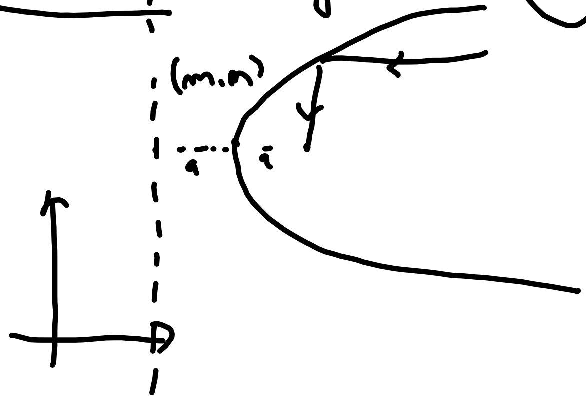
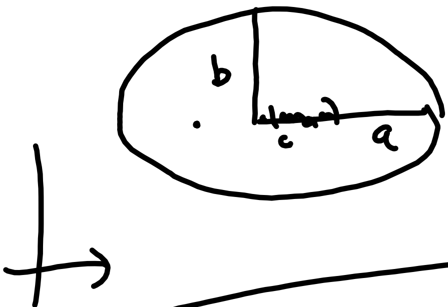
3.6.

Parabel $(y-n)^2 = 4a(x-m)$



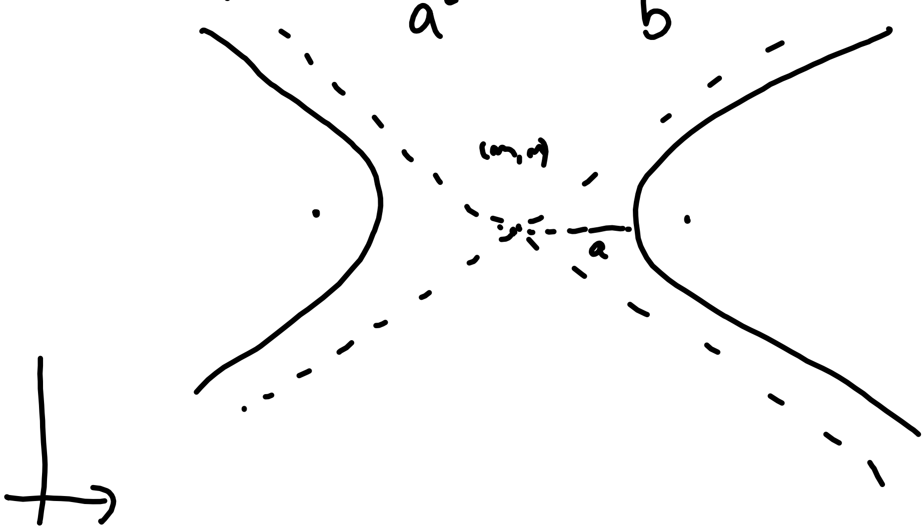
Ellipse $\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1$



