6.11.1.

a) $\iint_R xy\,dx\,dy \qquad R=[1,2]\times[2,4]$

$$= \int_2^4 \left(\int_1^2 xy\,dx \right) dy = \int_2^4 \left[\tfrac{1}{2}x^2 y\right]_1^2 dy = \int_2^4 \left(\tfrac{1}{2}\cdot 4y - \tfrac{1}{2}y\right) dy$$

$$= \int_2^4 \tfrac{3}{2}y\,dy = \tfrac{3}{4}y^2\Big|_2^4 = \tfrac{3}{4}(16-4) = 9.$$

b) $\iint_R (x+\sin y)\,dx\,dy \qquad R=[0,1]\times[0,\pi]$

$$\int_0^1 (x+\sin y)\,dx = \tfrac{1}{2}x^2 + x\sin y \Big|_0^1 = \tfrac{1}{2}+\sin y$$

$$\iint_R (x+\sin y)\,dx\,dy = \int_0^\pi \tfrac{1}{2}+\sin y\,dy = \tfrac{1}{2}y - \cos y\Big|_0^\pi =$$

$$\left(\tfrac{\pi}{2}-(-1)\right)-(0-1) = \tfrac{\pi}{2}+2$$

c) $$\int_0^1 \left(\int_{-1}^1 x^2 e^y\,dx \right) dy = \int_0^1 \left[\tfrac{1}{3}x^3 e^y\right]_{x=-1}^{x=1} dy = \int_0^1 \tfrac{2}{3}e^y\,dy = \tfrac{2}{3}e^y\Big|_0^1 = \tfrac{2}{3}(e-1)$$

d) $\iint_R x\cos(xy)\,dx\,dy \qquad R=[1,2]\times[\pi,2\pi]$

$$\int_1^2 \left(\int_\pi^{2\pi} x\cos(xy)\,dy \right) dx = \int_1^2 \left[x\cdot\tfrac{1}{x}\sin(xy)\right]_{y=\pi}^{y=2\pi} dx$$

$$= \int_1^2 \sin(2\pi x) - \sin \pi x\,dx = -\tfrac{1}{2\pi}\cos(2\pi x) + \tfrac{1}{\pi}\cos \pi x\Big|_1^2 =$$

$$-\tfrac{1}{2\pi}\cos 4\pi + \tfrac{1}{\pi}\cos 2\pi - \left(-\tfrac{1}{2\pi}\cos 2\pi + \tfrac{1}{\pi}\cos\pi\right)$$

$$= -\tfrac{1}{2\pi} + \tfrac{1}{\pi} + \tfrac{1}{2\pi} + \tfrac{1}{\pi} = \tfrac{2}{\pi}$$

e) $\iint xy\,e^{x^2y} = \int_1^2 \left(\int_0^2 xy\,e^{x^2y}\,dx \right) dy$

Indre

$$\int_0^2 xy\,e^{x^2y}\,dx = \int_0^{4y} e^u \cdot \tfrac{1}{2}\,du = \tfrac{1}{2} e^u \Big|_0^{4y} = \tfrac{1}{2}\left(e^{4y}-1\right)$$

$u = x^2y$

$du = 2xy\,dx$

$$\tfrac{1}{2}\int_1^2 \left(e^{4y}-1\right) dy = \tfrac{1}{2}\left(\tfrac{1}{4}e^{4y}-y\right)\Big|_1^2 = \tfrac{1}{2}\left[\left(\tfrac{1}{4}e^8-2\right)-\left(\tfrac{1}{4}e^4-1\right)\right]$$

$$= \tfrac{1}{2}\left(\tfrac{1}{4}e^8-\tfrac{1}{4}e^4-1\right) = \tfrac{1}{8}\left(e^8-e^4-4\right)$$

f) $\int_1^e \left(\int_1^e \ln(xy)\,dx\,dy = \int_1^e\int_1^e (\ln x + \ln y\;dx)\,dy = \right.$

$$\int_1^e (x\ln x - x + x\ln y)\Big|_1^e\,dy = \int_1^e \left[(e-e+e\ln y)-(-1+\ln y)\right.$$

$$= \int_1^e 1+(e-1)\ln y\,dy = (e-1)+(e-1)(y\ln y - y)\Big|_1^e$$

$$= (e-1)+(e-1)(e-e-(0-1)) = \underline{2(e-1)}$$

g) $\int_0^1 \frac{1}{1+x^2y}\,dy = \frac{1}{x^2}\ln(1+x^2y)\Big|_{y=0}^{y=1} = \frac{\ln(1+x^2)}{x^2}$

$$\int_1^{\sqrt{3}} \frac{\ln(1+x^2)}{x^2}\,dx = -\frac{1}{x}\ln(1+x^2) - \int_1^{\sqrt{3}} -\frac{1}{x}\cdot\frac{2x}{1+x^2}\,dx$$

