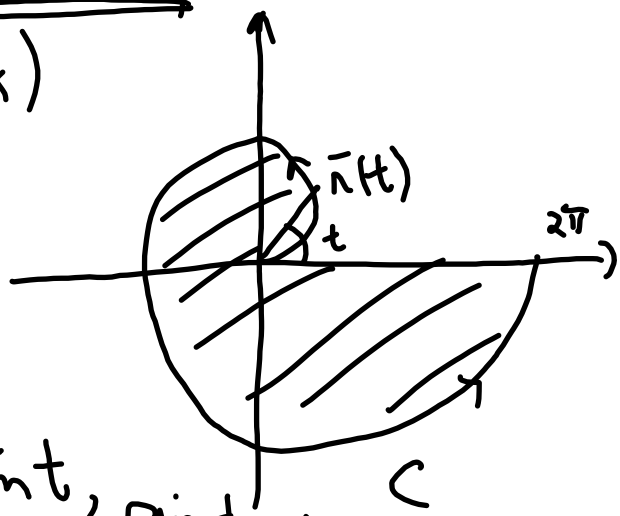
14 [illegible] 14/6-2017. – Opp 1.

$$\boxed{\bar{r}(t) = (t\cos t, t\sin t)} \qquad t \in [0, 2\pi]$$

$$\bar{F}(x,y) = (-y, x)$$

$$\int_C \bar{F}\cdot d\bar{r}$$



a)

$$\bar{r}'(t) = (\cos t - t\sin t, \sin t + t\cos t)$$

$$\underbrace{\int_C \bar{F}\cdot d\bar{r}}_{} = \int_0^{2\pi} (-t\sin t, t\cos t)\cdot(\cos t - t\sin t, \sin t + t\cos t)$$

$$= \int_0^{2\pi} -t\sin t\cancel{\cos t} + t^2\sin^2 t + t\cancel{\cos t\sin t} + t^2\cos^2 t \quad dt$$

$F = (-y, x)$

$$= \int_0^{2\pi} t^2(\sin^2 t + \cos^2 t)\,dt = \int_0^{2\pi} t^2\,dt = \frac{1}{3}t^3\Big|_0^{2\pi}$$

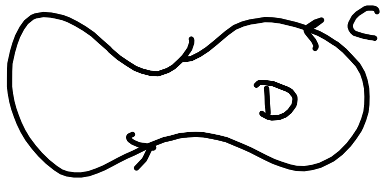
$$\underline{\underline{\frac{8}{3}\pi^3}}$$

b) 1) Polarkoordinater $|(t\sin t,\ t\cos t)| = \underline{t}.$

$$A(D) = \iint_D dx\,dy = \int_0^{2\pi}\left(\int_0^{|\vec{r}(t)|} r\,dr\right)dt = \int_0^{2\pi}\left(\int_0^{t} r\,dr\right)dt$$

$$= \int_0^{2\pi} \frac{1}{2}r^2\Big|_0^t \,dt = \int_0^{2\pi}\frac{1}{2}t^2\,dt = \frac{1}{2}\cdot\frac{1}{3}t^3\Big|_0^{2\pi} = \frac{8}{6}\pi^3 = \underline{\underline{\frac{4}{3}\pi^3}}.$$

2) Green's teorem.



$$A(D) = \int_C x\,dy = \int_C -y\,dx = \boxed{\frac{1}{2}\int_C -y\,dx + x\,dy}$$

Areal = halvparten av a)

dvs $\underline{\underline{\frac{4}{3}\pi^3}}$

18/8-2017 - Oppgave 1.

$$A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

a) Egenverdier:

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \lambda - \frac{1}{3} \end{vmatrix} = \left(\lambda - \frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2 = 0$$

$$\left(\lambda - \frac{1}{3}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$\lambda - \frac{1}{3} = \pm \frac{2}{3}$$

$$\lambda = \frac{1}{3} \pm \frac{2}{3} = \begin{cases} 1 \\ -\frac{1}{3} \end{cases}$$

Egenvektorer

$\lambda_1 = 1$: $\frac{2}{3}x - \frac{2}{3}y = 0$ y velges fritt $y = s$

$x = y$ $x = s$

Egenvektorene $\begin{pmatrix} s \\ s \end{pmatrix} = s \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $s \neq 0$.

y

$\lambda_2 = -\frac{1}{3}$

$-\frac{2}{3}x - \frac{2}{3}y = 0$

$x + y = 0$

$x = -y$

y velges fritt $y = t$, $x = -t$

Egenvektorene $\begin{pmatrix} -t \\ t \end{pmatrix} = t\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\cdot y$

b) $\underline{\{(x_n, y_n)\}}_{n=0}^{\infty}$ $\quad \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$\underline{\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}}$$

