Go 2

6.9.1. a) $\iiint\limits_A xyz\,dx\,dy\,dz \qquad A=\underset{x}{[0,1]}\times\underset{y}{[0,1]}\times\underset{z}{[0,1]}$

$$= \int_0^1 \Big(\int_0^1 \Big(\int_0^1 xyz\,dx\Big)dy\Big)dz = \Big(\int_0^1 x\,dx\Big)\Big(\int_0^1 y\,dy\Big)\Big(\int_0^1 z\,dz\Big)$$

Lov når faste grenser og funksjon som er produkt

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{8}}}$$

c) $\iiint\limits_A zy\underbrace{\cos(xy)}\,dx\,dy\,dz \qquad A=[1,2]\times[\pi,2\pi]\times[0,1].$

$$= \int_\pi^{2\pi}\Big(\int_1^2 \Big(\int_0^1 zy\cos(xy)\,dz\Big)dx\Big)dy = \int_\pi^{2\pi}\Big(\int_1^2 \frac{1}{2}y\cos(\underbrace{xy}_{u=xy})\,dx\Big)dy$$

$$= \int_\pi^{2\pi} \frac{1}{2}\sin(xy)\Big|_{x=1}^{x=2} dy = \frac{1}{2}\int_\pi^{2\pi} \sin 2y - \sin y\; dy$$

$$= \frac{1}{2}\Big(-\frac{1}{2}\cos 2y + \cos y\Big)\Big|_\pi^{2\pi} = \frac{1}{2}\Big(-\frac{1}{2}+1-\Big(-\frac{1}{2}+(-1)\Big)\Big) = \frac{1}{2}\cdot 2 = \underline{1.}$$

6.9.2.

$$\iiint_A (xy+z)\,dxdydz \qquad A=\{0\le x\le 1,\ 0\le y\le 2,\ 0\le z\le \underline{x^2y}\}.$$

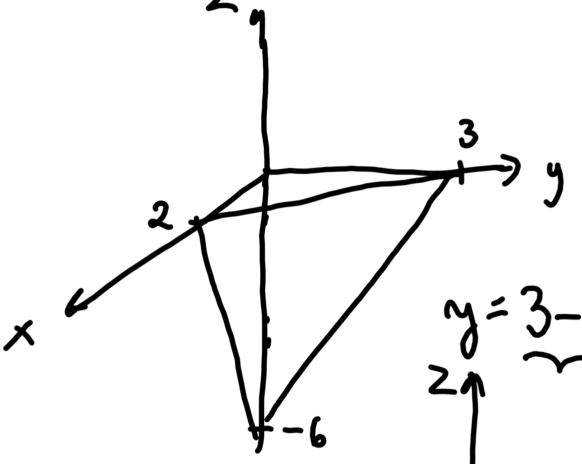
$$=\int_0^1\left(\int_0^2\left(\int_0^{x^2y}(xy+z)dz\right)dy\right)dx=\int_0^1\left(\int_0^2\left(\underline{xy}z+\frac{1}{2}z^2\right)\Big|_{z=0}^{z=x^2y}dy\right)dx$$

$$=\int_0^1\left(\int_0^2 x^3y^2+\frac{1}{2}x^4y^2\,dy\right)dx=\int_0^1\left(\frac{1}{3}x^3y^3+\frac{1}{2}\cdot\frac{1}{3}x^4y^3\right)\Big|_{y=0}^{y=2}dx$$

$$=\int_0^1\frac{8}{3}\underline{x^3}+\frac{4}{3}\underline{x^4}dx=\frac{8}{3}\cdot\frac{1}{4}+\frac{4}{3}\cdot\frac{1}{5}=\underline{\underline{\frac{14}{15}}}.$$

d) $\iiint_A \underline{3y^2}-\underline{3z}\,dx\,dy\,dz$

A = området avgrenset av xy-planet og planet $3x+2y-z=6$

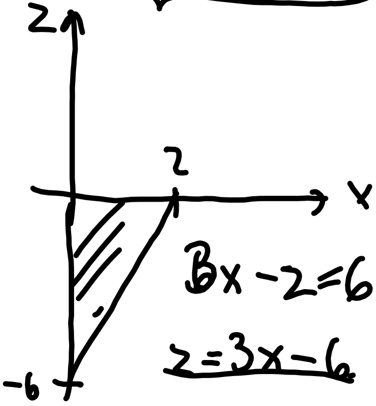


$\iiint 3y^2\,dx\,dy\,dz$

$=\int_0^2\left(\int_{3x-6}^0\left(\int_0^{\underbrace{3-\frac32x+\frac12z}} y^2\,dy\right)dz\right)dx$

