Løsningsforslag eksamen Mat 1100 09.12.2016

Oppgave 1

$$f(x,y) = x^3 y$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = \left(3x^2 y, x^3 \right)$$

$$\nabla f(3,1) = (27, 27)$$

$$f'(\vec{a};\vec{r}) = \vec{\nabla} f(\vec{a}) \cdot \vec{r} = (27,27)\cdot(1,-1) = \underline{\underline{0}}$$

$\boxed{A}$

Oppgave 2

$$\vec{a}\cdot\vec{b} = (2+i, 2i, 5)\cdot(1+4i, 3, 1-i)$$

$$= (2+i)(1-4i) + 2i\cdot 3 + 5(1+i)$$

$$= 2 + i - 8i + 4 + 6i + 5 + 5i = \underline{\underline{11 + 4i}}$$

$\boxed{C}$

Oppgave 3

$$\vec{F}(x,y) = (y \sin x, x^3 y) \quad \text{gir}$$

$$\vec{F}' = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} y \cos x & \sin x \\ 3x^2 y & x^3 \end{pmatrix}$$

$\boxed{A}$

Oppgave 4

$$\frac{1}{6} \begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & 4 \\ 3 & 0 & 3 \end{vmatrix} = \frac{1}{6}\left(1\cdot \begin{vmatrix} -2 & 4 \\ 0 & 3 \end{vmatrix} - 1\cdot \begin{vmatrix} 2 & 4 \\ 3 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \right)$$

$$= \frac{1}{6}\left(1\cdot(-6) - 1(6-12) + 1\cdot(0+6) \right)$$

$$= \frac{1}{6}(-6+6+6) = \underline{\underline{1}}$$

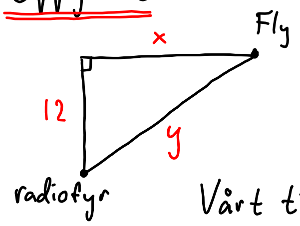
$\boxed{B}$

Oppgave 5

$$\begin{pmatrix} -5 & 2 \\ 8 & -3 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 8 & 5 \end{pmatrix} = \begin{array}{cc|cc} & & 3 & 2 \\ & & 8 & 5 \\ \hline -5 & 2 & 1 & 0 \\ 8 & -3 & 0 & 1 \end{array} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\boxed{D}$

Oppgave 6



$$12^2 + x^2 = y^2$$

$$2x \cdot x' = 2y \cdot y'$$

Vårt tidspunkt: $x = 9$

$$y = \sqrt{12^2 + 9^2} = 15$$

$$y' = 600$$

Innsatt: $18 \cdot x' = 30 \cdot 600$, så $x' = 1000$

$\boxed{D}$

Oppgave 7

$\boxed{D}$

Oppgave 8

$$\int \cos x \cdot \arctan(\sin x)\,dx = \int \arctan u \, du = \int (\arctan u) \cdot 1 \, du$$

$u = \sin x \qquad \frac{du}{dx} = \cos x$

$du = \cos x \, dx \qquad dx = \frac{1}{\cos x} du$

$$= u \cdot \arctan u - \int \frac{u}{1+u^2} du = u \cdot \arctan u - \frac{1}{2} \int \frac{1}{v} dv$$

Delvis $\quad F(u) = \arctan u \qquad G'(u) = 1$

$F'(u) = \frac{1}{1+u^2} \qquad G(u) = u$

$v = 1 + u^2 \qquad \frac{dv}{du} = 2u$

$dv = 2u\,du \qquad du = \frac{1}{2u} dv$

$$= u \cdot \arctan u - \frac{1}{2} \ln |v| + C$$

$$= \sin x \cdot \arctan(\sin x) - \frac{1}{2} \ln(1 + \sin^2 x) + C$$

Så

$$\int_0^{\pi/2} \cos x \cdot \arctan(\sin x)\,dx = \left[\sin x \cdot \arctan(\sin x) - \frac{1}{2}\ln(1+\sin^2 x)\right]_0^{\frac{\pi}{2}}$$

$$= \left[1 \cdot \arctan 1 - \frac{1}{2}\ln(1+1)\right] - \left[0 - 0\right] = \frac{\pi}{4} - \frac{1}{2}\ln 2$$

$\boxed{E}$

Oppgave 9

$$\int \frac{\cos(1/x)}{x^2}\,dx = \int \frac{\cos u}{x^2}(-x^2)\,du = -\int \cos u\,du$$

$u = \frac{1}{x}$ $\quad \frac{du}{dx} = -\frac{1}{x^2}$

$du = -\frac{1}{x^2}dx \quad dx = -x^2 du$

$$= -\sin u + C$$

$$= -\sin\left(\frac{1}{x}\right) + C$$

Så:

$$\int_1^\infty \frac{\cos(1/x)}{x^2}\,dx = \lim_{b\to\infty}\left[-\sin\left(\frac{1}{x}\right)\right]_1^b$$

$$= \lim_{b\to\infty}\left[-\sin\left(\underbrace{\frac{1}{b}}_{\to 0}\right) + \sin 1\right] = \sin 1$$

$\boxed{C}$

Oppgave 10

$\boxed{C}$

Oppgave 11

a) $(3+i)^2 = 9 + 6i + i^2 = 8 + 6i$

$(3+i)^3 = (3+i)(8+6i) = 24 + 8i + 18i - 6$

$= 18 + 26i$

$P(3+i) = 18 + 26i - 13(8+6i) + 52(3+i) - 70$

$= 18 + 26i - 104 - 78i + 156 + 52i - 70 = \underline{0}$

Altså er $3+i$ en rot til $P(z)$.

