~~Blokk~~ ~~8~~

6.1.1

b) $\iint_R (x + \sin y)\,dx\,dy \qquad R = [0,1] \times [0,\pi]$

$$= \int_0^{\pi} \left(\int_0^{1} (x + \sin y)\,dx \right) dy = \int_0^{\pi} \left(\left[\frac{1}{2}x^2 + x \sin y \right]_{x=0}^{x=1} \right) dy$$

$$= \int_0^{\pi} \frac{1}{2} + \sin y - (0+0)\,dy = \frac{1}{2}y - \cos y \Big|_0^{\pi} = \underline{\frac{\pi}{2} + 2.}$$

e) $\iint_R xy\, e^{x^2 y}\, dx\, dy \qquad R = [0,2] \times [1,2]$

$$= \int_1^2 \left(\int_0^2 \underbrace{xy\, e^{x^2 y}}\, dx \right) dy = \int_1^2 \left[\frac{1}{2} e^{x^2 y} \right]_{x=0}^{x=2} dy$$

$u = x^2 y$

$du = 2xy\, dx$

$$= \int_1^2 \frac{1}{2} e^{4y} - \frac{1}{2}\, dy$$

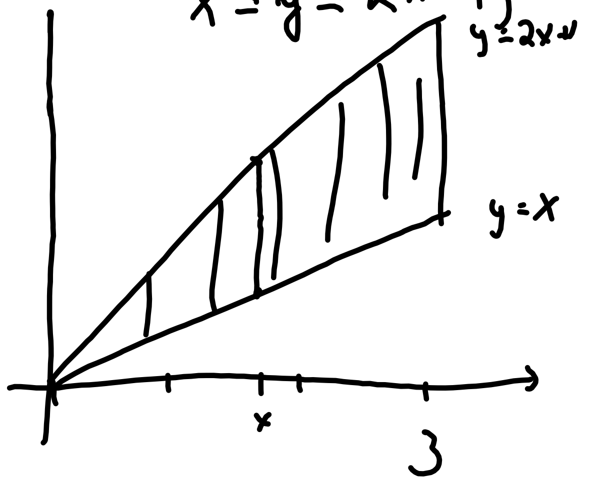
$$= \frac{1}{2} \cdot \frac{1}{4} e^{4y} - \frac{1}{2} y \Big|_1^2 = \frac{1}{8} \cdot e^8 - 1 - \left(\frac{1}{8} e^4 - \frac{1}{2} \right)$$

$$= \underline{\frac{1}{8} e^8 - \frac{1}{8} e^4 - \frac{1}{2}.}$$

6.2.1 b.

$$\iint_R (x+2y)\,dx\,dy$$

$$R = \{(x,y) : 0 \le x \le 3,\ x \le y \le 2x+1\}$$



$$= \int_0^3 \left(\int_x^{2x+1} (x+2y)\,dy \right) dx$$

$$= \int_0^3 \left[xy + y^2 \right]_{y=x}^{y=2x+1} dx =$$

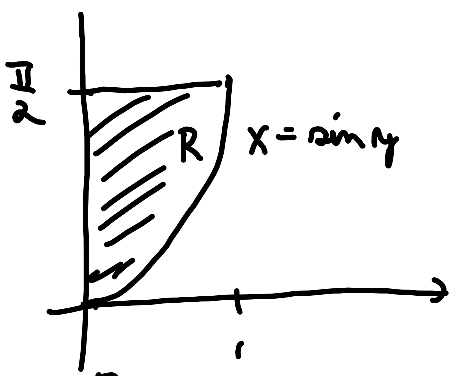
$$\int_0^3 x(2x+1) + (2x+1)^2 - (x^2 + x^2)\,dx$$

$$= \int_0^3 3x^3 + 5x^2 + 2x\,dx = \frac{459}{4}.$$

d) $\iint_R x \cos y \, dx\,dy$

$R = \{(x,y) : 0 \le y \le \frac{\pi}{2}$
$0 \le x \le \sin y\}$

$= \int_0^{\frac{\pi}{2}} \left(\int_0^{\sin y} x \cos y \, dx \right) dy$



$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} x^2 \cos y \right) \Big|_{x=0}^{x=\sin y} dy =$