$c = \sqrt{a^2 - b^2}$ $a > b.$

Hyperbel $\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1$

$c = \sqrt{a^2 + b^2}$



1. $4x^2 + 9y^2 + 32x - 18y + 37 = 0$

$4(x^2 + 8x) + 9(y^2 - \overset{2}{\cancel{18}}y) + 37$

$= 4((x+4)^2 - 16) + 9((y-1)^2 - 1) + 37$

$= 4(x+4)^2 - 64 + 9(y-1)^2 - 9 + 37$

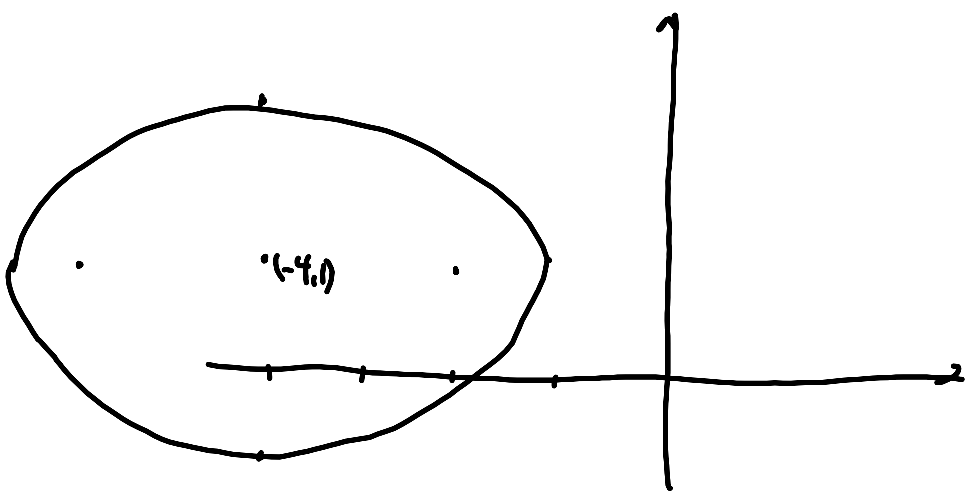
$= 4(x+4)^2 + 9(y-1)^2 - 36 = 0$

$\frac{(x+4)^2}{9} + \frac{(y-1)^2}{4} = 1$

Ellipse, sentrum (-4, 1), halvakser 3 og 2

$c = \sqrt{9-4} = \sqrt{5}$

Brennpunkter $(-4 \pm \sqrt{5}, 1)$



2) $\underline{y^2} - 4x - \underline{2y} - 7 = (y-1)^2 - 1 - 4x - 7 = 0$