$y=\underbrace{3-\frac32x+\frac12z}$

$=\int_0^2\left(\int_{3x-6}^0 \underbrace{\frac13\left(3-\frac32x+\frac12z\right)^3}\,dz\right)dx$



$3x-z=6$

$\underline{z=3x-6}$

$=\frac13\int_0^2\left(\cancel{\frac14}\left(3-\frac32x+\frac12z\right)^4\cdot\cancel{2}\right)\Big|_{\underbrace{z=3x-6}}^{z=0}\,dx$

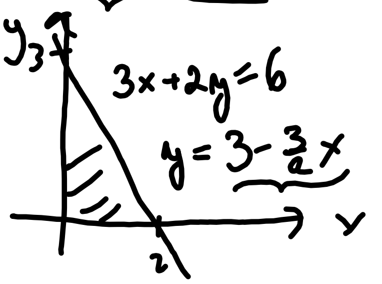
$=\frac13\cdot\frac12\int_0^2\left(3-\frac32x\right)^4dx = \frac16\cdot\frac15\underbrace{\left(3-\underbrace{\frac32}x\right)}^5\cdot\left(-\frac23\right)\Big|_0^2 = 0+\frac{81}{5}=\underline{\underline{\frac{81}{5}}}.$

$\iiint 3z\,dx\,dy\,dz =$

$\int_0^2\left(\int_0^{3-\frac32x}\left(\int_{6-3x-2y}^0 3z\,dz\right)dy\right)dX$

$3x+2y-z=6$

$z=\underbrace{6-3x-2y}$



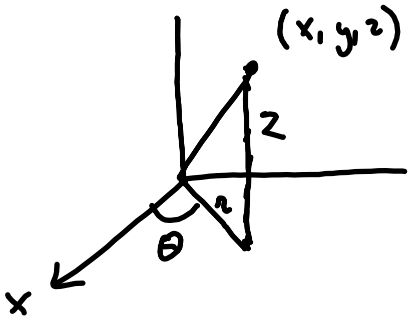
$3x+2y=6$

$y=\underbrace{3-\frac32x}$

$=-27.$

Svar: $\frac{81}{5}+27=\underline{\frac{216}{5}}.$

6.10. Sylinderkoordinater



$x = r\cos\theta$

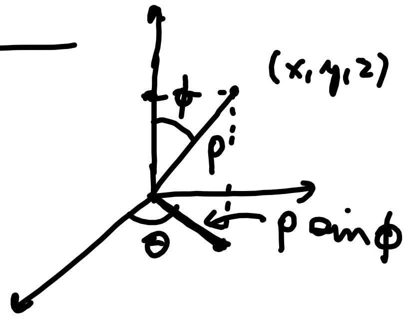
$y = r\sin\theta$

$z = z$

$$\iiint_A f\,dx\,dy\,dz = \iiint_D f(r\cos\theta, r\sin\theta, z)\,\underset{\cdot}{\underline{r}}\,dr\,d\theta\,dz.$$

D = beskrivelse av A i sylinderkoor.