$\int_0^{\frac{\pi}{2}} \frac{1}{2} \sin^2 y \cos y \, dy = \frac{1}{2} \cdot \frac{1}{3} \sin^3 y \Big|_0^{\frac{\pi}{2}} = \underline{\frac{1}{6}}$

$(u = \sin y)$

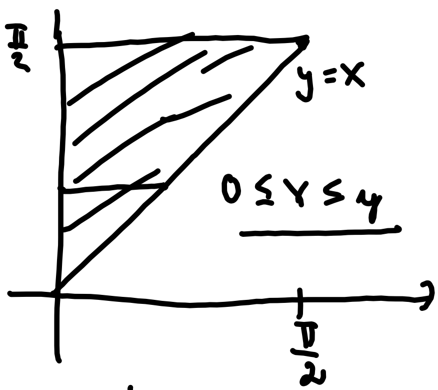
h) $\iint_R \frac{dx\,dy}{\sqrt{1-y^2}}$ R $0 \leq y \leq \sin x$

$0 \leq x \leq \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} \left(\int_0^{\sin x} \frac{dy}{\sqrt{1-y^2}} \right) dx = \int_0^{\frac{\pi}{2}} \arcsin y \Big|_0^{\sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} (x - 0)\,dx = \frac{1}{2}x^2 \Big|_0^{\frac{\pi}{2}} = \underline{\frac{\pi^2}{8}.}$$

6.2.3.



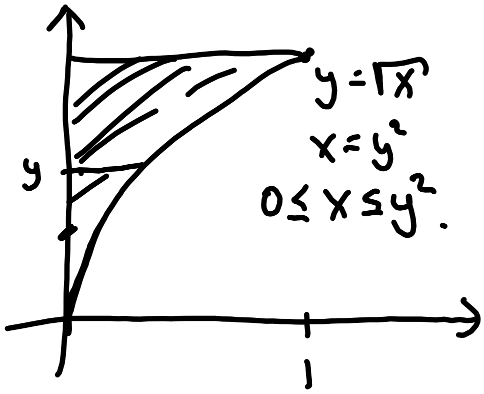
$$\int_0^{\frac{\pi}{2}}\left(\int_x^{\frac{\pi}{2}}\frac{\sin y}{y}\,dy\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\int_0^{y}\frac{\sin y}{y}\,dx\right)dy$$

$$\int_a^b c = \underline{c(b-a)}$$

$$=\int_0^{\frac{\pi}{2}} y\cdot\frac{\sin y}{y}\,dy = -\cos y\Big|_0^{\frac{\pi}{2}} = \underline{1}.$$

c) $\int_0^1 \left(\int_{\sqrt{x}}^1 e^{x/y^2} \, dy \right) dx$



$= \int_0^1 \left(\int_0^{y^2} e^{x/y^2} \, dx \right) dy$

$= \int_0^1 y^2 e^{x/y^2} \Big|_{x=0}^{x=y^2} dy =$

$\int e^{ax} \, dx = \frac{1}{a} e^{ax} \quad (+C)$

$\int_0^1 y^2 e - y^2 \, dy = \int_0^1 (e-1) y^2 \, dy = \underline{\frac{1}{3}(e-1)}.$

$\int_0^1 x^n \, dx = \frac{1}{n+1}$.

$\int_0^1 x^n \, dx = \frac{x^{n+1}}{n+1}\Big|_0^1 = \frac{1}{n+1}$

6.3. Skifte til polarkoordinater

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

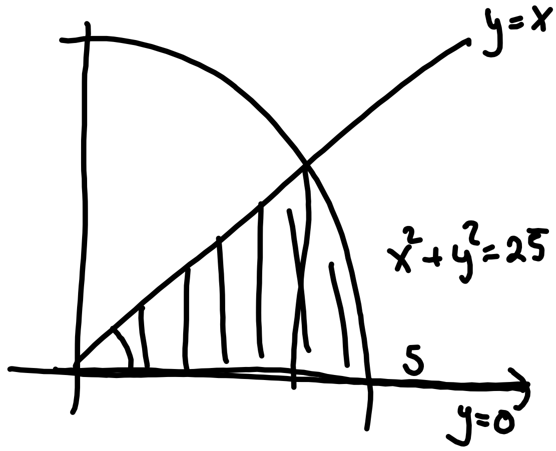
↑ NB!

