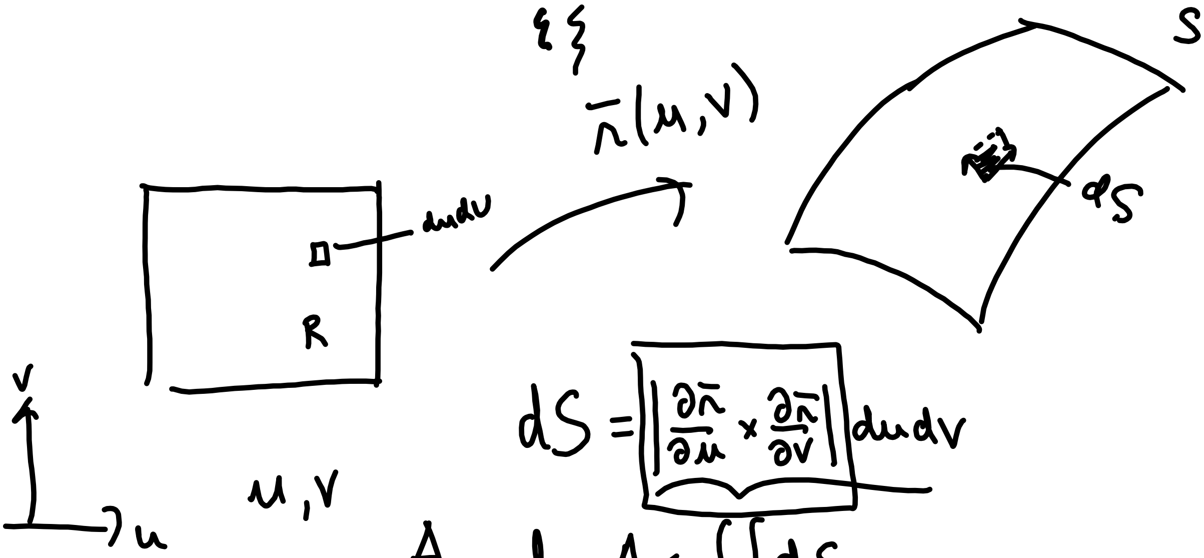
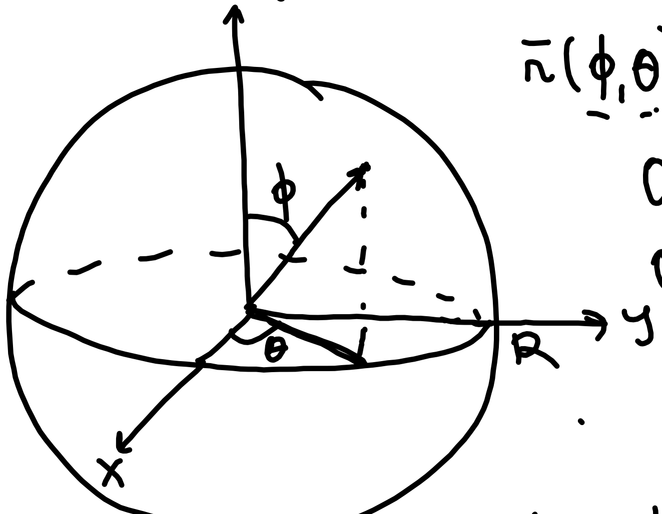


$$dS = \underbrace{\boxed{\left\| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right\|}}\, du\,dv$$

Areal $A = \iint_R dS$

$$\iint_S f\,dS = \iint_R f(\bar{r}(u,v))\,dS$$

6.4.4. Overflate av kule med radius R.



$$\bar{r}(\phi,\theta) = (R\sin\phi\cos\theta,\ R\sin\phi\sin\theta,\ R\cos\phi)$$

$$0 \le \theta \le 2\pi,$$

$$0 \le \phi \le \pi.$$

$$\frac{\partial \bar{r}}{\partial \phi} \times \frac{\partial \bar{r}}{\partial \theta} = \begin{vmatrix} i & j & k \\ R\cos\phi\cos\theta & R\cos\phi\sin\theta & -R\sin\phi \\ -R\sin\phi\sin\theta & R\sin\phi\cos\theta & 0 \end{vmatrix} =$$

$$(R^2\sin^2\phi\cos\theta,\ R^2\sin^2\phi\sin\theta,\ R^2\sin\phi\cos\phi)$$

$$|\quad|^2 = R^4\sin^4\phi\cos^2\theta + R^4\sin^4\phi\sin^2\theta + R^4\sin^2\phi\cos^2\phi)$$