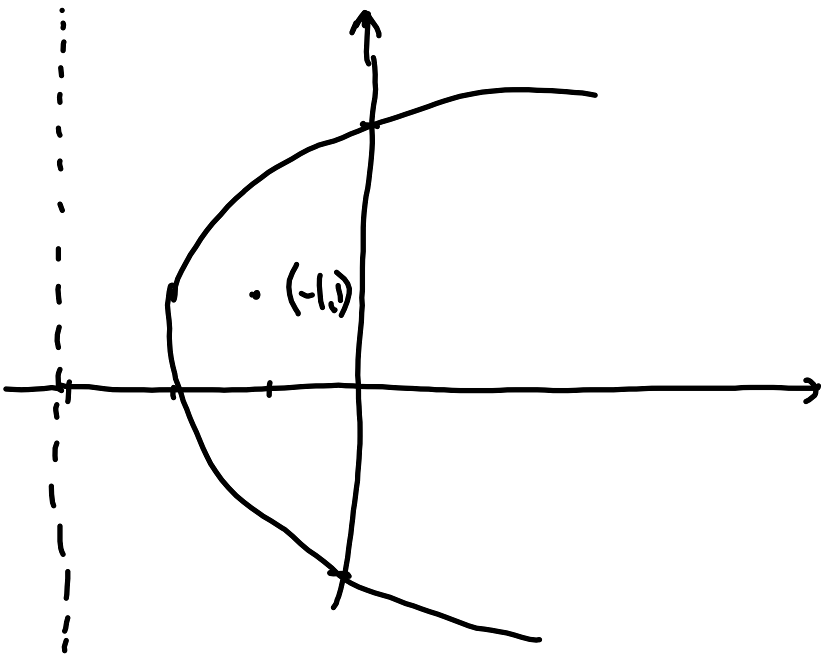
$(y-1)^2 = 4x + 8 = 4(x+2)$

Toppunkt $(-2,1)$

$\underline{a = 1.}$ Brennvidde

$x = 0, \quad y - 1 = \pm\sqrt{8}$

$y = 1 \pm \sqrt{8}$



(3) $x^2 - y^2 - 2x + 4y - 7 =$

$(x-1)^2 - 1 - \underbrace{(y^2 - 4y)} - 7 =$

$(x-1)^2 - 8 - \left((y-2)^2 - 4\right) =$

$(x-1)^2 - (y-2)^2 - 4 = 0$

$$\frac{(x-1)^2}{4} - \frac{(y-2)^2}{4} = 1$$

Hyperbel,

sentrum $(1, 2)$

Halvakse 2

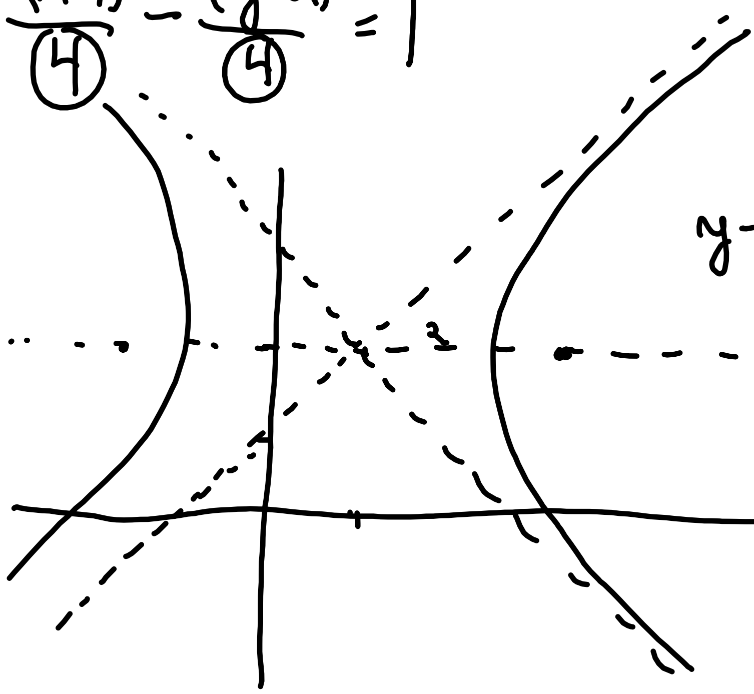
Brennvidde $c = \sqrt{4+4}$

$= 2\sqrt{2} \approx 2.8$

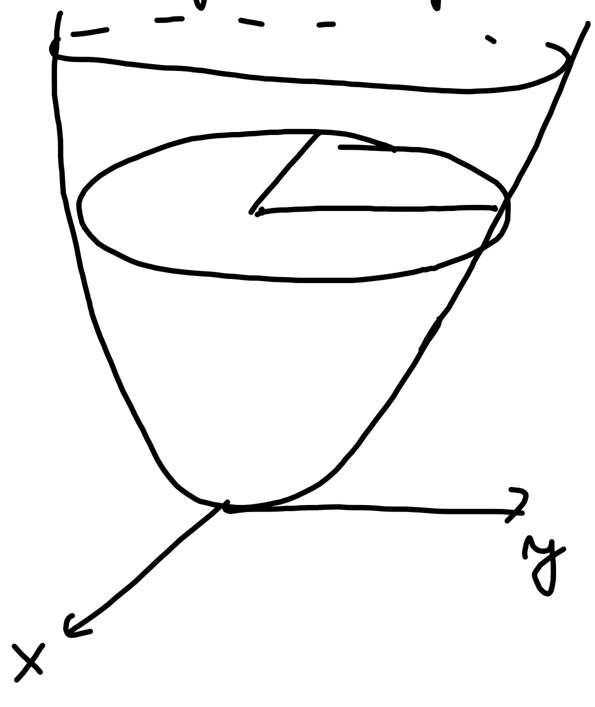
Asymptoter

$y - 2 = \pm 1 (x - 1)$