S beskrivelsen av R i polarkoordinater.

$$\left(\int_{\theta_0}^{\theta_1} \int_{r_0}^{r_1} g(r)\, r\, dr\, d\theta = (\theta_1 - \theta_0) \int_{r_0}^{r_1} g(r)\, r\, dr. \right)$$

6.3.1

b) $\iint_R (x^2+y^2)\,dx\,dy$



$S=[0,5]\times[0,\frac{\pi}{4}]$

$= \int_0^{\frac{\pi}{4}} \left(\int_0^5 r^2 \cdot r\,dr \right) d\theta = \frac{\pi}{4} \cdot \left[\frac{1}{4}r^4\right]_0^5 = \frac{625\pi}{16}.$

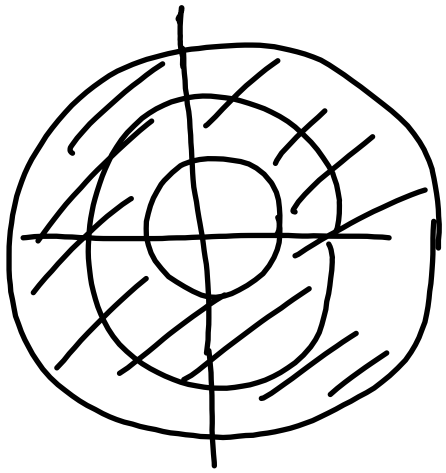
c) $\iint_R e^{(x^2+y^2)}\,dx\,dy$

$R = \{1 \le |(x,y)| \le 4\}$

$= \int_0^{2\pi} \left(\int_0^4 e^{r^2} \cdot r\, dr \right) d\theta$

$= 2\pi \cdot \left[\frac{1}{2} e^{r^2} \right]_1^4$

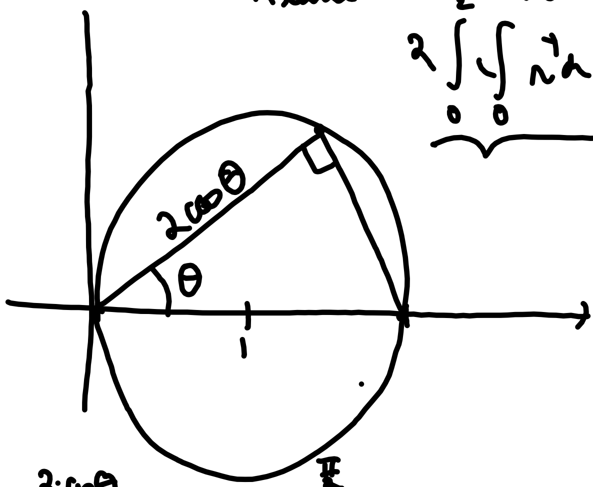
$= \pi \left(e^{16} - e^{1} \right)$



$S = [1,4] \times [0, 2\pi]$

g) $\iint_R (\underbrace{x^2+y^2})^{3/2}\,dx\,dy$

$S = \{(r,\theta);\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\cos\theta\}$



Kunne tatt $2\int_0^{\frac{\pi}{2}}\left(\int_0^{2\cos\theta} r^4\,dr\right)d\theta$.

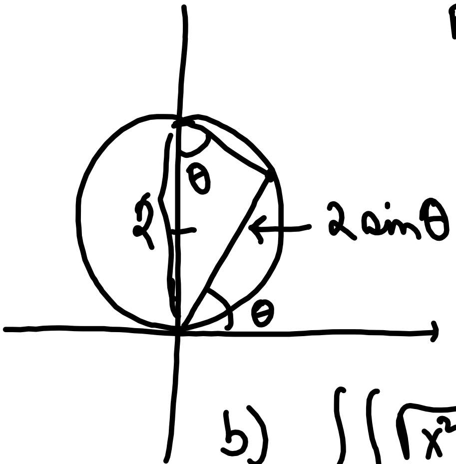
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\int_0^{2\cos\theta} r^3 \cdot r\,dr\right) d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{5} r^5 \Big|_0^{2\cos\theta} d\theta = \frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^5\theta\, d\theta$$

$$\frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\left(1-\sin^2\theta\right)^2 d\theta = \frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta - 2\cos\theta\sin^2\theta + \cos\theta\sin^4\theta\, d\theta$$

$$= \frac{32}{5}\left(\sin\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \underline{\underline{\frac{512}{75}}}$$