Kulekoordinater



$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$$\iiint_A f\,\underbrace{dx\,dy\,dz} = \iiint_D f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\underbrace{\rho^2\sin\phi}_{\uparrow}\,d\rho\,d\phi\,d\theta$$

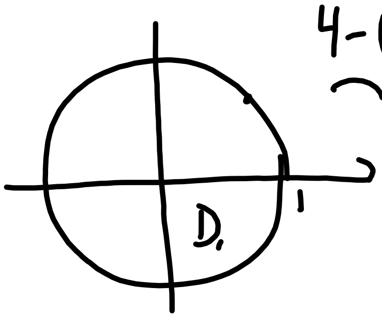
k 10.1 b, Sylindr

$$\iiint_A xy\,dx\,dy\,dz$$

$$A=\{x^2+y^2\le 1,\ 0\le z\le 4-x-y\}$$

$$\underbrace{4-(x+y)}$$

$$=\int_0^1\left(\int_0^{2\pi}\left(\int_0^{4-\underline{r\cos\theta-r\sin\theta}}\underbrace{r\cos\theta\, r\sin\theta\, r}_{\underline{r^3\sin\theta\cos\theta}}\,dz\right)d\theta\right)dr$$

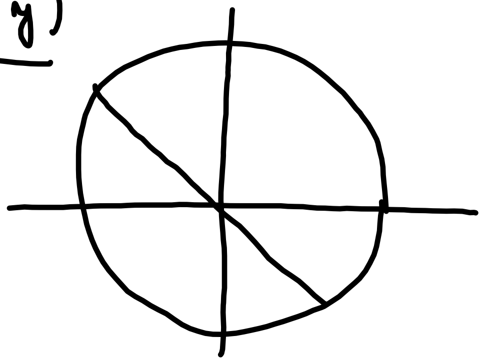


$$=\int_0^1\left(\int_0^{2\pi}4r^3\underbrace{\sin\theta\cos\theta}-r^4\underbrace{\sin\theta\cos^2\theta}-r^4\,\underline{\sin^2\theta\cos\theta}\,d\theta\right)dr$$

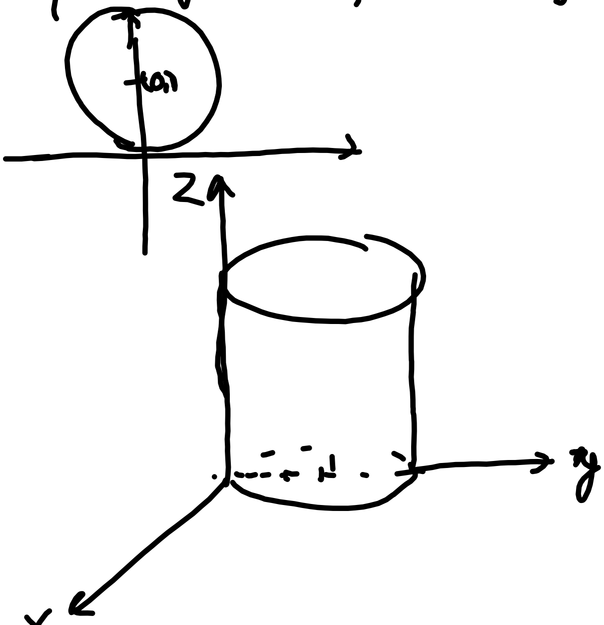
$$=\int_0^1 4r^3\cdot\underline{\tfrac{1}{2}\sin^2\theta}-r^4\left(-\tfrac{1}{3}\cos^3\theta\right)-r^4\left(\underline{\tfrac{1}{3}\sin^3\theta}\right)\Big|_0^{2\pi}dr=\int_0^1 0\,dr=\underline{\underline{0}}.$$

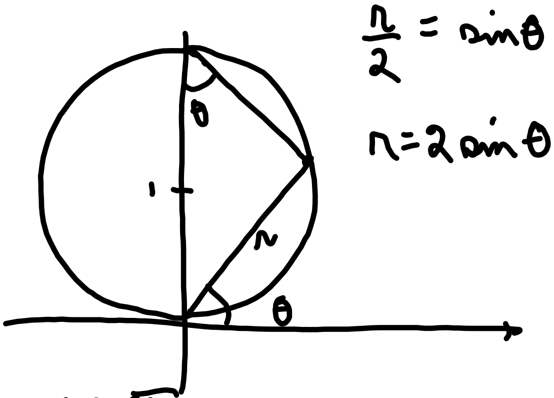
$$\underline{z}=\underbrace{4-(x+y)}$$

$$\iiint \underline{xy}$$



c)' $\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz$ $\qquad A=\{x^2+(y-1)^2\le 1,\ 0\le z\le 2\}$