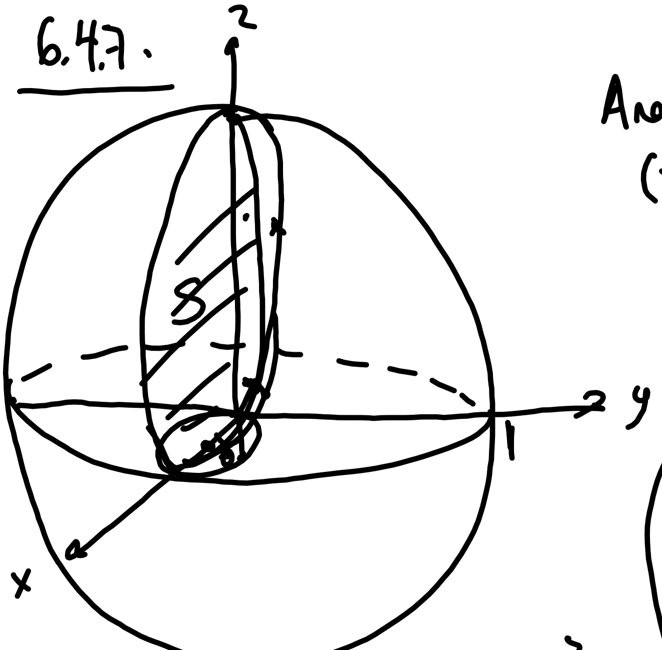
$$= R^4\sin^2\phi\sin^2\phi + R^4\sin^2\phi\cos^2\phi = R^4\sin^2\phi$$

$$|\quad| = R^2\sin\phi \qquad \boxed{dS = R^2\sin\phi\, d\phi\, d\theta}$$

$$A = \iint_S dS = \int_0^{2\pi}\left(\int_0^{\pi} R^2 \sin\phi \, d\phi\right) d\theta = \int_0^{2\pi} 2R^2 \, d\theta = 2R^2 \cdot 2\pi$$

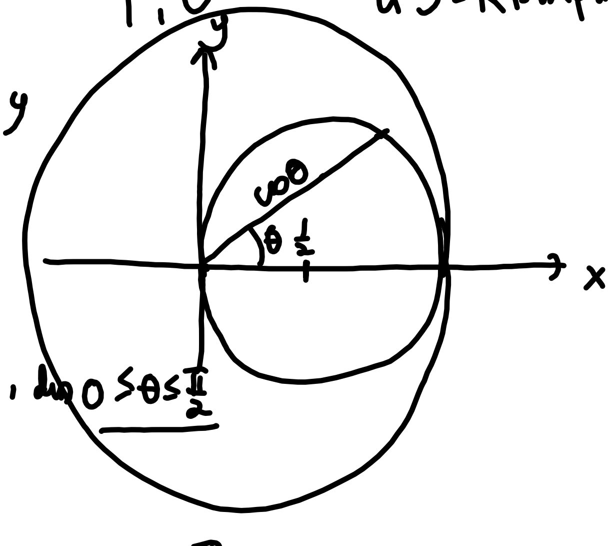
$$= \underline{\underline{4\pi R^2}}$$

6.4.7.