$u = \ln(1+x^2)$

$$= -\frac{1}{x}\ln(1+x^2) + \int_1^{\sqrt{3}} \frac{2}{1+x^2}\,dx$$

$u' = \frac{2x}{1+x^2}$

$$= -\frac{1}{x}\ln(1+x^2) + 2\arctan x\Big|_1^{\sqrt{3}} =$$

$v' = \frac{1}{x^2}$

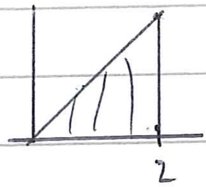
$v = -\frac{1}{x}$

$$= -\frac{1}{\sqrt{3}}\ln 4 + 2\cdot\frac{\pi}{3} - \left(-\ln 2 + 2\cdot\frac{\pi}{4}\right)$$

$$= \left(1-\frac{2}{\sqrt{3}}\right)\ln 2 + \frac{2}{3}\pi - \frac{1}{2}\pi = \left(1-\frac{2}{\sqrt{3}}\right)\ln 2 + \frac{\pi}{6}$$

6.2.1

a) $\iint_R x^2 y\,dx\,dy$ $\quad R=\{(x,y);\ 0\le x\le 2,\ 0\le y\le x\}$



$$=\int_0^2\left(\int_0^x x^2 y\,dy\right)dx=\int_0^2\left[\frac12 x^2 y^2\right]_{y=0}^{y=x}dx$$

$$=\int_0^2 \frac12 x^4\,dx=\frac{1}{10}x^5\Big|_0^2=\frac{32}{10}=\frac{16}{5}=\underline{3.2.}$$

b) $\iint_R (x+2xy)\,dx\,dy$ $\quad R=\{(x,y);\ 0\le x\le 3,\ x\le y\le 2x+1\}.$

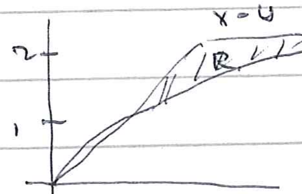
$$\int_0^3\left(\int_x^{2x+1}(x+2xy)\,dy\right)dx=\int_0^3\left[(xy+xy^2)\right]_x^{2x+1}dx$$

$$=\int_0^3 x(2x+1)+x(2x+1)^2-(x^2+x^3)\,dx$$

$$=\int_0^3 2x^2+x+4x^3+4x^2+x-x^2-x^3\,dx=\int_0^3 3x^3+5x^2+2x\,dx$$

$$=\frac34 x^4+\frac53 x^3+x^2\Big|_0^3=\frac{243}{4}+45+9=\frac{459}{4}$$

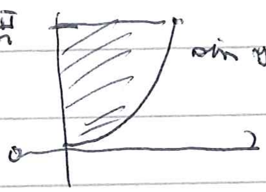
c) $\iint y\,dx\,dy$ $\quad R=\{1\le y\le 2;\ y\le x\le y^2\}$



$$=\int_1^2\left(\int_y^{y^2} y\,dx\right)dy=\int_1^2(y^3-y^2)\,dy=\frac14 y^4-\frac13 y^3\Big|_1^2=$$

$$\frac14 16-\frac13 8-\left(\frac14-\frac13\right)=4-\frac83-\frac14+\frac13=\frac{48-32-\cancel{12}3+4}{12}=\frac{17}{12}$$

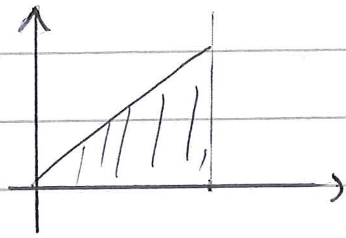
d) $\iint_R x\cos y\,dx\,dy$, $\quad R=\{(x,y),\ 0\le y\le\frac{\pi}{2},\ 0\le x\le\sin y\}$



$$=\int_0^{\frac{\pi}{2}}\left(\int_0^{\sin y} x\cos y\,dx\right)dy=\int_0^{\frac{\pi}{2}}\frac12\sin^2 y\cos y\,dy=\frac16\sin^3 y\Big|_0^{\frac{\pi}{2}}$$