Siden $P(z)$ har reelle koeffisienter, er også $3-i$ en rot.

$(z-(3+i))\cdot(z-(3-i)) = (z-3-i)\cdot(z-3+i)$

$= z^2 - 3z + iz - 3z + 9 - 3i - iz + 3i + 1 = \underline{z^2 - 6z + 10}.$

Polynomdivisjon:

$$(z^3 - 13z^2 + 52z - 70) : (z^2 - 6z + 10) = \underline{z - 7}$$

$$\begin{array}{l} z^3 - 6z^2 + 10z \\ \hline -7z^2 + 42z - 70 \\ -7z^2 + 42z - 70 \\ \hline 0 \end{array}$$

De øvrige røttene er:

$\underline{\underline{z = 3 - i}}$ og $\underline{\underline{z = 7}}$

b) Kompleks faktorisering:

$$P(z) = \underline{\underline{(z-(3+i))\cdot(z-(3-i))\cdot(z-7)}}$$

Reell faktorisering:

$$P(z) = \underline{\underline{(z^2-6z+10)(z-7)}}$$

Oppgave 12

a) $$f(x) = x^3\begin{vmatrix} 0 & 0 \\ x & -1\end{vmatrix} - 3x\begin{vmatrix} e^{x^3} & 0 \\ x^2 & -1\end{vmatrix} + 1\begin{vmatrix} e^{x^3} & 0 \\ x^2 & x\end{vmatrix}$$

$$= x^3\cdot 0 - 3x\left(-e^{x^3}\right) + xe^{x^3} = 4xe^{x^3}$$

$$V = \int_0^a \pi\left[f(x)\right]^2 dx = \int_0^a \pi\, 16x^2\left(e^{x^3}\right)^2 dx$$

$$= \underline{\underline{16\pi\int_0^a x^2 e^{2x^3}\,dx}}.$$

b) $$V = 16\pi\int_0^a x^2 e^{2x^3}\,dx = 16\pi\int_0^{2a^3}\frac{1}{6}e^u\,du$$

$$\boxed{\begin{array}{ll} u = 2x^3 & \frac{du}{dx} = 6x^2 \\ du = 6x^2\,dx & dx = \frac{1}{6x^2}du \\ x = 0 \text{ gir } u = 0 & \\ x = a \text{ gir } u = 2a^3 & \end{array}}$$

$$= \frac{16}{6}\pi\left[e^u\right]_0^{2a^3}$$

$$= \underline{\underline{\frac{8}{3}\pi\left(e^{2a^3}-1\right)}}$$

Oppgave 13

a) $$\lim_{x\to 0^+} x\cdot(\ln x)^2 \overset{[0\cdot\infty]}{=} \lim_{x\to 0^+} \frac{(\ln x)^2}{\frac{1}{x}}$$

$$\overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{x\to 0^+} \frac{2\ln x\cdot\frac{1}{x}}{-\frac{1}{x^2}} = -\lim_{x\to 0^+} \frac{2\ln x}{\frac{1}{x}}$$

$$\overset{\left[\frac{\infty}{\infty}\right]}{=} -\lim_{x\to 0^+} \frac{\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 2x = \underline{\underline{0}}$$

b) Siden $\ln x$ er kontinuerlig og ulik 0 for $x\in(0,1)$, er $f(x)$ kontinuerlig på $(0,1)$. At f er kontinuerlig på $(-\infty,0)$ fås fordi den er konstant der. Videre:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 0 = 0 = f(0)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{\underbrace{(\ln x)^2}_{\to +\infty}} = 0 = f(0)$$

Altså er f kontinuerlig i $x=0$ også. Så f er kontinuerlig.

c) Siden f er kontinuerlig, er $g'(x) = f(x)$ for alle $x\in(-\infty,1)$ ved fundamentalteoremet.
Vi får da:

$$g''(0) = \lim_{h\to 0} \frac{g'(0+h)-g'(0)}{h} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

Men vi har

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{\frac{1}{(\ln h)^2}-0}{h}$$

$$= \lim_{h\to 0^+} \frac{1}{\underbrace{h(\ln h)^2}_{\to 0 \text{ ved a)}}} = +\infty$$

<u>Så $g''(0)$ eksisterer ikke</u>