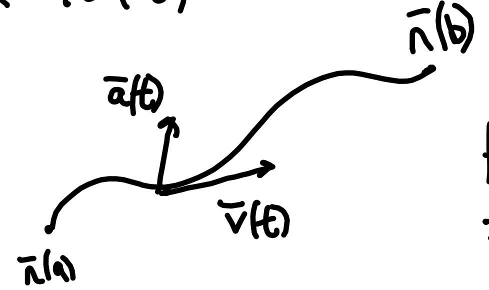
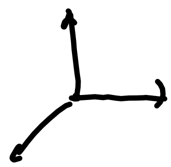
Kurve $\bar{r}(t)$





$\bar r(t)$ posisjonsvektoren

Hastighet $\bar v(t) = \bar r'(t)$

Fart $v(t) = |\bar v(t)|$

Akselerasjon $\bar a(t) = \bar v'(t)$

Baneakselerasjon $a(t) = v'(t)$

$\bar a(t) = a(t)\bar T(t) + v(t)\bar T'(t)$

Buelengde $L = \int_c ds = \int_a^b v(t)\,dt$

3.1.1 $\bar r(t) = (t^3, t^2)$

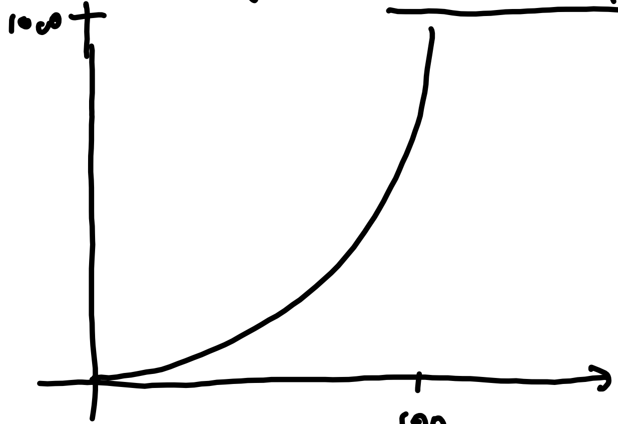
$\bar v(t) = \bar r'(t) = (3t^2, 2t)$ $\quad v(t) = |\bar v(t)| = \underbrace{\sqrt{9t^4 + 4t^2}}$

$\bar a(t) = \bar v'(t) = (6t, 2)$

$$a(t) = \frac{d}{dt}\left((9t^4 + 4t^2)^{\frac{1}{2}} \right) = \frac{1}{2}(9t^4 + 4t^2)^{-\frac{1}{2}} \cdot (36t^3 + 8t)$$

$$= \frac{18t^3 + 4t}{\sqrt{9t^4 + 4t^2}}$$

8. Buelengde $\bar{r}(t) = (t^2, t^3)$ $t \in [0, 10]$



$\bar{v}(t) = \bar{r}'(t) = (2t, 3t^2)$

$v(t) = |\bar{v}(t)| = \sqrt{4t^2 + 9t^4}$

$= t\sqrt{4+9t^2}$

$L = \int_0^{10} t\sqrt{\underbrace{4+9t^2}} \, dt = \int_4^{904} u^{1/2} \frac{1}{18} du = \frac{1}{18}\frac{2}{3} u^{3/2} \Big|_4^{904}$

$u = 4 + 9t^2$

$du = 18t \, dt$

$t = 0 \Rightarrow u = 4$

$t = 10 \Rightarrow u = 904$

$= \frac{1}{27}(904^{3/2} - 8) \approx \underline{100.6.}$

10. $\bar{r}(t) = (2\cos t, \underbrace{\sqrt{2}\sin t}, \underbrace{\sqrt{2}\sin t})$

a) $\bar{v}(t) = (-2\sin t, \sqrt{2}\cos t, \sqrt{2}\cos t)$

$v(t) = \sqrt{4\sin^2 t + 2\cos^2 t + 2\cos^2 t} = \sqrt{4} = \underline{2}$

$a = 0$

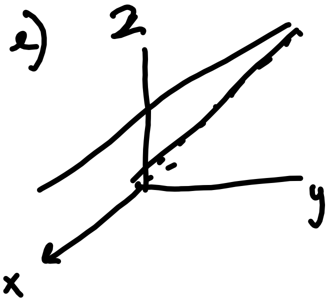
$\bar{a} = (-2\cos t, -\sqrt{2}\sin t, -\sqrt{2}\sin t) = -\bar{r}(t)$