6.3.3. a)

$$\iint_R f\,dx\,dy = \int_0^{\pi}\left(\int_0^{2\sin\theta} f(r\cos\theta, r\sin\theta)\, r\,dr\right)d\theta$$

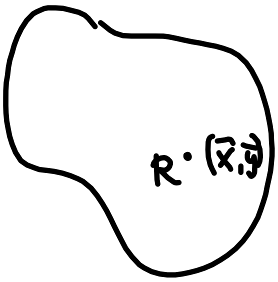


b) $$\iint_R \sqrt{x^2+y^2}\,dx\,dy = \int_0^{\pi}\left(\int_0^{2\sin\theta} r\cdot r\,dr\right)d\theta$$

$$= \int_0^{\pi} \frac{1}{3}r^3\Big|_0^{2\sin\theta} d\theta = \frac{8}{3}\int_0^{\pi} \sin^3\theta\, d\theta = \frac{8}{3}\int_0^{\pi} \sin\theta\,(1-\cos^2\theta)\,d\theta$$

$$\frac{8}{3}\cdot\left(2 - 2\cdot\frac{1}{3}\right) = \frac{32}{9}.$$

6.4. Anvendelser.



$R \subset \mathbb{R}^2$.

Areal $A = \iint_R 1 \cdot dx\,dy$

Massemiddelpunkt

$$\bar{x} = \frac{1}{A} \iint_R x\,dx\,dy$$

$$\bar{y} = \frac{1}{A} \iint_R y\,dx\,dy$$

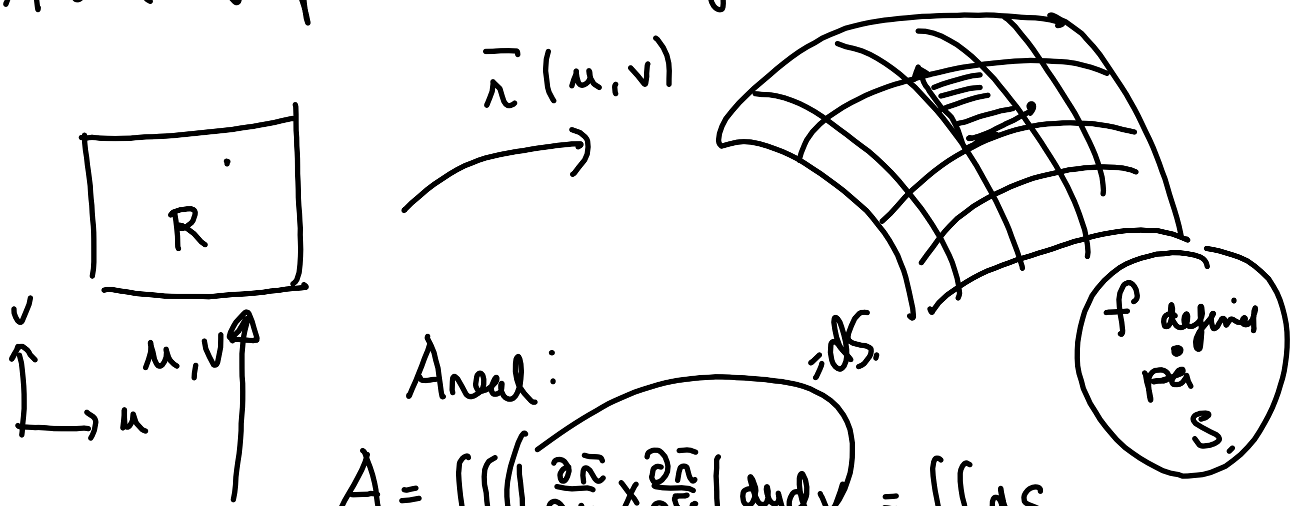
Hvis vi har tetthetsfunksjon $f(x,y)$:

Total masse $M = \iint_A f\,dx\,dy$

$$\bar{x} = \frac{1}{M} \iint x f(x,y)\,dx\,dy$$

$$\bar{y} = \frac{1}{M} \iint y f(x,y)\,dx\,dy.$$

Areal av parametrisert flate



Areal:

$$A = \iint_R \left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right| du\,dv = \iint_S dS.$$

($\left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right| du\,dv = dS$.)