$\frac{r}{2}=\sin\theta$

$r=2\sin\theta$

$0\le\theta\le\pi.$

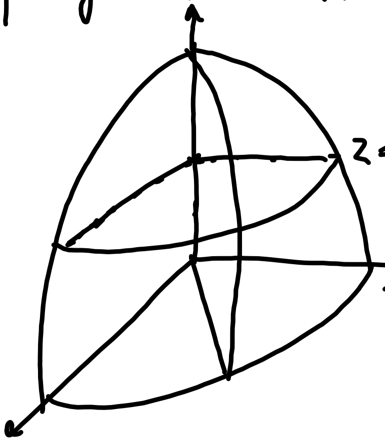
$$\iiint z\sqrt{x^2+y^2}\,dx\,dy\,dz=\int_0^{\pi}\left(\int_0^{2\sin\theta}\left(\int_0^{2} z\,r\cdot r\,dz\right)dr\right)d\theta$$

$$=\int_0^{\pi}\left(\int_0^{2\sin\theta}2r^2\,dr\right)d\theta=\int_0^{\pi}\frac{2}{3}r^3\Big|_0^{2\sin\theta}d\theta=\int_0^{\pi}\frac{16}{3}\sin^3\theta\,d\theta$$

$$=\frac{16}{3}\int_0^{\pi}\underline{\sin}\theta\left(1-\underline{\cos^2}\theta\right)d\theta=\frac{16}{3}\left(-\cos\theta+\frac{1}{3}\cos^3\theta\right)\Big|_0^{\pi}=\underline{\underline{\frac{64}{9}}}$$

6.10.26. Kulekoordinater

$A = \{ x, y \geq 0 ,\ z \geq \frac{1}{2} ,\ x^2 + y^2 + z^2 \leq 1 \}$



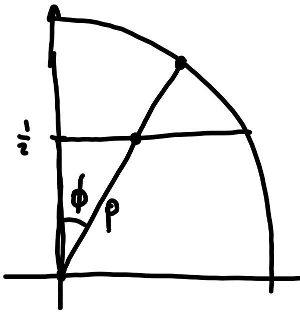
ϕ, θ, ρ

$0 \leq \theta \leq \frac{\pi}{2}$

$\cos\phi_0 = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

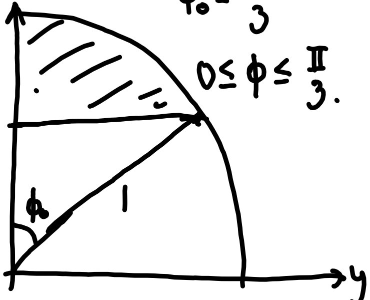
$\phi_0 = \frac{\pi}{3}$

$0 \leq \phi \leq \frac{\pi}{3}.$



$\cos\phi = \frac{\frac{1}{2}}{\rho}$

$\rho = \frac{1}{2\cos\phi}$



$\frac{1}{2\cos\phi} \leq \rho \leq 1$

$$\iiint_A x\,dx\,dy\,dz = \underbrace{\int_0^{\frac{\pi}{2}} \left(\int_0^{\frac{\pi}{3}} \left(\int_{\frac{1}{2\cos\phi}}^{1} \underline{\rho}\, \cancel{\sin\phi \cos\theta}\, \underline{\rho}^2 \sin\phi \, d\rho \right) d\phi \right) d\theta}$$

$$\boxed{\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta = 1.}$$

$$\int_0^{\frac{\pi}{3}} \left(\int_{\frac{1}{2\cos\phi}}^{1} \rho^3 \sin^2\phi \, d\rho \right) d\phi = \int_0^{\frac{\pi}{3}} \left(\frac{1}{4} \rho^4 \sin^2\phi \right) \Bigg|_{\underline{\rho = \frac{1}{2\cos\phi}}}^{\rho = 1}$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{3}} \underline{\sin^2\phi} - \frac{1}{16} \frac{\sin^2\phi}{\cos^4\phi}\, d\phi =$$

$$\frac{1}{4} \int_0^{\frac{\pi}{3}} \frac{1}{2} - \frac{1}{2}\cos 2\phi - \frac{1}{16} \underset{u}{\sin\phi} \cdot \underbrace{\frac{\sin\phi}{\cos^4\phi}}_{v'} \, d\phi$$

