UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT1110 — Calculus and linear algebra.

Day of examination: Wednesday, June 10, 2015.

Examination hours: 09.00 – 13.00.

This problem set consists of 2 pages.

Appendices: Formula sheet.

Permitted aids: Approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All items (Problem 1a, 1b, 2, 3a, 3b etc.) count 10 points.

Problem 1. In this problem $f : \mathbb{R}^2 \to \mathbb{R}$ is the function

$$f(x, y) = 2x^2y + 2xy + y^2$$

a) (10 points) Find the stationary points of f.

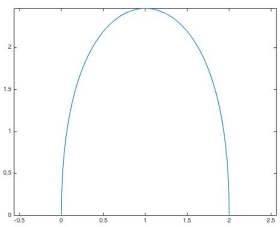
b) (10 points) Decide whether the stationary points are saddle points, local minima or local maxima.

Problem 2. (10 points) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n2^n}$$

Problem 3. The figure shows a MATLAB-plot of a curve $\mathcal{C}$ with parametrization

$$\mathbf{r}(t) = (1 + \cos t)\,\mathbf{i} + t(\pi - t)\,\mathbf{j}\,, \quad \text{where } t \in [0, \pi]$$



a) (10 points) Explain that the area A of the region between the curve and the x-axis is given by

$$A = \int_{\mathcal{C}} x\,dy + \int_{\mathcal{D}} x\,dy$$

where $\mathcal{D}$ is the line segment from $(0, 0)$ to $(2, 0)$.

(Continued on page 2.)

b) (10 points) Calculate A.

Problem 4. In this problem, V is the volume of the region bounded by the two paraboloids

$$z = x^2 + 2x + y^2 - 4y$$
$$z = 6 - x^2 - 2x - y^2 - 4y$$

a) (10 points) Explain that

$$V = 2\iint_D (3 - x^2 - y^2 - 2x)\,dxdy$$

where D is a region in the xy-plane. What region is D?

b) (10 points) Calculate V.

Problem 5. In this problem you can without proof use the following consequence of the spectral theorem: If $\mathbf{v}_1$ is an eigenvector of a symmetric $n \times n$-matrix A, there is an orthogonal basis of eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ of A that contains $\mathbf{v}_1$ (a basis is *orthogonal* if all the vectors are perpendicular to each other).

Throughout the problem, A_n is the $n \times n$-matrix where all the elements are 1, i.e.

$$A_n = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ \vdots & \vdots & \ldots & \vdots \\ 1 & 1 & \ldots & 1 \end{pmatrix}$$

a) (10 points) Show that

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

is an eigenvector for A_n. What is the eigenvalue? Show also that all non-zero vectors perpendicular to $\mathbf{v}_1$ are eigenvectors. How many different eigenvalues does A_n have, and what is their multiplicity?

b) (10 points) Find an orthogonal basis of eigenvectors of A_3.

c) (10 points) For each real number a, we let $A_n(a)$ be the matrix

$$A_n(a) = \begin{pmatrix} a & 1 & \ldots & 1 \\ 1 & a & \ldots & 1 \\ \vdots & \vdots & \ldots & \vdots \\ 1 & 1 & \ldots & a \end{pmatrix} = (a-1)I_n + A_n$$

Show that the eigenvectors of A_n are also eigenvectors of $A_n(a)$. What are the eigenvalues of $A_n(a)$, and what multiplicity do they have?

THE END