$\vec{a}$ $\bar{a}$

$\bar{T} : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ linear

er alltid gitt ved matrise ($m \times n$ matrise)

$$A = m\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$
$$n$$

$$\bar{T}(\bar{x}) = A\bar{x}$$

A er gitt ved $\begin{pmatrix} \bar{T}(\bar{e}_1) & \bar{T}(\bar{e}_2) & \cdots & T(\bar{e}_n) \\ \cdot & \cdot & & \cdot \end{pmatrix}$

<u>1.91</u> $\bar{T}$ er gitt ved $\bar{T} : \mathbb{R}^3 \to \mathbb{R}^2$

$$T(x, y, z) = \begin{pmatrix} 2x - 1y + 1z \\ -1x + 1y - 3z \end{pmatrix}$$

$$A = \begin{pmatrix} \bar{T}(\bar{e}_1) & \bar{T}(\bar{e}_2) & \bar{T}(\bar{e}_3) \end{pmatrix}$$
$$= \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}$$

2 $\quad \bar{T} : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$

$$\bar{T}(\bar{e}_1) = \begin{pmatrix} -1 \\ 2 \\ -3 \\ 4 \end{pmatrix} \qquad \bar{T}(\bar{e}_2) = \begin{pmatrix} 0 \\ -2 \\ 4 \\ 7 \end{pmatrix}$$

$$A = \left(\bar{T}(\bar{e}_1) \;\; \bar{T}(\bar{e}_2) \right) = \begin{pmatrix} -1 & 0 \\ 2 & -2 \\ -3 & 4 \\ 4 & 7 \end{pmatrix}$$

---

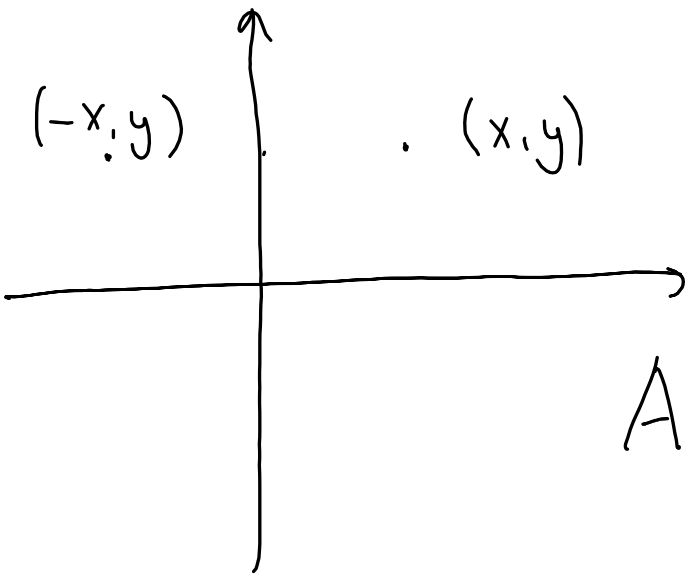
3) $\bar{T} : \mathbb{R}^2 \longrightarrow \mathbb{R}^{\cancel{3}2}$ lineær

$$\bar{T}(\bar{a}) = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \qquad \bar{T}(\bar{b}) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\bar{T}(3\bar{a} - 2\bar{b}) = 3\bar{T}(\bar{a}) - 2\bar{T}(\bar{b})$$

$$= 3\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6 \\ -3 \end{pmatrix}}}$$

4. $\bar{T}: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad \bar{T}(\bar{a})$ = speilbildet om y-aksen



$$\bar{T}(x,y) = \begin{pmatrix} -x \\ y \end{pmatrix}$$

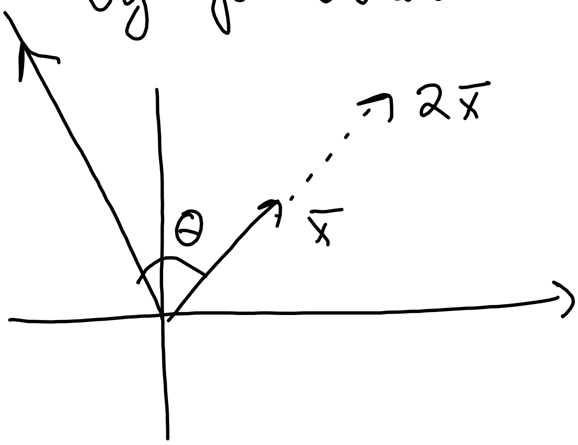
$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \left(\bar{T}(\bar{e}_1)\ \bar{T}(\bar{e}_2)\right)$$

5) $\bar{T}: \mathbb{R}^2 \to \mathbb{R}^2$ fordobler annen komp
bevarer første komp