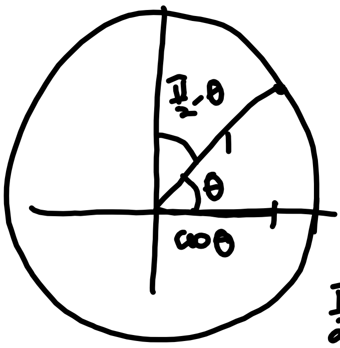


Arealet over disken
$(x-\frac{1}{2})^2 + y^2 \le \frac{1}{4}$

ϕ, θ $\quad dS = R^2 \sin\phi \, d\phi \, d\theta$



$y \ge 0$, så $\underline{0 \le \theta \le \frac{\pi}{2}}$

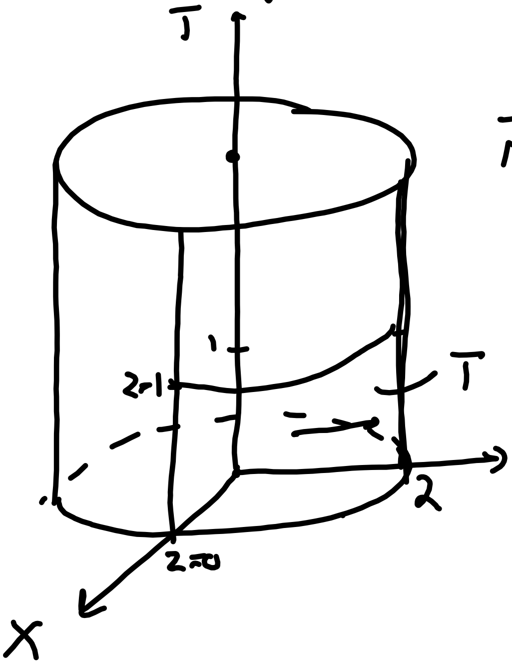


$0 \le \phi \le \frac{\pi}{2} - \theta$

$$A = \iint_S dS = 2\int_0^{\frac{\pi}{2}}\left(\int_0^{\frac{\pi}{2}-\theta} \sin\phi \, d\phi\right) d\theta = 2\int_0^{\frac{\pi}{2}} \left(-\cos\phi\right)\Big|_0^{\frac{\pi}{2}-\theta} d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} -\sin\phi + 1 \, d\theta = 2\left(\frac{\pi}{2} - 1\right) = \underline{\underline{\pi - 2}}$$

6.4.10. $\iint_T xyz^2\,dS$ T del av sylinderflaten $x^2+y^2=4$ der $x\geq 0$, $y\geq 0$ og $0\leq z\leq 1$



$$\bar{r}(\theta,z)=(\underline{2\cos\theta},\underline{2\sin\theta},\underline{z})$$

$$0\leq\theta\leq\frac{\pi}{2}$$

$$0\leq z\leq 1.$$

$$\underbrace{\frac{\partial\bar{r}}{\partial\theta}}\times\frac{\partial\bar{r}}{\partial z}=\begin{vmatrix} i & j & k\\ -2\sin\theta & 2\cos\theta & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$=(2\cos\theta,2\sin\theta,0)$$

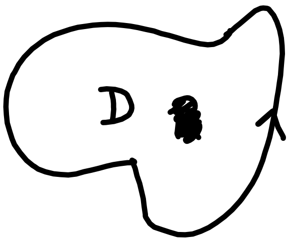
$$dS=\left|\frac{\partial\bar{r}}{\partial\theta}\times\frac{\partial\bar{r}}{\partial z}\right|d\theta\,dz=2\,d\theta\,dz$$

xyz^2.

$$\iint_T xyz^2\,dS=\int_0^{\frac{\pi}{2}}\left(\int_0^1 2\cos\theta\cdot 2\sin\theta\, z^2 2\,dz\right)d\theta=\int_0^{\frac{\pi}{2}}\frac{8}{3}\sin\theta\cos\theta\,d\theta=$$

$$\frac{8}{3}\left(\frac{1}{2}\sin^2\theta\right)\Big|_0^{\frac{\pi}{2}}=\frac{8}{3}\cdot\frac{1}{2}=\underline{\underline{\frac{4}{3}}}$$

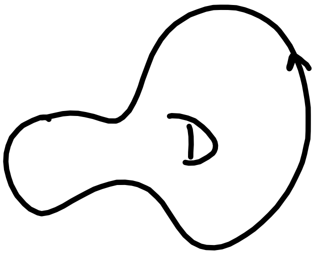
6.