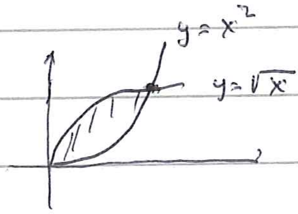
$$=\frac16.$$

e) $\iint e^{x^2}\,dx\,dy$ R = 1. kvadrant, avgrenset av x-aksen, $x=1$, $y=x$



$$\int_0^1 \Big(\int_0^x e^{x^2}\,dy \Big)\,dx$$

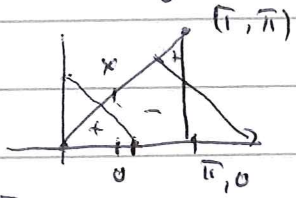
$$= \int_0^1 x e^{x^2}\,dx = \frac{1}{2} e^{x^2} \Big|_0^1 = \frac{1}{2}(e-1).$$



f) $\iint x^2 y\,dx\,dy$ R avgrenset av $y=x^2$, $y=\sqrt{x}$

$$\int_0^1 \Big(\int_{x^2}^{\sqrt{x}} x^2 y\,dy \Big)\,dx = \int_0^1 \Big[\frac{1}{2}x^2y^2\Big]_{y=x^2}^{y=\sqrt{x}} = \frac{1}{2}\int_0^1 x^3 - x^6\,dx = \frac{1}{2}\Big(\frac{1}{4}-\frac{1}{7}\Big) = \frac{1}{2}\cdot\frac{3}{28} = \underline{\underline{\frac{3}{56}}}$$

g) $\iint_R x\cos(x+y)\,dx\,dy$



$$= \int_0^\pi \Big(\int_0^x x\cos(x+y)\,dy \Big)\,dx = \int_0^\pi \Big[x\sin(x+y)\Big]_{y=0}^{y=x} = \int_0^\pi \underset{u}{x}\,(\underset{v'}{\sin 2x - \cancel{x}\sin x})\,dx$$

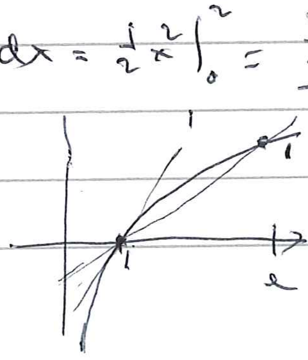
$u'=1$, $v = \cos x - \frac{1}{2}\cos 2x$

$$= x\Big(\cos x - \frac{1}{2}\cos 2x\Big)\Big|_0^\pi - \int_0^\pi \cos x - \frac{1}{2}\cos 2x$$

$$= x\Big(\cos x - \frac{1}{2}\cos 2x\Big) - \Big(\sin x - \frac{1}{4}\sin 2x\Big)\Big|_0^\pi = \pi\Big(-1-\frac{1}{2}\Big) = -\frac{3}{2}\pi$$

h) $\iint_R \frac{dx\,dy}{\sqrt{1-y^2}}$ R: $0 \le y \le \sin x$ $0 \le x \le \frac{\pi}{2}$.

$$= \int_0^{\frac{\pi}{2}} \Big(\int_0^1 \frac{dy}{\sqrt{1-y^2}} \Big)\,dx = \int_0^{\frac{\pi}{2}} \Big[\arcsin y\Big]_{y=0}^{y=\sin x} dx = \int_0^{\frac{\pi}{2}} x\,dx = \frac{1}{2}x^2\Big|_0^{\frac{\pi}{2}} = \frac{\pi^2}{8}$$



i) $\iint x\,dx\,dy$ $y=\ln x$ og $y=\frac{x-1}{e-1}$

= $x=e^y$ $x = 1+(e-1)y$

i) $\iint x \, dx \, dy = \int_0^1 \left(\int_{e^y}^{1+(e-1)y} x \, dx \right) dy = \int_0^1 \left[\frac{1}{2} x^2\right]_{e^y}^{1+(e-1)y} dy$