Integral av skalarfelt definert på S:

$$\iint_S f\,dS = \iint_R f(\bar{r}(u,v)) \left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right| du\,dv.$$

Volum under $z \leq f(x,y)$ $\quad (f(x,y) \geq 0)$

$$V = \iint_R f(x,y)\,dx\,dy.$$

6.4.1 a) $E = \{(x,y,z) ; 0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq x + y^2\}$,

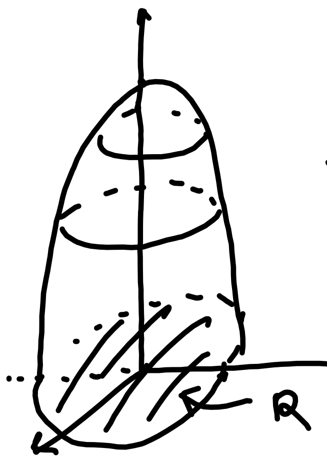
$$V = \int_0^2 \left(\int_0^1 x + y^2 \, dy \right) dx = \int_0^2 xy + \frac{1}{3}y^3 \Big|_0^1 \, dx = \int_0^2 x + \frac{1}{3} \, dx$$

$$= \frac{8}{3}.$$

d) E området over xy planet under $z = \sqrt{32-2x^2-2y^2}$

$z = 0$ når $32-2x^2-2y^2=0$

$x^2+y^2=16.$ ←



$z=\sqrt{32-2r^2}$ $\quad r=0 : z=4\sqrt{2}.$

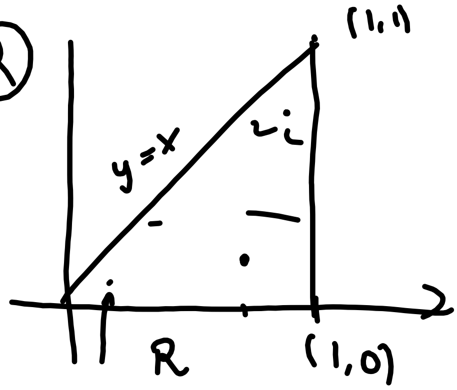
$$V = \iint_R z\,dx\,dy = \int_0^{2\pi}\left(\int_0^4 (32-2r^2)^{\frac{1}{2}}\cdot r\,dr\right)d\theta$$

$S=[0,4]\times[0,2\pi]$ $\quad u = 32-2r^2$

$$= 2\pi \cdot \frac{2}{3}\frac{1}{(-4)}(32-2r^2)^{3/2}\Big|_0^4 = -\frac{\pi}{3}(32-2r^2)^{3/2}\Big|_0^4$$

$$= -\frac{\pi}{3}\left(0-32^{3/2}\right) = \frac{\pi}{3}\cdot 32\cdot 4\sqrt{2} = \frac{128\sqrt{2}\,\pi}{3}$$

(2)



Tetthet x.

Total masse M =

$$\iint_R x\,dx\,dy = \int_0^1\left(\int_0^x x\,dy\right)dx = \int_0^1 x^2\,dx = \frac{1}{3}.$$

$$\bar{x} = \frac{1}{M}\iint_R x\cdot x\,dx\,dy = 3\int_0^1 x^3\,dx = \frac{3}{4}.$$

$$\bar{y} = \frac{1}{M}\iint y x\,dx\,dy = 3\int_0^1\left(\int_0^x xy\,dy\right)dx = 3\int_0^1 \frac{1}{2}x^3\,dx = \frac{3}{2}\cdot\frac{1}{4} = \frac{3}{8}.$$