\left(1 + 2^3 + 3^3\right) = \frac{36}{256}$$

$\boxed{A}$