Skriv $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ som ~~som~~ lineær kombinasjon av egenvektorer:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = s \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} + t \underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \qquad \left.\begin{aligned} s - t &= 1 \\ s + t &= 0 \end{aligned}\right\} \begin{aligned} s &= \tfrac{1}{2} \\ t &= -\tfrac{1}{2} \end{aligned}$$

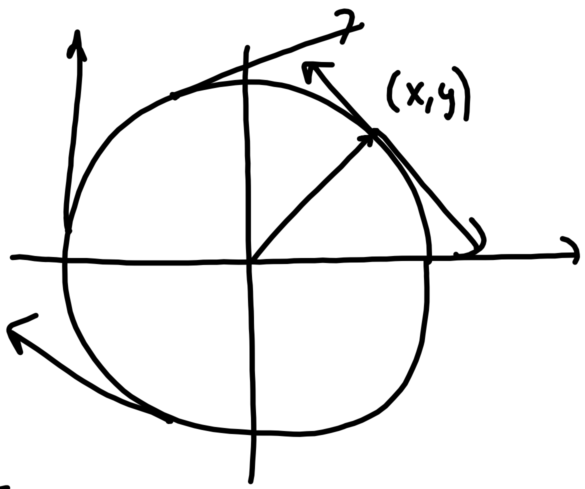
$$\underbrace{\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}} = A^n \left(\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{2}\underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \right)$$

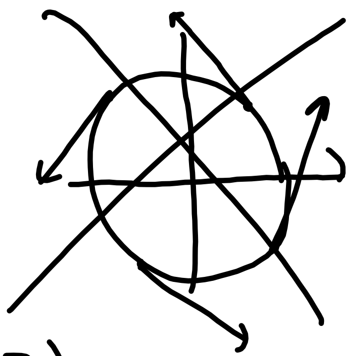
$$= \frac{1}{2} \cdot 1^n \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{2} \underbrace{\left(-\frac{1}{3}\right)^n} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow[n \to \infty]{} \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}}}$$

Oppgave 4.

$$\bar{F} = \left(\frac{y}{x^2+y^2+1}, -\frac{x}{x^2+y^2+1} \right) =: (F_1, F_2)$$

$$\frac{1}{r^2+1}\left(\underbrace{y, -x}\right)$$





Jacobimatrisen

$$\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix}$$

$$\bar{F} = \left(\frac{\cancel{x}\,y}{x^2+y^2+1}, \frac{-x}{x^2+y^2+1} \right)$$

$$\qquad F_1 \qquad\qquad F_2$$

$$\frac{\partial F_1}{\partial x} = \frac{0\cdot(x^2+y^2+1)-2x\,y}{(x^2+y^2+1)^2} = \frac{-2xy}{(x^2+y^2+1)^2}.$$

$$\frac{\partial F_1}{\partial y} = \frac{1\cdot(x^2+y^2+1)-2y\cdot y}{(x^2+y^2+1)^2} = \frac{x^2-y^2+1}{(x^2+y^2+1)^2}.$$

$$\frac{\partial F_2}{\partial x} = -\frac{y^2-x^2+1}{(x^2+y^2+1)^2} = \frac{x^2-y^2-1}{(x^2+y^2+1)^2}.$$

$$\frac{\partial F_2}{\partial y} = \frac{2xy}{(x^2+y^2+1)^2}.$$

$$\bar{F}'(x,y) = \frac{1}{(x^2+y^2+1)^2}\begin{pmatrix} -2xy & x^2-y^2+1 \\ x^2-y^2-1 & 2xy \end{pmatrix}$$

Linearisering om $(0,0)$: $\bar{F}(0,0)+\bar{F}'(0,0)\begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \underline{\underline{\begin{pmatrix} y \\ -x \end{pmatrix}}}.$$

Eller $(y,-x)$ hvis vi vil.

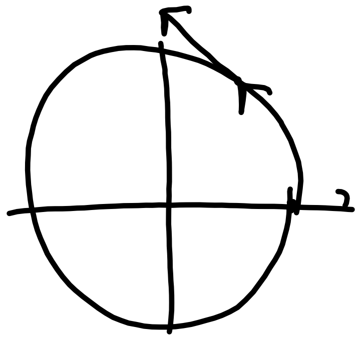
b) $\bar{F}$ konservativ? Må ha $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$?