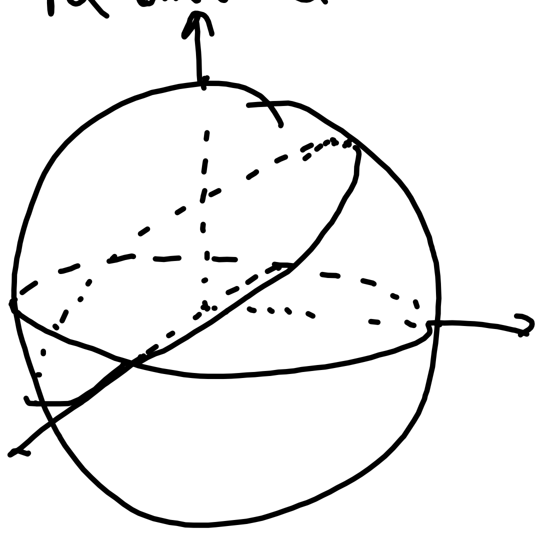
b) $L = \int_0^{2\pi} v(t)\,dt = \int_0^{2\pi} 2\,dt = 2 \cdot 2\pi = \underline{4\pi}.$

c) $|\bar{r}(t)| = \sqrt{4\cos^2 t + 2\sin^2 t + 2\sin^2 t} = 2$

d) $y - z = \sqrt{2}\sin t - \sqrt{2}\sin t = 0$

e)





21.

$$\bar{F} = m\bar{a}$$

$$\bar{a} = \underbrace{k(t)\bar{r}(t)}$$

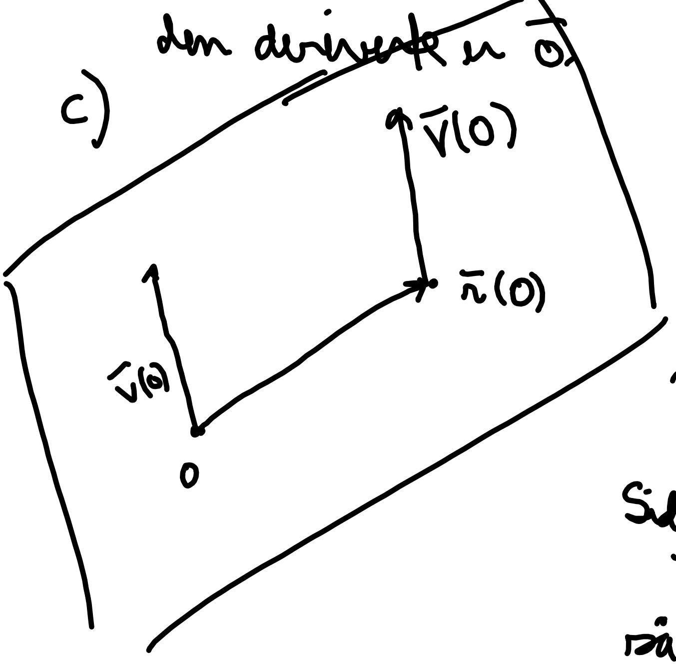
a) Vis at $\frac{d}{dt}\left(\bar{r}(t)\times\bar{v}(t)\right) = \bar{0}$

Svar: $\frac{d}{dt}\left(\bar{r}(t)\times\bar{v}(t)\right) = \bar{v}(t)\times\bar{v}(t) + \bar{r}(t)\times\left(k(t)\bar{r}(t)\right)$

$= \bar{0} + k(t)\left(\bar{r}(t)\times\bar{r}(t)\right) = \bar{0} + \bar{0} = \bar{0}.$

b) $\bar{r}(t)\times\bar{v}(t) = \bar{c}$, konstant vektor, siden den deriverte er $\bar{0}$.

c)



Partikkel ligger i planet utspent av $\underline{\bar{r}(0)\text{ og }\bar{v}(0)}$:

$$\bar{r}(0)\times\bar{v}(0) = \bar{c}$$

Planet er $\{\bar{x}\in\mathbb{R}^3 \mid \bar{x}\cdot\bar{c} = 0\}$

Siden

$\bar{r}(t)\cdot\bar{c} = \bar{r}(t)\cdot\underbrace{\left[\bar{r}(t)\times\bar{v}(t)\right]} = 0$

så er $\bar{r}(t)$ i planet.