$\left(\frac{b}{a}\right)$



2a) $z = f(x,y) = 2x^2 + y^2$



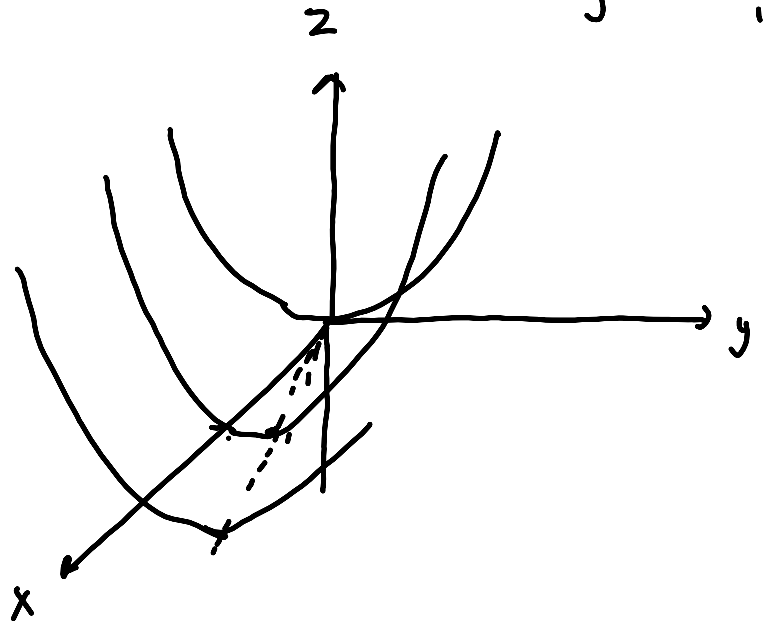
$2x^2 + y^2 = c$

x-retning : $\sqrt{\frac{c}{2}}$

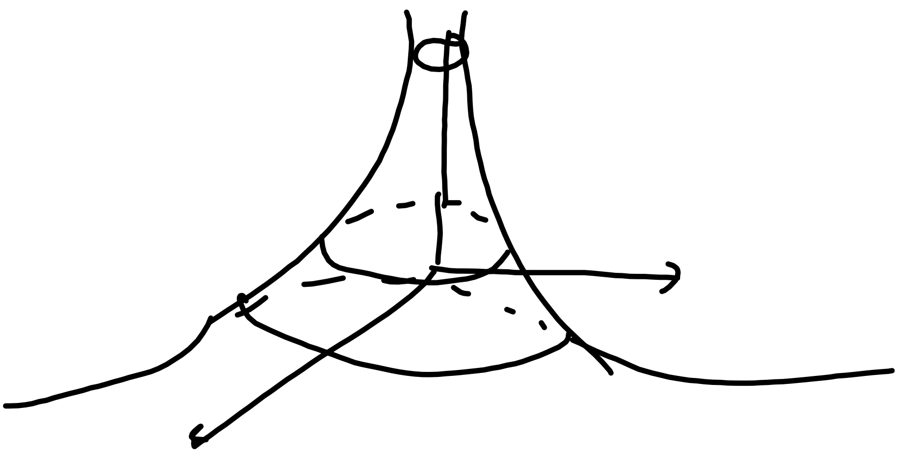
y-retning : $\sqrt{c}$,

b) $z = y^2 - x$

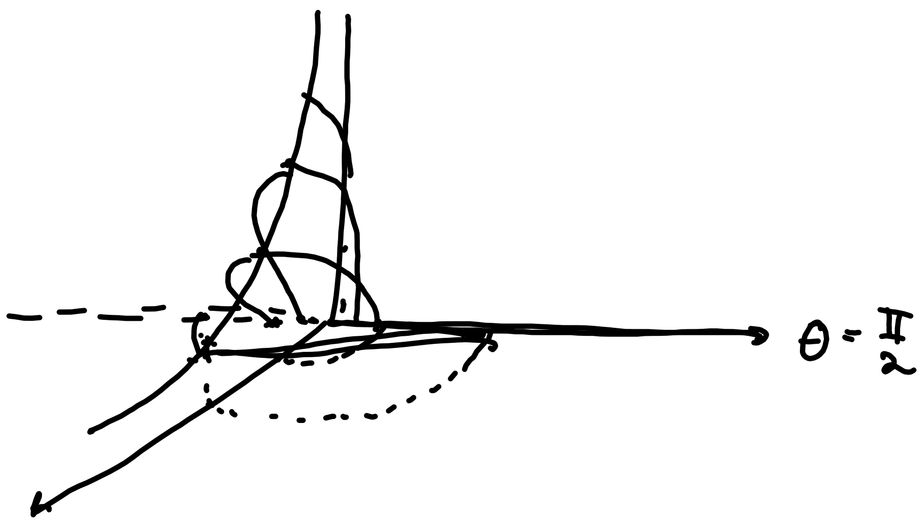
$x = 0$, $z = y^2$
$y = 0$, $z = -x$



3a) $z = \dfrac{1}{\sqrt{x^2+y^2}} = \dfrac{1}{r}$



b) $z = f(x, y) = \dfrac{x}{x^2 + y^2} = \dfrac{r\cos\theta}{r^2} = \dfrac{\cos\theta}{r}$



5 a) Tangentplan til $z = x^2 y$ i $\underbrace{(1,-2),\ z=-2}$

$F = z - x^2 y = 0$

Gradienten til F står $\perp$ på flaten.