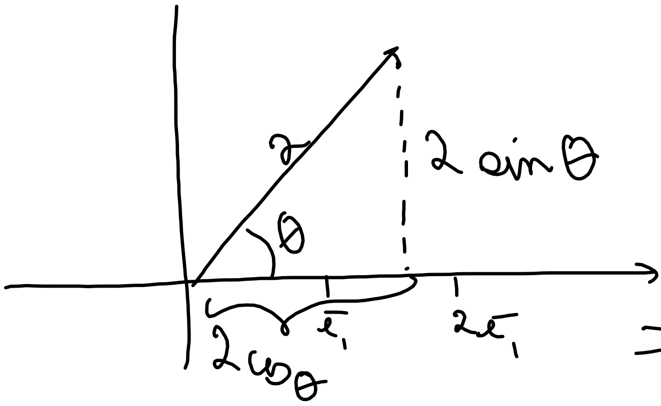
$$\bar{T}(x,y) = \begin{pmatrix} x \\ 2y \end{pmatrix} \quad A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

b) $T: \mathbb{R}^2 \to \mathbb{R}^2$

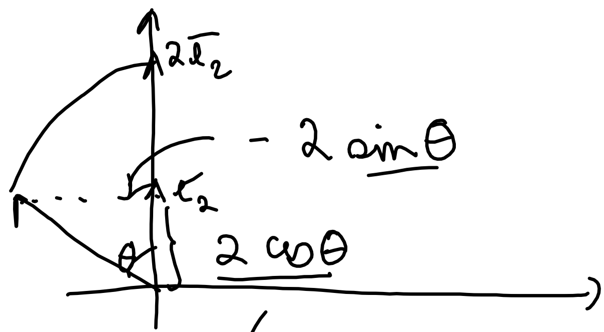
T dreier vinkel θ mot klokka og fordobler lengden.



$$T(\bar{e}_1) = \begin{pmatrix} 2\cos\theta \\ 2\sin\theta \end{pmatrix}$$



$$T(\bar{e}_2) = \begin{pmatrix} -2\sin\theta \\ 2\cos\theta \end{pmatrix}$$



$$A = \begin{pmatrix} 2\cos\theta & -2\sin\theta \\ 2\sin\theta & 2\cos\theta \end{pmatrix} = 2\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

7) $T: \mathbb{R}^3 \to \mathbb{R}^3$ avbilder vektorer på projeksjonen i xy-planet

$$T(x,y,z) = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} T(\bar{e}_1) & T(\bar{e}_2) & T(\bar{e}_3) \end{pmatrix}$$

Affin avbildning

$$\bar{F}: \mathbb{R}^n \to \mathbb{R}^m \quad , \quad A \ m \times n \ \text{matrise}$$

$$\bar{F}(\bar{x}) = \underline{A}\bar{x} + \bar{c} \qquad \bar{c} \in \mathbb{R}^m$$

$$\bar{F}(\bar{0}) = \bar{c}$$

1) $$\bar{F}(x, y, z) = \begin{pmatrix} 2x - 3y + z - 7 \\ -x + z - 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2x - 3y + z \\ -x + z \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \underbrace{\begin{pmatrix} -7 \\ -2 \end{pmatrix}}_{\bar{c}}$$

$\underline{3}$ $\bar{F}: \mathbb{R}^2 \to \mathbb{R}^2$ affin

$\bar{F}(0,0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \bar{c}$ $\quad \bar{F}(\underset{}{\overset{\bar{e}_1 \\ \|}{1,0}}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\bar{F}(\underset{\bar{e}_2}{\underset{\|}{0,1}}) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

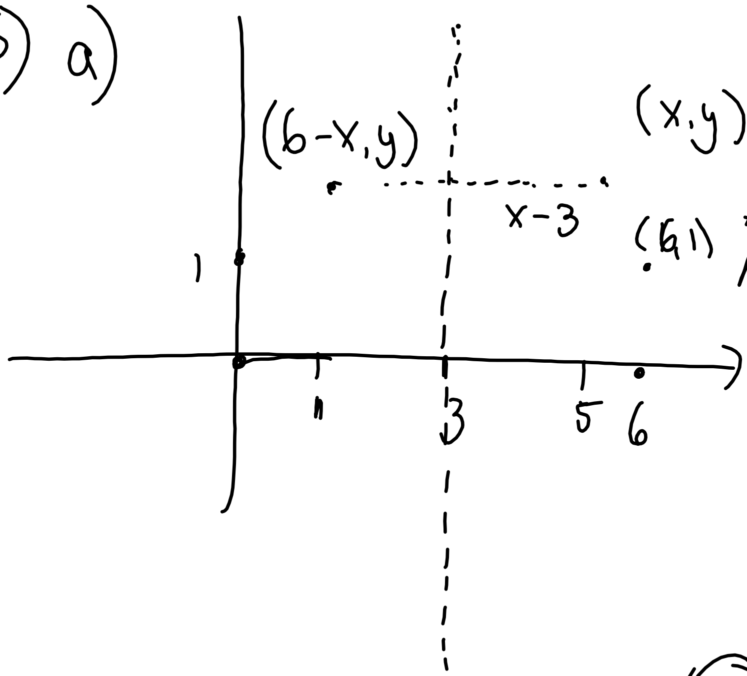
$\bar{F}(\bar{x}) = A\bar{x} + \bar{c} \qquad A\bar{x} = \bar{F}(\bar{x}) - \bar{c}$

$A\bar{e}_1 = \bar{F}(\bar{e}_1) - \bar{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$A\bar{e}_1 = \bar{F}(\bar{e}_2) - \bar{c} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & -2 \\ 4 & 1 \end{pmatrix}$

5) a)



$\bar{F}(0,0) = \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \bar{c}$

$\bar{F}(\bar{e}_1) = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$A\bar{e}_1 = \bar{F}(\bar{e}_1) - \bar{c} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$3-(x-3)$

$= 6-x$

$$\bar{F}(x,y) = \begin{pmatrix} -x & +6 \\ y & \end{pmatrix}$$

$$\bar{c} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

4.1.4.

$$1 \cdot x - 2y + 3z = 1$$
$$-x + y - 2z = 0$$
$$-3x + 5y - 8z = 2$$

$$\begin{pmatrix} 1 & -2 & 3 & 1 \\ -1 & 1 & -2 & 0 \\ -3 & 5 & -8 & 2 \end{pmatrix} \overset{\begin{subarray}{c} II + I \\ III + 3I \end{subarray}}{\sim} \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 5 \end{pmatrix} \; III - II$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$0x + 0y + 0z = 4$$

Umulig. Ingen løsning

4.1.1.

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & -3 & -1 \\ -1 & 2 & 3 & 1 \end{pmatrix} \underset{\sim}{\substack{\mathrm{II}-2\mathrm{I} \\ \mathrm{III}+\mathrm{I}}} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & -1 & -7 \\ 0 & 4 & 2 & 4 \end{pmatrix} \underset{\sim}{(-1)\mathrm{II}}$$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 4 & 2 & 4 \end{pmatrix} \underset{\sim}{\mathrm{III}-4\mathrm{II}} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & -2 & -24 \end{pmatrix} \underset{(-\frac{1}{2})\mathrm{III}}{\sim}$$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 12 \end{pmatrix} \underset{\sim}{\substack{\mathrm{I}+\mathrm{III} \\ \mathrm{II}-\mathrm{III}}} \begin{pmatrix} 1 & 2 & 0 & 15 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 12 \end{pmatrix} \underset{\sim}{\mathrm{I}-2\mathrm{II}}$$

Trappeform.

$$\begin{pmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 12 \end{pmatrix}$$

Redusert trappeform:

$x = 25$
$y = -5$
$z = 12$

Entydig lösning

<u>41.3.</u> $\begin{pmatrix} 2 & -4 & 6 & -2 \\ -3 & 2 & -1 & 8 \\ 1 & -6 & 11 & 4 \end{pmatrix} \overset{\cdot \frac{1}{2} I}{\sim} \begin{pmatrix} 1 & -2 & 3 & -1 \\ -3 & 2 & -1 & 8 \\ 1 & -6 & 11 & 4 \end{pmatrix} \begin{matrix} II + 3I \\ III - I \\ \sim \end{matrix}$

$\begin{pmatrix} 1 & -2 & 3 & -1 \\ 0 & -4 & 8 & 5 \\ 0 & -4 & 8 & 5 \end{pmatrix} \overset{III - II}{\sim} \begin{pmatrix} 1 & -2 & 3 & -1 \\ 0 & -4 & 8 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \cdot \left(-\frac{1}{4}\right) II \\ \sim \end{matrix}$

$\begin{pmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -2 & -5/4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \overset{I + 2 \cdot II}{\sim} \begin{pmatrix} \boxed{1 \quad 0 \quad -1 \quad -7/2} \\ \boxed{0 \quad 1 \quad -2 \quad -5/4} \\ 0 \quad 0 \quad 0 \quad 0 \end{pmatrix}$

Redusert trapp.

z velges fritt

$y - 2z = -5/4 \qquad y = 2z - 5/4$

$x - z = -7/2 \qquad x = z - 7/2.$

4.1.6. x = sannsynlighet for at A vinner når det er deuce

y = ——— " ——— hvis det er fordel A

z = ——— " ——— B.

$$x = 0.6y + 0.4z$$

$$y = 0.6 + 0.4x$$

$$z = 0.6x$$

$$\begin{pmatrix} 1 & -0.6 & -0.4 & 0 \\ -0.4 & 1 & 0 & 0.6 \\ -0.6 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.6923 \\ 0.8769 \\ 0.4154 \end{pmatrix}$$

Lewes.

Leuiy		A	B	C
A	x	$0.6x$	$0.3x$	$0.1x$
B	y	$0.3y$	$0.5y$	$0.2y$
C	z	$0.6z$	$0.1z$	$0.3z$

$$x + y + z = 120$$

$$x = 0.6x + 0.3y + 0.6z$$

$$y = 0.3x + 0.5y + 0.1z$$

$$z = 0.1x + 0.2y + 0.3z$$

$$x = 60$$

$$y = 40$$

$$z = 20.$$