⑦ Temperaturen $f(x, y, t) = 20 + 2t - x^2 + y^2$

$\bar{r}(t) = \left(\underbrace{3t - \frac{t^2}{4}}_{x}, \underbrace{2t + \frac{t^2}{8}}_{y}\right)$ Avta/öka ved $t = 1$?

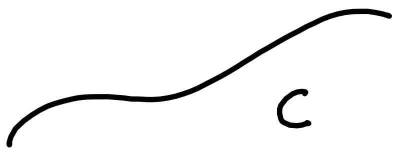
$T(t) = f(\bar{r}(t), t) = 20 + 2t - \left(3t - \frac{t^2}{4}\right)^2 + \left(2t + \frac{t^2}{8}\right)^2$

$T' = 2 - 2\left(3t - \frac{t^2}{4}\right)\left(3 - \frac{1}{2}t\right) + 2\left(2t + \frac{t^2}{8}\right)\left(2 + \frac{1}{4}t\right)$

$T'(1) = -2.1875$ Avtagende.

$T'(t) = \frac{\partial f}{\partial x}(\bar{r}(t))\frac{dx}{dt} + \frac{\partial f}{\partial y}(\bar{r}(t)) \cdot \frac{dy}{dt} + \frac{\partial f}{\partial t}(\bar{r}(t))$

<u>Linjeintegral</u> av <u>skalarfelt</u>



f definert på C

$$\int_C f\,ds = \int_a^b f(\bar{r}(t))\underbrace{v(t)\,dt}_{ds}$$

<u>5.</u> $\bar{r}(t) = (t\sin t, t\cos t, t) \quad t \in [0, 2\pi]$

$\bar{v}(t) = (\sin t + t\cos t, \cos t - t\sin t, 1)$

$v(t) = |\bar{v}(t)| = \sqrt{(\sin t + t\cos t)^2 + (\cos t - t\sin t)^2 + 1}$

$= \sqrt{\sin^2 t + t^2\cos^2 t + \cos^2 t + t^2\sin^2 t + 1}$

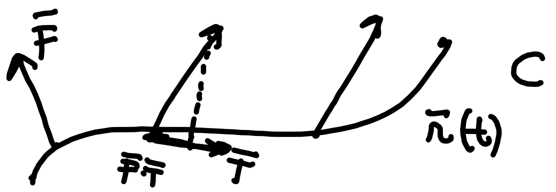
$= \sqrt{2 + t^2}$

$f = z$

$$\int_C z\,ds = \int_0^{2\pi} t\cdot\sqrt{2+t^2}\,dt = \dots = \frac{1}{3}\left((2+4\pi^2)^{3/2} - 2\sqrt{2}\right)$$

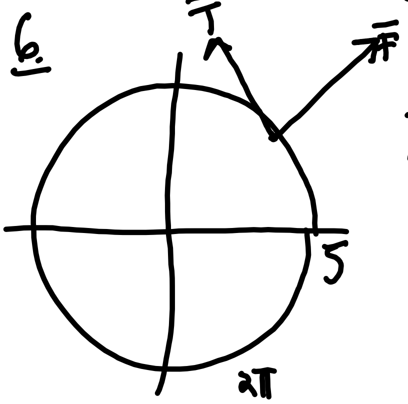
$u = 2 + t^2$

Linjeintegral av vektorfelt.



$$\int_C \bar F \cdot d\bar r = \underbrace{\int_a^b \bar F(\bar r(t)) \cdot \bar r'(t)\,dt}_{} = \int_C \bar F \cdot \bar T \, ds$$

6.



$\bar F \cdot \bar T = 0.$

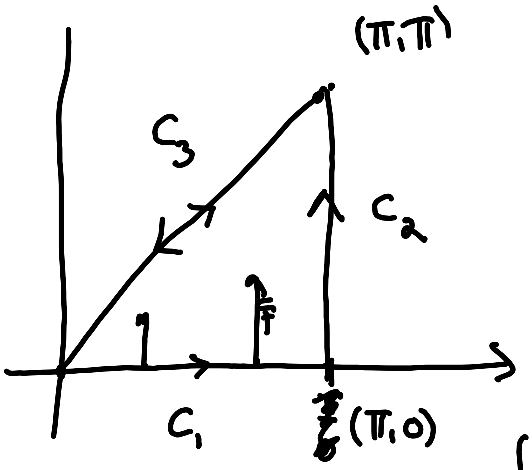
$\bar r(t) = (5\cos t, 5\sin t)$ $t \in [0, 2\pi]$