$\underline{\nabla F = (-2xy, -x^2, 1)}$

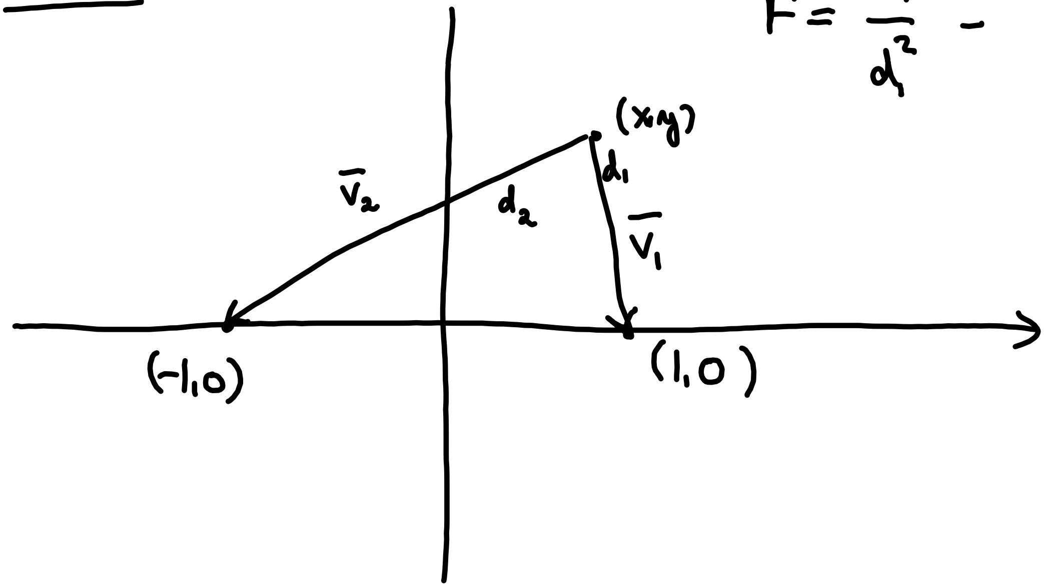
$\nabla F(\underbrace{1,-2,-2}) = (\underbrace{4,-1,1})$ Normalvektor.