$= \frac{1}{2} \int_0^1 (e-1)^2 y^2 + 2(e-1)y + 1 - e^{2y} \, dy$

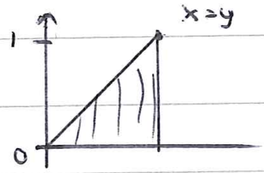
$= \frac{1}{2} \left(\frac{1}{3}(e-1)^2 + \frac{1}{2}\cancel{2}(e-1) + 1 - \frac{1}{2} e^{2y} \Big|_0^1 \right)$

$= \frac{1}{2} \left(\frac{1}{3} e^2 - \frac{2}{3} e + \frac{1}{3} + \cancel{\frac{2}{2}} e - \cancel{1} + 1 - \frac{1}{2} e^2 + \frac{1}{2} \right)$

$= \frac{1}{2} \left(-\frac{1}{6} e^2 + \frac{1}{3} e + \frac{5}{6} \right) = \underline{-\frac{1}{12} e^2 + \frac{1}{6} e + \frac{5}{12}}.$

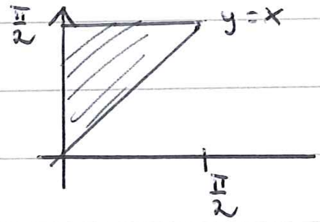
6.2.3

a) $\int_0^1 \left(\int_y^1 e^{x^2} dx \right) dy$



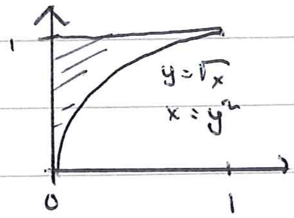
$= \int_0^1 \left(\int_0^x e^{x^2} d\cancel{y} \right) dx = \int_0^1 x e^{x^2} dx = \frac{1}{2} e^{x^2} \Big|_0^1 = \underline{\frac{1}{2}(e-1)}$

b) $\int_0^{\frac{\pi}{2}} \left(\int_x^{\pi/2} \frac{\sin y}{y} dy \right) dx$



$= \int_0^{\frac{\pi}{2}} \left(\int_0^y \frac{\sin y}{y} dx \right) dy = \int_0^{\frac{\pi}{2}} \sin y \, dy = [-\cos y]_0^{\frac{\pi}{2}} = 1.$

c) $\int_0^1 \left(\int_{\sqrt{x}}^1 e^{x/y^2} dy \right) dx$



$= \int_0^1 \left(\int_0^{y^2} e^{x/y^2} d\cancel{x} \right) dy = \int_0^1 \left[y^2 \cdot e^{x/y^2} \right]_{x=0}^{x=y^2} d\cancel{x}y$

$\int e^{ax} dx = \frac{1}{a} e^{ax}$

$= \int_0^1 y^2 (e-1) \, dy = \frac{1}{3}(e-1).$

6.3. Skifte til polarkoordinater

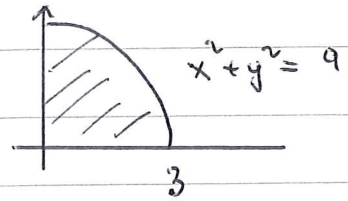
$$\iint_R f(x,y)\,dx\,dy = \iint_S f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

↑ NB!

Må finne RS!

6.3.1 a)

$$\iint_R x y^2\,dx\,dy$$

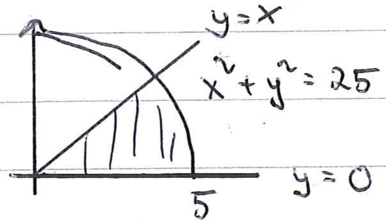


$S=[0,3]\times[0,\frac{\pi}{2}]$

$$= \int_0^{\frac{\pi}{2}}\left(\int_0^3 r\cos\theta\,r^2\sin^2\theta\,r\,dr\right)d\theta = \int_0^{\frac{\pi}{2}}\left[\frac{1}{5}r^5\cos\theta\sin^2\theta\right]_{r=0}^{r=3}d\theta$$

$$= \frac{243}{5}\int_0^{\frac{\pi}{2}}\cos\theta\sin^2\theta\,d\theta = \frac{243}{5}\cdot\left[\frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{2}} = \underline{\underline{\frac{81}{5}}}$$