$\bar F(x,y) = (x, y)$

$$\int_C \bar F \cdot d\bar r = \int_0^{2\pi} (5\cos t, 5\sin t) \cdot (-5\sin t, 5\cos t)\,dt$$

$$= \int_0^{2\pi} 0\,dt = 0 .$$

8. $\bar{F} = (\cos x \sin y, x)$



$C_1 : \bar{r}_1(t) = (t, 0) \qquad 0 \le t \le \pi$

$$\int_{C_1} \bar{F}\cdot d\bar{r}_1 = \int_0^\pi (\cos t \cdot 0, t)\cdot(1,0)\,dt = \int_0^\pi (0\cdot 1 + t\cdot 0)\,dt = \underline{0.}$$

$C_2 : \bar{r}_2(t) = (\pi, t) \qquad 0 \le t \le \pi$

$$\int_{C_2} \bar{F}\cdot d\bar{r}_2 = \int_0^\pi (\cos\pi \sin t, \pi)\cdot(0,1)\,dt = \int_0^\pi \pi\,dt = \underline{\pi^2}$$

Skifter retning på C_3:

$\bar{r}_3(t) = (t,t) \qquad 0 \le t \le \pi$

$$\int_{-C_3} \bar{F}\cdot d\bar{r}_3 = \int_0^\pi (\cos t \sin t, t)\cdot(1,1)\,dt = \int_0^\pi \cos t \sin t + t\,dt$$

$$= \frac{1}{2}\sin^2 t + \frac{1}{2}t^2\Big|_0^\pi = \frac{1}{2}\pi^2$$

$$\int_C \bar{F}\cdot d\bar{r} = \int_{C_1} \bar{F}\,d\bar{r}_1 + \int_{C_2} \bar{F}\cdot d\bar{r}_2 - \int_{-C_3} \bar{F}\cdot d\bar{r}_3 = 0 + \pi^2 - \frac{1}{2}\pi^2 = \underline{\frac{1}{2}\pi^2}.$$

C_3 nedover: $\bar{r}(t) = (\pi - t, \pi - t) \qquad 0 \le t \le \pi.$

Gradienter / Konservative felt.

ϕ funksjon

$$\nabla \phi = \left(\frac{\partial \phi}{\partial x_1}, \dots, \frac{\partial \phi}{\partial x_n} \right)$$ Vektorfelt.

$$\int_C \nabla \phi \cdot d\bar{r} = \phi(\bar{b}) - \phi(\bar{a}).$$

Et slikt felt kalles konservativt.
(eller gradientfelt).

- $\bar{F} = (F_1, \dots, F_n)$ er konservativt $\Rightarrow \frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}$

  x y z $\Leftarrow$ Hvis vi er i område uten hull.

- $n = 3.$ $\bar{F} = (F_1, F_2, F_3)$

  $$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$ o.s.v.

Hvis $\bar{F} = \nabla \phi$. ϕ kalles et potensial for $\bar{F}$.

2. $\bar{F} = (2xe^y, x^2e^y + x)$ Konservativ?

$\frac{\partial F_1}{\partial y} = 2xe^y$

$\frac{\partial F_2}{\partial x} = 2xe^y + 1$

Nei!

10) $\int \bar{F} \cdot d\bar{r}$ $\bar{F} = (y^2z + 2xy, 2xyz + x^2, xy^2 + 1)$

$\bar{r} = (t, t^2, \tan\frac{\pi t}{2})$ $t \in [0,1]$

Prøver å finne potensial for $\bar{F}$:

$\frac{\partial \phi}{\partial x} = y^2z + 2xy \Rightarrow \phi = xy^2z + x^2y + C(y,z)$.

$\frac{\partial \phi}{\partial y} = 2xyz + x^2 \Rightarrow \phi = xy^2z + x^2y + D(x,z)$,

$\frac{\partial \phi}{\partial z} = xy^2 + 1 \Rightarrow \phi = xy^2z + z + E(x,y)$.

$\phi = xy^2z + x^2y + z$

$\int \bar{F} \cdot d\bar{r} = \phi(1,1,1) - \phi(0,0,0) = 3$