$4(x-1) - 1\cdot(y-(-2)) + 1\cdot(z-(-2)) = 0$

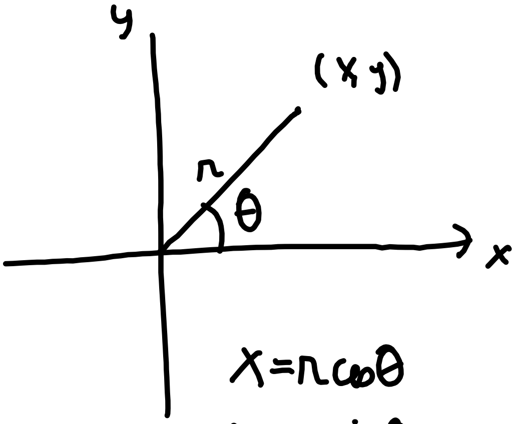
$\underline{4x - y + z = 4.}$

3.8.1c

$$\bar{F} = \frac{\bar{v}_1}{d_1^2} - \frac{\bar{v}_2}{d_2^2}$$



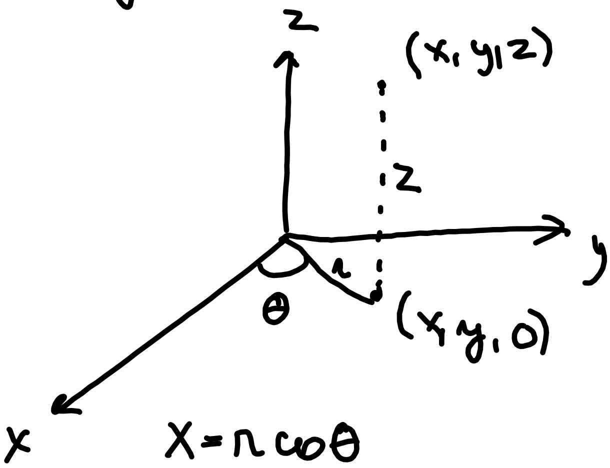
3.9. Polarkoordinater



$$x = r\cos\theta$$
$$y = r\sin\theta$$

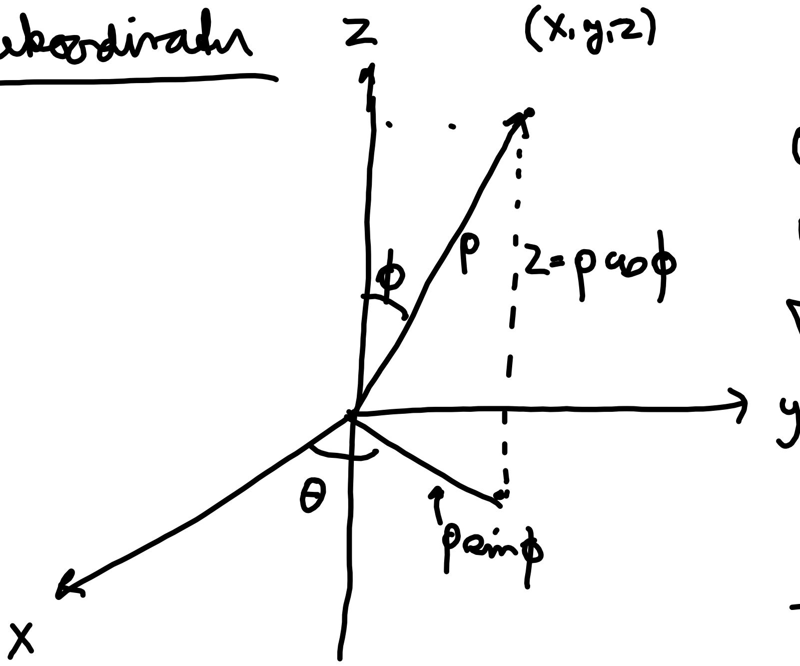
$$r = \sqrt{x^2 + y^2}$$
θ fra ligning

Sylinderkoordinater r, θ, z



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z$$

Kulekoordinater



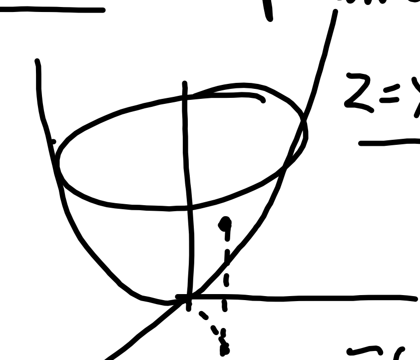
$\rho \geq 0$

$0 \leq \phi \leq \pi$

$0 \leq \theta \leq 2\pi$

$$\boxed{\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}}$$

3.9.1. To parameteriseringer av $z = x^2 + y^2$



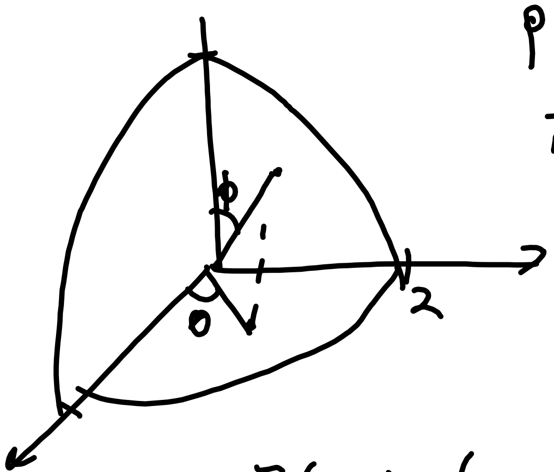
$z = x^2 + y^2$.

$$\bar{r}(x,y) = (x, y, x^2 + y^2)$$

$$z = x^2 + y^2 = r^2$$

$$\underline{\bar{r}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)}$$

2) Kuleflaten $\underline{x^2+y^2+z^2=4}$ i 1. oktant.