C orientert mot klokka.

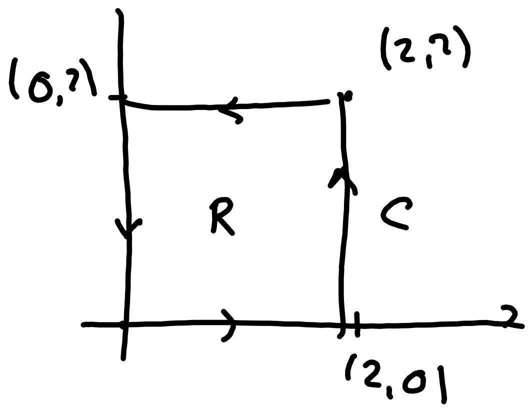
$$\int_C P\,dx + Q\,dy = \iint_D \overbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}\,dx\,dy.$$

$$\| \quad 1$$



$$A = \int_C x\,dy = -\int_C y\,dx$$

6.5.1. a)

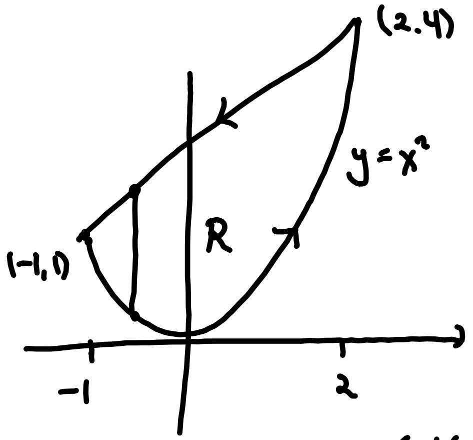


$$\int_C \underbrace{(x^2+y)}_{P}\,dx + \underbrace{x^2 y}_{Q}\,dy$$

$$= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \iint_R 2xy - 1 \; dx\,dy = \dots = \underline{\underline{4.}}$$

d) $\int_C (\underline{x^2 y + xe^x})\,dx + (\underline{xy^3 + e^{\sin y}})\,dy$

$P \qquad Q$



$= \iint_R y^3 - x^2 \, dx\, dy$

$= \int_{-1}^{2} \left(\int_{x^2}^{x+2} y^3 - x^2 \, dy \right) dx$

$y - 1 = \frac{3}{3}(x+1)$

$y < x+2$

$-1 \le x \le 2$

$x^2 \le y \le x+2$

$= \int_{-1}^{2} \frac{1}{4}y^4 - x^2 y \Big|_{y=x^2}^{y=x+2} = \int_{-1}^{2} \frac{1}{4}(x+2)^4 \underline{- x^2(x+2)} - \frac{1}{4}x^8 + x^4 \, dx$

$= \frac{1}{20}(x+2)^5 - \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{36}x^9 + \frac{1}{5}x^5 \Big|_{-1}^{2} = \cdots = \underline{\underline{\frac{135}{4}}}.$

6.5.2. $\vec{r}(t) = (\underbrace{t\sin t}, \underbrace{2\pi t - t^2}_{t(2\pi - t)})$ $\quad 0 \leq t \leq 2\pi$

$$A = \int_C x\,dy = \int_0^{2\pi} t\sin t\,(2\pi - 2t)\,dt = \int_0^{2\pi} \underbrace{(2\pi t - 2t^2)}_{u} \underbrace{\sin t}_{v'}\,dt$$

$u' = 2\pi - 4t \quad v = -\cos t$

$$= (2\pi t - 2t^2)(-\cos t)\Big|_0^{2\pi} + \int_0^{2\pi} \underbrace{(2\pi - 4t)}_{u} \underbrace{\cos t}_{v'}\,dt$$

