Plenum 8/4 -15

6.9: 2abd

6.10: 1ab, 2ab, 3acde, $\boxed{7}$

6.11: 1, 6

4.1: 1, 3, 4, $\boxed{6}$

4.2: 1, $\boxed{2bc}$, 3, $\boxed{4}$, $\boxed{5}$, 10

Her: 6.10: 7
4.1: 6.
4.2: 2bc, 4, 5

Resten i bok fra i fjor

6.10: Skifte av variable i trippelintegraler

7.) A: kule sentrum origo, radius R. Anta $a > R$.

VIS: $$\iiint_A \frac{1}{\sqrt{x^2+y^2+(z-a)^2}}\,dx\,dy\,dz = \frac{4\pi R^3}{3a}$$

Kulekoordinater: $x = \rho \sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$
og $z = \rho\cos\phi$, $\phi \in [0,\pi]$, $\theta \in [0, 2\pi]$, $\rho \in [0,R]$

Jacobideterminant: $\rho^2 \sin\phi$

$$\iiint_A \frac{1}{\sqrt{x^2+y^2+(z-a)^2}}\,dx\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{\rho^2\sin\phi}{\sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta + (\rho\cos\phi - a)^2}}\,d\rho\,d\phi\,d\theta$$

$\sin^2 + \cos^2 = 1$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{\rho^2\sin\phi}{\sqrt{\rho^2\sin^2\phi + \rho^2\cos^2\phi - 2\rho\cos\phi\, a + a^2}}\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{1}{\sqrt{\rho^2 - 2\rho\cos(\phi)a + a^2}}\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= (2\pi - 0) \int_0^{R}\int_0^{\pi} \frac{\rho^2 \sin\phi}{\sqrt{\rho^2 - 2\rho\cos(\phi)a + a^2}}\, d\phi\, d\rho$$

ingen θ i det vi integrerer

$$= 2\pi \int_0^{R} \int_{\rho^2 - 2\rho a + a^2}^{\rho^2 + 2\rho a + a^2} \frac{\rho}{2a} \frac{1}{\sqrt{u}}\, du\, d\rho$$

$u = \rho^2 - 2\rho\cos(\phi)a + a^2$

$du = -2\rho\sin(\phi)a\, d\phi$

$\frac{du}{2\rho\sin(\phi)a} = d\phi$

$\frac{1}{\sqrt{u}} = u^{-\frac{1}{2}}$

$\phi = 0 \Rightarrow u = \rho^2 - 2\rho a + a^2$

$\phi = \pi \Rightarrow u = \rho^2 + 2\rho a + a^2$

$$= \frac{\pi}{a} \int_0^{R} \rho \left[2u^{\frac{1}{2}}\right]_{u=\rho^2 - 2\rho a + a^2}^{\rho^2 + 2\rho a + a^2} d\rho$$

$$= \frac{2\pi}{a} \int_0^{R} \rho\left(\sqrt{\rho^2 + 2\rho a + a^2} - \sqrt{\rho^2 - 2\rho a + a^2}\right) d\rho$$

$$= \frac{2\pi}{a} \int_0^{R} \rho\left(\sqrt{(\rho+a)^2} - \sqrt{(\rho-a)^2}\right) d\rho$$

$$= \frac{2\pi}{a} \int_0^{R} \rho\left(\rho + \cancel{a} - \cancel{a} + \rho\right) d\rho$$

$a > R > 0 \Rightarrow \rho + a > 0$

$\rho \in [0, R]$ & $a > R \Rightarrow \rho - a < 0 \Rightarrow \sqrt{(\rho - a)^2} = |\rho - a| = -(\rho - a) = a - \rho$

$$= \frac{2\pi}{a} \int_0^{R} 2\rho^2\, d\rho = \frac{4\pi}{a}\left[\frac{1}{3}\rho^3\right]_{\rho=0}^{R}$$

$$= \frac{4\pi R^3}{3a}$$

som var det vi ville vise.