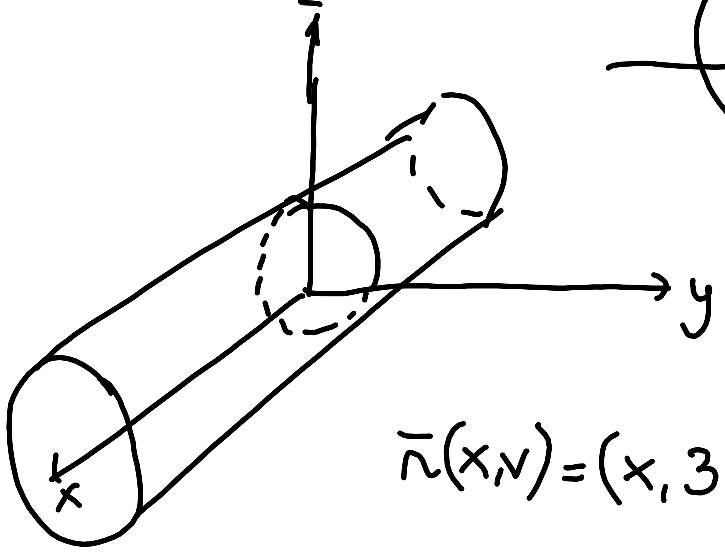
$\rho = 2$

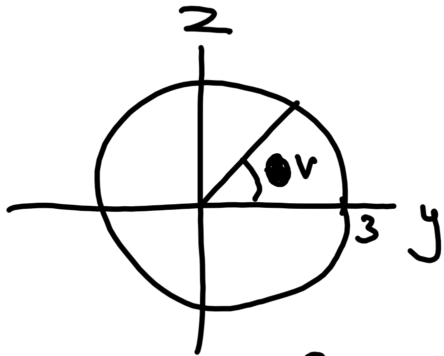
$$\vec{r}(\phi,\theta) = \underline{(2\sin\phi\cos\theta,\ 2\sin\phi\sin\theta,\ 2\cos\phi)}$$

$$\left.\begin{aligned} 0 &\le \phi \le \frac{\pi}{2} \\ 0 &\le \theta \le \frac{\pi}{2}. \end{aligned}\right\}$$