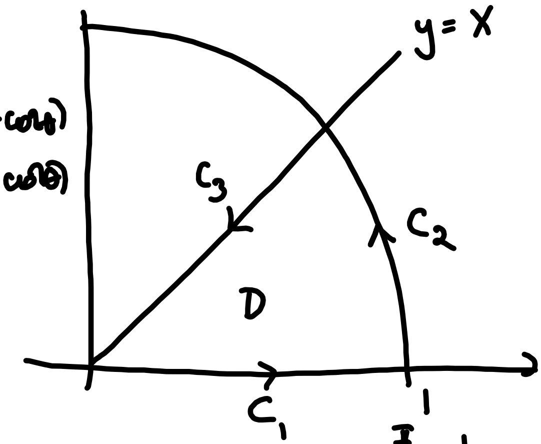
$u' = -4 \quad v = \sin t$

$$= \underbrace{(4\pi^2 - 8\pi^2)(-1)} + \underbrace{(2\pi - 4t)\sin t\Big|_0^{2\pi}} + \underbrace{\int_0^{2\pi} 4\sin t\,dt} = 4\pi^2 + 0 + 0 = \underline{\underline{4\pi^2}}.$$

6.5.10.

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$



$C = C_1 \cup C_2 \cup C_3$

$0 \le \theta \le \frac{\pi}{4}$

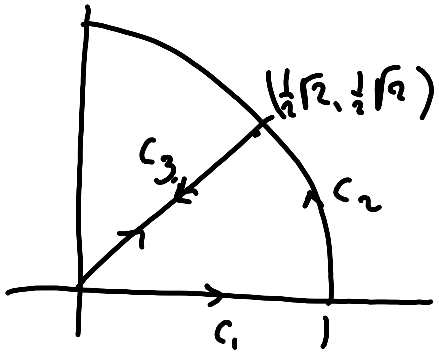
$0 \le r \le 1$

$$I = \iint_D (x+y^2)\,dx\,dy = \int_0^{\frac{\pi}{4}} \left(\int_0^1 (r\cos\theta + r^2\sin^2\theta)\, r\, dr \right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos\theta + \frac{1}{4}\sin^2\theta \, d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos\theta + \frac{1}{4}\frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{3}\sin\theta + \frac{1}{8}\theta - \frac{1}{16}\sin 2\theta \Big|_0^{\frac{\pi}{4}} = \underline{\underline{\frac{1}{6}\sqrt{2} + \frac{\pi}{32} - \frac{1}{16}}}.$$

$\iint (\underbrace{x+y^2}) \, dx\,dy$



b) Finn P, Q slik at

$$\underline{x+y^2 = \frac{\partial Q}{\partial x}} - \cancel{\frac{\partial P}{\partial y}}$$

$\int P\,dx + Q\,dy$

$-C_3 \quad (t,t)$

$0 \le t \le \frac{1}{2}\sqrt{2}$

Velger $P=0$,

$$Q = \frac{1}{2}x^2 + xy^2$$

$$I = \int_C \left(\frac{1}{2}x^2 + xy^2\right) dy$$

$C_1: \ \bar{r}(t) = (t, 0) \Rightarrow dy = 0\,dt \qquad \int_{C_1} Q\,dy = 0$

$C_2: \ \bar{r}(\theta) = (\underline{\cos\theta}, \underline{\sin\theta}), \ 0 \le \theta \le \frac{\pi}{4}$

$$\int_{C_2} \left(\frac{1}{2}x^2 + xy^2\right) dy = \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\underline{\cos^2\theta} + \sin^2\theta\cos\theta\right) \cdot \cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2} - \frac{1}{2}\sin^2\theta\right)\cos\theta + \sin^2\theta\cos^2\theta \, d\theta =$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}\cos\theta - \frac{1}{2}\sin^2\theta\cos\theta + \frac{1}{4}(\sin 2\theta)^2 \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}\cos\theta - \underline{\frac{1}{2}\sin^2\theta\cos\theta} \quad + \underline{\frac{1}{4}\cdot\frac{1}{2}(1} - \cos 4\theta)\, d\theta$$

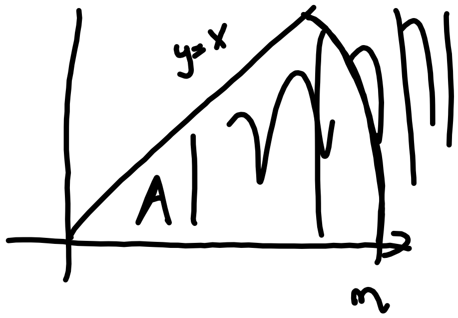
$$= \frac{1}{2}\sin\theta - \frac{1}{2}\cdot\frac{1}{3}\sin^3\theta + \frac{1}{8}\theta - \frac{1}{8}\cdot\frac{1}{4}\sin 4\theta \Big|_0^{\frac{\pi}{4}} = \frac{5}{24}\sqrt{2} + \frac{\pi}{32}$$

$C_3: \ \bar{r}(t) = (t,t), \ 0 \le t \le \frac{1}{2}\sqrt{2}$, parametrisk motsatt vei.

$$\int_{-C_3} \left(\frac{1}{2}x^2 + xy^2\right) dy = -\int_0^{\frac{1}{2}\sqrt{2}} \left(\frac{1}{2}t^2 + t^3\right) dt = \dots = -\frac{1}{24}\sqrt{2} - \frac{1}{16}.$$

<u>6.8.1</u>

$$\iint_A e^{-(x^2+y^2)}\,dx\,dy$$



$$= \lim \underbrace{\iint_{A\cap B(0,n)}}\, e^{-(x^2+y^2)}\,dx\,dy$$

$0 \le \theta \le \frac{\pi}{4}$

$0 \le r \le \infty$

$$= \lim_{n\to\infty} \int_0^n \left(\int_0^{\pi/4} e^{-r^2} r \, d\theta \right) dr = \lim_{n\to\infty} \int_0^n \frac{\pi}{4} \underbrace{r e^{-r^2}}\, dr$$

$$= \lim \frac{\pi}{4}\left(-\frac{1}{2}e^{-r^2}\right)\Big|_0^n = \lim_{n\to\infty} \frac{\pi}{4}\left(\underline{-\frac{1}{2}e^{-n^2}} + \frac{1}{2}\right) = \frac{\pi}{4}\cdot\frac{1}{2} = \underline{\frac{\pi}{8}}$$

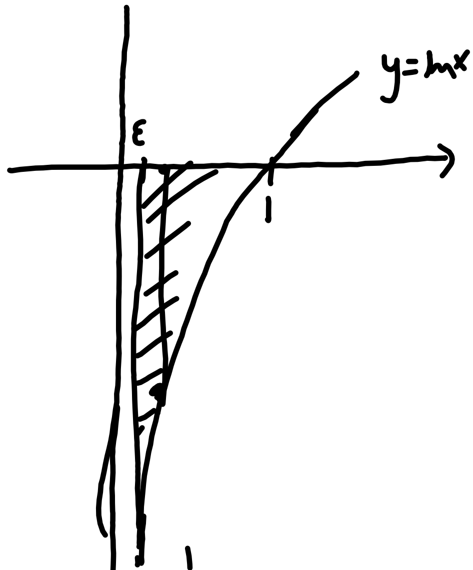
6.8.3.

$$\iint_A x\,dx\,dy$$

$$= \lim_{\varepsilon \to 0} \int_{\varepsilon}^{1} \left(\int_{\ln x}^{0} x\,dy \right) dx$$

$$= \lim_{\varepsilon \to 0} \int_{\varepsilon}^{1} \underbrace{-x}_{v'} \underbrace{\ln x}_{u}\,dx = \dots = \lim_{\varepsilon \to 0} \left(-\frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 \right) \Big|_{\varepsilon}^{1}$$

$v = -\frac{1}{2}x^2 \quad u' = \frac{1}{x}$



$$\lim_{\varepsilon \to 0} \left(\frac{1}{4} + \frac{1}{2}\varepsilon^2 \ln \varepsilon - \frac{1}{4}\varepsilon^2 \right) = \frac{1}{4} + 0 + 0 = \underline{\underline{\frac{1}{4}.}}$$

$$\frac{\ln \varepsilon}{\frac{1}{\varepsilon^2}}$$