Nei, disse er forskjellige.

c) $C : \bar{r}(t) = (\underbrace{\cos t, \sin t}_{r=1}) \qquad t \in [0, 2\pi]$

$\bar{F}(\bar{r}(t)) \cdot \bar{r}'(t)$

Enhetssirkelen.



$$= \left(\frac{\sin t}{2}, \frac{-\cos t}{2} \right) \cdot (-\sin t, \cos t)$$

$$= -\frac{1}{2}\sin^2 t - \frac{1}{2}\cos^2 t = -\frac{1}{2}$$

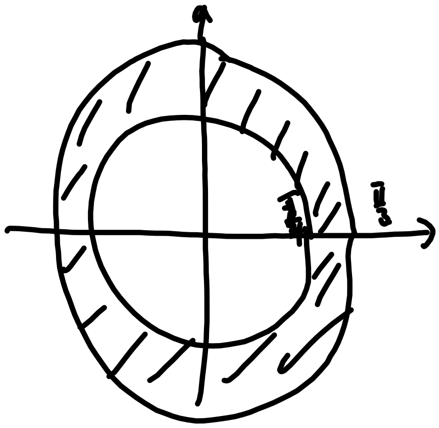
$$\int_C \bar{F} \cdot d\bar{r} = \int_0^{2\pi} -\frac{1}{2}\, dt = -\frac{1}{2} \cdot 2\pi = \underline{\underline{-\pi}}.$$

Dette gir også "nei" i b), siden integralet ikke er 0.

Se nettsiden for bruk av Green.

5) $A = \left\{ (x,y) : \frac{\pi}{4} \le \sqrt{x^2+y^2} \le \frac{\pi}{3} \right\}$ $\qquad \frac{\pi}{4} \le r \le \frac{\pi}{3}$

Annulus



$$\iint_A \frac{\tan\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\, dx\, dy$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\int_0^{2\pi} \frac{\tan r}{r}\, r\, d\theta \right) dr \qquad \ln a - \ln b = \ln\frac{a}{b}$$

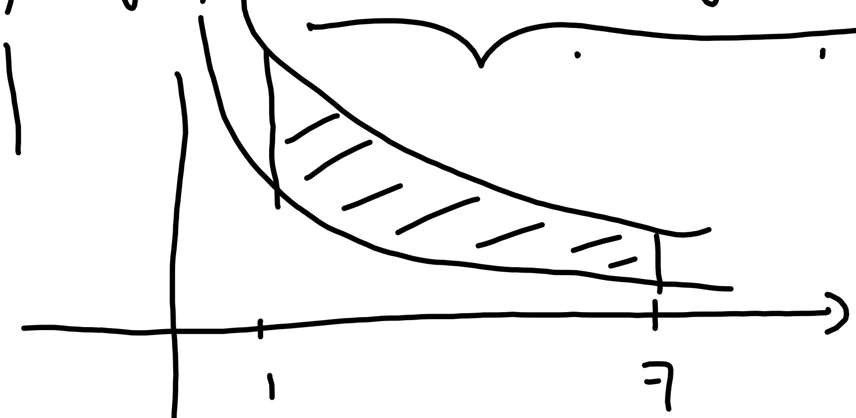
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\int_0^{2\pi} \tan r\, d\theta \right) dr$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2\pi \tan r\, dr = 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin r}{\cos r}\, dr = 2\pi \left(-\ln\cos r\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= 2\pi\left(\ln\cos\frac{\pi}{4} - \ln\cos\frac{\pi}{3}\right) = 2\pi\left(\ln\frac{1}{2}\sqrt{2} - \ln\frac{1}{2}\right)$$

$$= 2\pi \ln\sqrt{2} = 2\pi \ln 2^{1/2} = 2\pi \cdot \frac{1}{2}\ln 2 = \underline{\underline{\pi \ln 2}}$$

b) $B = \{(x,y) \mid 1 \le x \le 7,\ \frac{1}{x} \le y \le \frac{2}{x}\}$.



$$\iint_B xy^2\,dx\,dy = \int_1^7 \Big(\underbrace{\int_{\frac{1}{x}}^{\frac{2}{x}} xy^2\,dy}\Big)\, dx = \int_1^7 \left(\frac{1}{3} xy^3 \Big|_{y=\frac{1}{x}}^{y=\frac{2}{x}} \right) dx$$

$$= \int_1^7 \frac{1}{3} x\left(\frac{8}{x^3}-\frac{1}{x^3}\right) dx = \int_1^7 \frac{7}{3} x\cdot\frac{1}{x^3}\, dx = \int_1^7 \frac{7}{3} x^{-2}\, dx =$$

$$\frac{7}{3}\left(-x^{-1}\right)\Big|_1^7 = \frac{7}{3}\left(\frac{1}{1}-\frac{1}{7}\right) = \frac{7}{3}\cdot\frac{6}{7} = \underline{2}$$