$$\vec{r}(x,y) = (x, y, \sqrt{4-x^2-y^2})$$

8. Sylinderen $y^2 + z^2 = 9$





$y = 3\cos v$

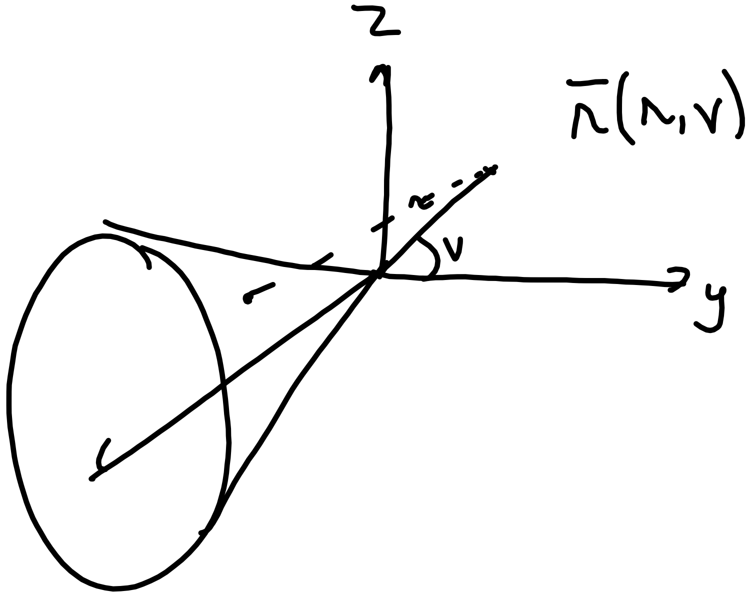
$z = 3\sin v$

x

$\bar{r}(x,v) = (x, 3\cos v, 3\sin v)$

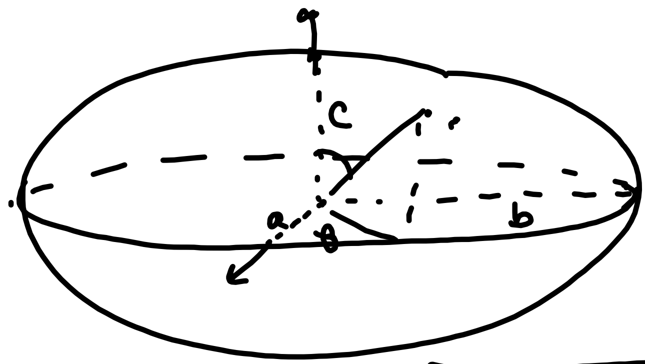
5) Kjeglefaten $\underline{x} = \sqrt{y^2 + z^2} = r$ Polarkoord i y, z

$\bar{r}(r, v) = (r, r\cos v, r\sin v)$



6. Ellipsoide

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

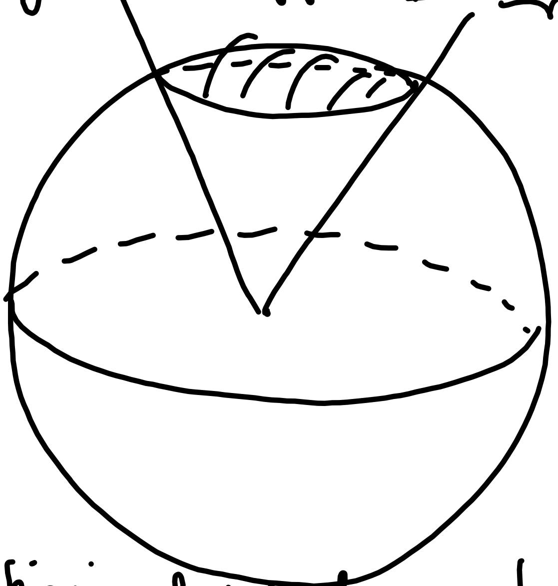


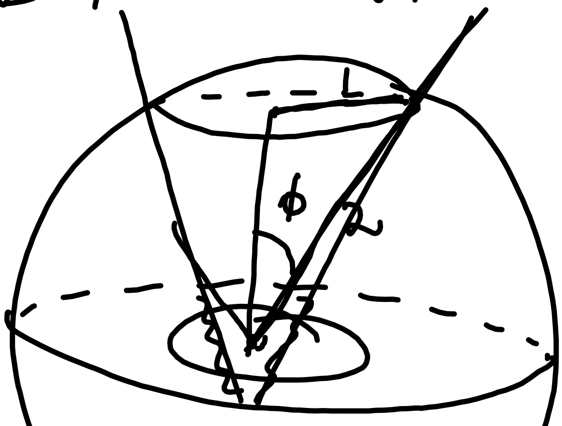
Parametre ϕ, θ, $0 \le \phi \le \pi$
$0 \le \theta \le 2\pi$

ϕ, θ.
ikke de geometriske vinklene

$$\tilde{r}(\phi,\theta)=(a\sin\phi\cos\theta,\ b\sin\phi\sin\theta,\ c\cos\phi).$$

8. Den delen av $x^2+y^2+z^2=4$, over xy planet og inni kjeglen $z^2=3(x^2+y^2)$, $z=\sqrt{3}\,|(x,y)|$





Skjæring kuleflate og kjegle: $x^2+y^2+3(x^2+y^2)=4$

$$4(x^2+y^2)=4$$
$$x^2+y^2=1$$

$\sin\phi=\frac{1}{2}$, $\phi=\frac{\pi}{6}$

$$\left.\begin{array}{l} 0\le\phi\le\frac{\pi}{6}. \\ 0\le\theta\le 2\pi \end{array}\right\}$$

$$\bar{r}(\phi,\theta)=(2\sin\phi\cos\theta,\ 2\sin\phi\sin\theta,\ 2\cos\phi)$$