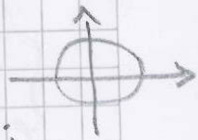
[Trig formel: se Rane]

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\cos\theta + \frac{1}{4}\sin^2\theta\right) d\theta = \frac{1}{3}\left[\sin\theta\right]_{\theta=0}^{\frac{\pi}{4}}$$

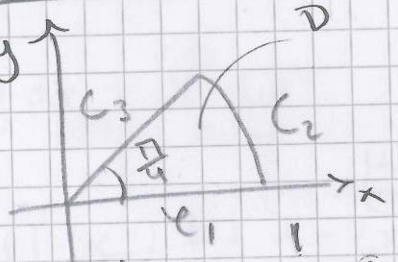
$$+ \frac{1}{4}\int_0^{\frac{\pi}{4}} \frac{1-\cos 2\theta}{2}\, d\theta = \frac{1}{3}\left(\frac{\sqrt{2}}{2} - 0\right) + \frac{1}{4}\frac{1}{2}\frac{\pi}{4} - \frac{1}{4}\frac{1}{2}\left[\frac{1}{2}\sin(2\theta)\right]_{\theta=0}^{\frac{\pi}{4}}$$

$$= \frac{\sqrt{2}}{6} + \frac{\pi}{32} - \frac{1}{16}(1-0) = \underline{\underline{\frac{\sqrt{2}}{6} + \frac{\pi}{32} - \frac{1}{16}}}$$



Fra fig. forrige side:

b)



Int. over randen til D, $\mathcal{C}$:

$$\int_{\mathcal{C}} = \int_{\mathcal{C}_1} + \int_{\mathcal{C}_2} + \int_{\mathcal{C}_3}$$

Parametriserer alle kurvene orientert mot klokka (pos. orientert):

$$\vec{r}_1(t) = \mathcal{C}_1 = (t, 0), \quad t \in [0, 1]$$

$$\vec{r}_2(t) = \mathcal{C}_2 = (\cos t, \sin t), \quad t \in \left[0, \frac{\pi}{4}\right]$$

$$\vec{r}_3(t) = \mathcal{C}_3 = (t, t), \quad t \in \left[\frac{\sqrt{2}}{2}, 0\right]$$

ditte er $\cos\frac{\pi}{4} = \sin\frac{\pi}{4}$

MERK: $\iint_D (x+y^2)\,dx\,dy = \int_{\mathcal{C}} P\,dx + Q\,dy$

randa til D er enkel, lukket, stykkvis glatt kurve orientert mot klokka

hvis $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = x + y^2$ fra Greens thm.

Velger (f. eks.) $P = 0$, $\frac{\partial Q}{\partial x} = x + y^2$

Mange muligheter

$$\Downarrow$$

$$Q = \frac{1}{2}x^2 + xy^2$$

Så

$$\iint_D (x+y^2)\,dx\,dy = \int_{\mathcal{C}} \left(\frac{1}{2}x^2 + xy^2\right) dy$$

Greens teorem

$$= \int_{\mathcal{C}_1} \left(\frac{1}{2}x^2 + xy^2\right) dy + \int_{\mathcal{C}_2} \left(\frac{1}{2}x^2 + xy^2\right) dy$$

$$+ \int_{\mathcal{C}_3} \left(\frac{1}{2}x^2 + xy^2\right) dy$$

$dy = 0\,dt$

$$\mathcal{C}_1 : I_1 = \int_0^1 \left(\frac{1}{2}t^2 + t\cdot 0^2\right)\cdot 0\,dt = 0$$

$dy = \cos t\,dt$

$$\mathcal{C}_2 : I_2 = \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\cos^2 t + \cos t \sin^2 t\right)\cos t\,dt$$

Del i 2. Tas v/ hhv. $\cos^2 = 1 - \sin^2$ og substitusjon $u = 1 - \sin^2$ samt $\cos^2 t = \frac{1+\cos 2t}{2}$ og $\sin^2 t = \frac{1-\cos 2t}{2}$ og $\cos^2 2t = \frac{1+\cos 4t}{2}$

$dy = 1\,dt$

$$\mathcal{C}_3 : I_3 = \int_{\frac{\sqrt{2}}{2}}^{0} \left(\frac{1}{2}t^2 + t^3\right)\cdot 1\,dt = \ldots$$

13.) <u>Vis</u>: $\vec{F}(x,y) = (P(x,y), Q(x,y))$ konservativ

$$\Downarrow$$

$\int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$ for enkle, lukkede, stykkevis glatte kurver.

<u>Pf</u>: $\vec{F}$ konservativ $\Rightarrow \exists\ \phi : \mathbb{R}^2 \to \mathbb{R}$ s.a. $\vec{F} = \nabla\phi$

$$= \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right) \Rightarrow P = \frac{\partial\phi}{\partial x},\ Q = \frac{\partial\phi}{\partial y}.$$

La $\vec{r}(t)$ være en parametrisering av $\mathcal{C}$.