$\underline{u' = \cos\phi}$ $\underline{v = \frac{1}{3}\cos^{-3}\phi}$

$$= \frac{1}{4} \left(\frac{1}{2}\phi - \frac{1}{2}\sin 2\phi \cdot \frac{1}{2} - \frac{1}{16} \left(\underline{\frac{\sin\phi}{3\cos^3\phi}} - \int \frac{1}{3} \frac{1}{\cos^2\phi}\, d\phi \right) \right) \Bigg|_{\phi=0}^{\phi=\frac{\pi}{3}}$$

$$= \frac{1}{7} \left(\frac{1}{2}\phi - \frac{1}{4}\sin 2\phi - \frac{1}{48} \frac{\sin\phi}{\cos^3\phi} - \frac{1}{3}\tan\phi \right) \Bigg|_{\phi=0}^{\phi=\frac{\pi}{3}} = \underline{\frac{\pi}{24} - \frac{3\sqrt{3}}{64}.}$$

3c $\iiint\limits_A e^{-\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$

A kule sentrum origo, radius 1.
$0 \le \rho \le 1$, $0 \le \phi \le \pi$,
$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\left(\int_0^{\pi}\left(\int_0^1 e^{-\rho}\rho^2 \sin\phi\,d\rho\right)d\phi\right)d\theta =$$

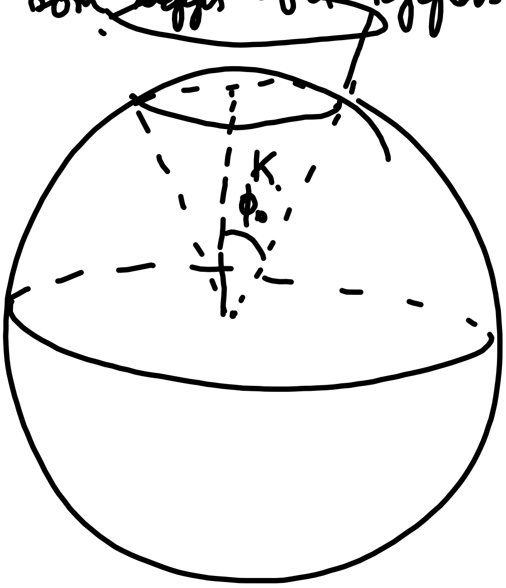
(Som i første oppgave)

$$\left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin\phi\,d\phi\right)\left(\int_0^1 \rho^2 e^{-\rho}\,d\rho\right)$$

$$= 2\pi \cdot 2 \cdot \left((-\rho^2-2\rho-2)\,e^{-\rho}\right)\Big|_0^1 = \underline{\underline{4\pi\left(2-\frac{5}{e}\right).}}$$

3. Volum av den delen av kulen $x^2+y^2+z^2 \le R^2$

som ligger over kjeglen $z = \sqrt{\frac{x^2+y^2}{3}}$



Skjæres hverandre.

$$x^2+y^2+\frac{x^2+y^2}{3} = R^2$$

$x^2+y^2 = \frac{3}{4}R^2$, sirkel radius $\frac{1}{2}\sqrt{3}R$.

$$z = \sqrt{\frac{\frac{3}{4}R^2}{3}} = \frac{1}{2}R.$$

Kulekoord. $0 \le \rho \le R$
$0 \le \theta \le 2\pi$.

$\cos\phi_0 = \frac{1}{2}$, $\phi_0 = \frac{\pi}{3}$. $0 \le \phi \le \frac{\pi}{3}$.

$$V = \iiint_K dx\,dy\,dz = \int_0^{2\pi}\left(\int_0^{\frac{\pi}{3}}\left(\int_0^R \rho^2 \sin\phi \, d\rho\right) d\phi\right) d\theta$$

$$= 2\pi \cdot \frac{1}{3}R^3 \cdot (-\cos\phi)\Big|_0^{\frac{\pi}{3}} = \underline{\frac{\pi}{3}R^3},$$