c) $A = \{(x,y,z) : x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,$

$z = \rho\cos\phi,\ \phi \in [0,\pi],\ \theta \in [0,2\pi],$

$\rho \in [0,1]\}$

$$\iiint_A e^{-\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$$

$$= \int_0^1 \int_0^\pi \int_0^{2\pi} e^{-\rho}\rho^2 \sin\phi \, d\theta\, d\phi\, d\rho$$

$$= 2\pi \int_0^\pi \sin\phi\, d\phi \int_0^1 e^{-\rho}\rho^2\, d\rho$$

<u>M:</u> $\int_0^1 e^{-\rho}\rho^2\,d\rho = [-\rho^2 e^{-\rho}]_{\rho=0}^1$

Delvis int. $u = \rho^2,\ u' = 2\rho$
$v' = e^{-\rho},\ v = -e^{-\rho}$

$+ 2\int_0^1 \rho e^{-\rho}\,d\rho = -\frac{1}{e} + 2\Big[[-\rho e^{-\rho}]_{\rho=0}^1$

$u = \rho,\ u' = 1$
$v' = e^{-\rho}\ \ v = -e^{-\rho}$

$+ \int_0^1 e^{-\rho}\,d\rho\Big]$

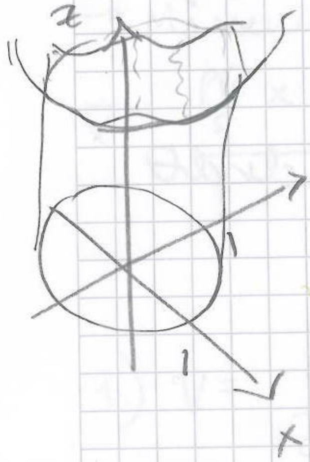
$$= -\frac{1}{e} - 2\left(\frac{1}{e} + [-e^{-\rho}]_{\rho=0}^1\right)$$

$$= -\frac{3}{e} - 2\left(\frac{1}{e} + 1\right) = \underline{2 - \frac{5}{e}}$$

Sett dette inn over + litt regning ⇒ svar!

d) $\iiint_A \sqrt{x^2+y^2}\, dx\,dy\,dz$, A inni sylinder

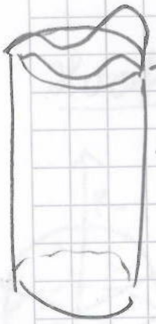
$x^2+y^2=1$, mellom xy-plan og $z=(x^2+y^2)^{\frac{3}{2}}$



Sylinderkoordinater: $\theta \in [0, 2\pi]$, $r \in [0,1]$, $z \in [0, r^3]$

$$\int_0^{2\pi}\int_0^1\int_0^{r^3} r \cdot r \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left[r^2 z\right]_{z=0}^{r^3} dr\, d\theta$$



A: En sylinder med ujevn topp

$$= \int_0^{2\pi}\int_0^1 r^5\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[\frac{1}{6} r^6\right]_{r=0}^{1} d\theta$$

$$= \frac{1}{6}\int_0^{2\pi} 1\, d\theta = \frac{2\pi}{6} = \underline{\underline{\frac{\pi}{3}}}$$

e) $x^2-2x+y^2=1 \Leftrightarrow (x-1)^2+y^2=2$;

sirkel, sentrum $(1,0)$, radius $\sqrt{2}$.

Sett $u=x-1$, $v=y$, $w=z$ ⇒

Jacobideterminant $=1$.

La D være sylinderen mellom $w=0$ og $w=2$: