4.3.2

$\mathrm{II} - 3\cdot\mathrm{I}$

a) $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} \underset{-3\mathrm{I}}{\sim} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \underset{\cdot -1}{\sim} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \overset{-2\mathrm{II}}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4.3.3 b)

4.3.4.

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix} \overset{\text{ref}}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5.) $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 3 & -4 & -1 \end{pmatrix} \overset{\text{ref}}{\sim} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

b)
$$\begin{array}{c}\begin{pmatrix} 2 & -1 & 1 & 3 & -4 \\ -1 & 2 & 4 & 3 & 2 \\ -2 & 1 & 3 & -4 & -1 \end{pmatrix} \\ \begin{matrix} x & y & z & u & \end{matrix}\end{array} \overset{\text{mye}}{\sim} \begin{array}{c}\begin{pmatrix} 1 & 0 & 0 & 3.5 & 0.5 \\ 0 & 1 & 0 & 3.75 & 3.75 \\ 0 & 0 & 1 & -0.25 & -1.25 \end{pmatrix} \\ \begin{matrix} x & y & z & u & \end{matrix}\end{array}$$

u velges fritt

$$z - 0{,}25u = -1.25 \qquad z = -1.25 + 0.25u$$

$$y = 3{,}75 - 3.75u$$

$$x = 0{,}5 - 3.5u$$

$$\begin{pmatrix} 0.5 - 3.5u \\ 3{,}75 - 3.75u \\ -1{,}25 + 0.25u \\ u \end{pmatrix}$$

4.41 b) $A\bar{x} = \bar{b}$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ 2 & -2 & 1 & 0 \end{pmatrix} \overset{\text{rref}}{\sim} \begin{pmatrix} 1 & 0 & 0 & \boxed{2} \\ 0 & 1 & 0 & \boxed{4/3} \\ 0 & 0 & 1 & \boxed{-4/3} \end{pmatrix}$$

$$\bar{x} = \begin{pmatrix} 2 \\ 4/3 \\ -4/3 \end{pmatrix}$$

c) 3×4 matrise, $\bar{x}$ er 4-vektor

$$A\bar{x} = \bar{b} \qquad \bar{x} = \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & -1 & 2 & 3 \\ 1 & 1 & -1 & 2 & -1 \\ -1 & 1 & 2 & 1 & -2 \end{pmatrix} \overset{\text{rref}}{\sim} \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5/3 & -8/3 \\ 0 & 0 & 1 & -1/3 & 7/3 \end{pmatrix} \\ \begin{matrix} x & y & z & u & \end{matrix} \end{array}$$

u velges fritt

$$z = 7/3 + \tfrac{1}{3}u \qquad y = -8/3 - 5/3\,u$$

$$x = 4$$

5a) <u>Trappform</u>

$$\begin{array}{c}\begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & a^2-a & 3 \\ -1 & 1 & -3 & a \end{pmatrix}\begin{array}{l} \\ -2\mathrm{I} \\ +\mathrm{I} \end{array} \\ \begin{array}{ccc} x & y & z \end{array}\end{array} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 1 & -2 & a+1 \end{pmatrix}\begin{array}{l} \\ \\ -\mathrm{II} \end{array} \sim$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & \overset{1}{\cancel{a^2-a}} & \overset{-\frac{a}{a^2-a}}{-a} \end{pmatrix}$$

Oändlig många: Nollrad nederst

~~Ingen lös~~

$-a=0$, $\underline{\underline{a=0}}$

Ingen lösningar: Rad $(0 \cdots 0 \ c)$, $c \neq 0$

$a^2-a = a\underbrace{(a-1)}$ $\underline{\underline{a=1}}$

$a \neq 0, 1$ $a^2-a \neq 0$. Entydig lösning.

1 a) Finn inversen!

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{pmatrix} \underset{-3\mathrm{I}}{\sim} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -7 & -3 & 1 \end{pmatrix} \cdot -\tfrac{1}{7} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{3}{7} & -\frac{1}{7} \end{pmatrix} -2\mathrm{II}$$

$$\begin{pmatrix} 1 & 0 & 1/7 & 2/7 \\ 0 & 1 & 3/7 & -1/7 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 1/7 & 2/7 \\ 3/7 & -1/7 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

2a) $A = \begin{pmatrix} & & \\ & & \end{pmatrix}$ $\quad \text{inv}(A) = \begin{pmatrix} 0 & 0.6 & 0.2 \\ -1 & 0.2 & 0.6 \\ 1 & 0.2 & -0.4 \end{pmatrix}$.

b) $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & 0 \\ 4 & 16 & -6 \end{pmatrix}$ $\quad \det B = 0$

Har ikke invers.

Matlab $\det(B) = \cdots 10^{-15}$ $\quad 0.\ldots\ldots \quad 0\ldots$

$\text{inv}(B) =$ Matrix close to singular

Bare tilnærmelsesløsning.

4. Løs $\underbrace{A\bar{x} = \bar{b}}$ ved kommando $\bar{x} = A\backslash b$

$$\text{"}\frac{b}{A}\text{"} = A^{-1}b$$

4.6.2.

$$\bar{a}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \bar{a}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \bar{a}_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad \bar{b} = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$$

Vil løse $x\bar{a}_1 + y\bar{a}_2 + z\bar{a}_3 = \bar{b}$

$$\begin{pmatrix} 1 & 1 & 3 & -2 \\ 0 & 2 & -1 & 5 \\ -1 & 1 & 2 & 1 \end{pmatrix} \overset{\text{ref}}{\sim} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\bar{b} = -\bar{a}_1 + 2\bar{a}_2 - \bar{a}_3$$

3b)

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 3 & -1 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \leftarrow$$

Mangler pivot. Nei! V spenner ikke $\mathbb{R}^3$.

7.

7. Lineært uavhengige $\Leftrightarrow$ Hver søyle skal være pivotsøyle.

c) $\begin{pmatrix} \boxed{\begin{matrix}1\\-2\\3\end{matrix}} & \boxed{\begin{matrix}0\\1\\3\end{matrix}} & \begin{matrix}2\\-3\\9\end{matrix} \end{pmatrix} \overset{\text{ref}}{\sim} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

↑ Ikke pivot. Ikke lineært uavh.

d) $\begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 5 \end{pmatrix} \overset{\text{ref}}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Hver søyle pivot! Lineært uavh.

a) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Lineært avhengig: Den ene et multiplum av den andre.

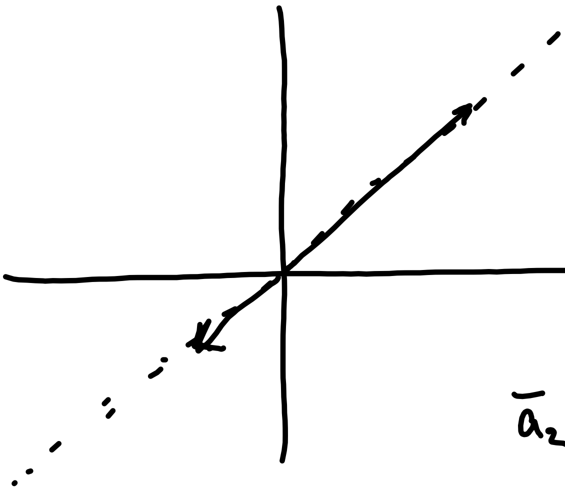
Lineært uavh.

b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ Lineært avh. $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

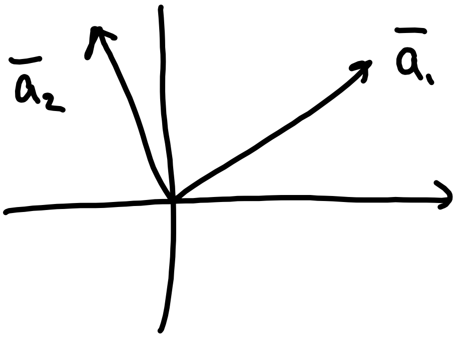
8c Lineært uavhengig delmengde

$$\begin{pmatrix} 4 & -2 & 3 & 1 \\ 1 & 3 & -1 & 2 \\ -3 & 4 & 2 & -5 \\ 2 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

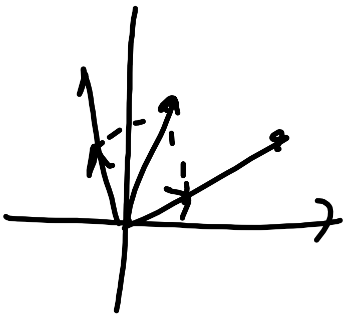
F.eks. 1, 2 og 3.



Lineært avh.



Lineært uavh.



3 vektorer i $\mathbb{R}^2$ er lineært avhengige.

c) $\begin{vmatrix} -1 & 4 \\ 1 & 2 \end{vmatrix} \neq 0.$ 2 lineært uavh. vektorer i $\mathbb{R}^2$, dvs basis.

a) $\begin{vmatrix} -1 & 2 & -1 \\ 3 & 0 & 3 \\ -2 & 1 & 2 \end{vmatrix} \neq 0.$ $\xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Lineært uavh.

3 L.U. i $\mathbb{R}^3$, basis.

4.8.2. Skriv som produkt av elementære.

Elementære Start med $I = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$

1) Bytte om to rader. Invers: ~~Bytte rader~~ Gjøre det samme.

2) Mult rad med $c \neq 0$ Invers: Mult. samme rad med $1/c$.

3) Legg til multiplum av en rad til en annen Invers: Trekk fra samme rad.

$$M \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}_{+\mathrm{I}} \longrightarrow \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}_{\cdot \frac{1}{3}} \longrightarrow \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \xrightarrow{-2\mathrm{II}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

A fra venstre, $\qquad B \qquad C$

$$CBAM = I \qquad M = A^{-1}B^{-1}C^{-1}$$

$$= \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

4.9.1

a)

$$\begin{vmatrix} 1 & -2 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = +1 \cdot \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - (-2) \begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\uparrow \quad \uparrow \quad \uparrow \quad = 1 + 2 \cdot 5 - (-1) = \underline{12}$$

2a)
$$\begin{vmatrix} 1 & -3 & 0 \\ 2 & -1 & -2 \\ 1 & -1 & 1 \end{vmatrix} \begin{matrix} \\ +2\mathrm{I} \\ -\mathrm{I} \end{matrix} = \begin{vmatrix} 1 & -3 & 0 \\ 0 & 5 & -2 \\ 0 & 2 & 1 \end{vmatrix} =$$

$$5 \begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & -0.4 \\ 0 & 2 & 1 \end{vmatrix} \begin{matrix} \\ \\ -2\mathrm{II} \end{matrix} = 5 \begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & -0.4 \\ 0 & 0 & 1.8 \end{vmatrix} = 5 \cdot 1 \cdot 1 \cdot 1.8$$

$$= \underline{9},$$

5.c) $$\begin{vmatrix} 3 & 1 & 0 & 4 \\ 2 & 1 & 2 & 0 \\ 0 & 0 & 1 & -2 \\ 1 & 2 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 0 & 4 \\ 2 & 1 & 2 & 4 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 4 \end{vmatrix}$$

$\uparrow$ $2S_3$

$\underline{\det A^T = \det A.}$

$$= \begin{vmatrix} 3 & 1 & 4 \\ 2 & 1 & 4 \\ 1 & 2 & 4 \end{vmatrix} -\mathrm{II} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 4 \\ 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 2 & 4 \end{vmatrix} = \underline{-4}$$