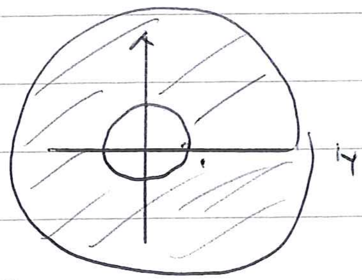
b) $$\iint_R (x^2+y^2)\,dx\,dy$$



$S=[0,5]\times[0,\frac{\pi}{4}]$

$$= \int_0^{\frac{\pi}{4}}\left(\int_0^5 r^2\cdot r\,dr\right)d\theta = \int_0^{\frac{\pi}{4}}\left[\frac{1}{4}r^4\right]_0^5 d\theta = \frac{625}{4}\cdot\frac{\pi}{4} = \underline{\underline{\frac{625\pi}{16}}}$$

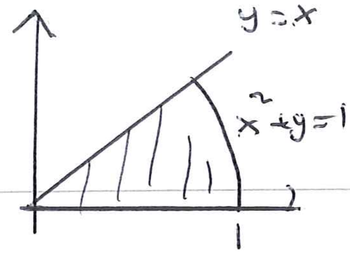
c) $$\iint_R e^{(x^2+y^2)}\,dx\,dy$$



$S=[1,4]\times[0,2\pi]$

$$= \int_0^{2\pi}\left(\int_1^4 e^{r^2}\cdot r\,dr\right)d\theta = \int_0^{2\pi}\left[\frac{1}{2}e^{r^2}\right]_{r=1}^{r=4}d\theta = \frac{1}{2}(e^{16}-e)\cdot 2\pi = \underline{(e^{16}-e)\pi}$$

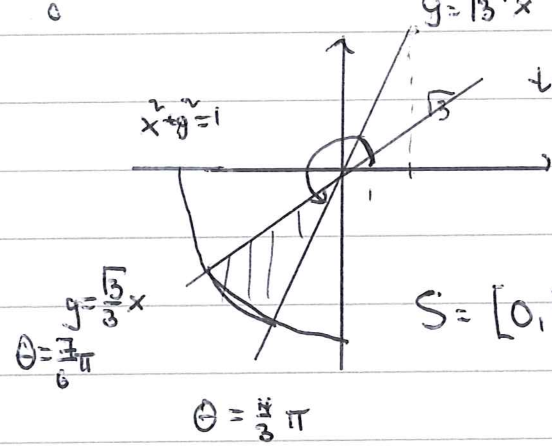
d) $\iint_R xy\,dx\,dy$



$S=[0,1]\times[0,\frac{\pi}{4}]$

$$= \int_0^{\frac{\pi}{4}} \left(\int_0^1 r^2 \cos\theta \sin\theta \cdot r\,dr \right) d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{4} \cos\theta \sin\theta \, d\theta = \frac{1}{8} \sin^2\theta \Big|_0^{\frac{\pi}{4}} = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}.$$

e) $\iint_R (x^2-y^2)\,dx\,dy$



$\tan\varphi=\sqrt{3}$
$\varphi=\frac{\pi}{3}$

(= 0 pga symmetri)

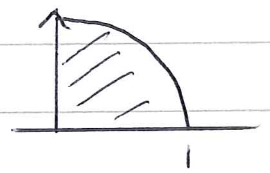
$S=[0,1]\times[\frac{7}{6}\pi,\frac{4}{3}\pi]$

$$= \int_{\frac{7}{6}\pi}^{\frac{4}{3}\pi} \left(\int_0^1 r^2(\cos^2\theta - \sin^2\theta)\, r\,dr\,d\theta \right)$$

$$= \int_{\frac{7}{6}\pi}^{\frac{4}{3}\pi} \frac{1}{4}\cos 2\theta\, d\theta = \left[\frac{1}{8}\sin 2\theta\right]_{\frac{7}{6}\pi}^{\frac{4}{3}\pi} = \frac{1}{8}\left(\sin\frac{8}{3}\pi - \sin\frac{7}{3}\pi\right) = \frac{1}{8}\left(\sin\frac{2}{3}\pi - \sin\frac{1}{3}\pi\right)$$
$$= 0.$$

f) $\iint_R \sqrt{2-(x^2+y^2)}\,dx\,dy$

R:

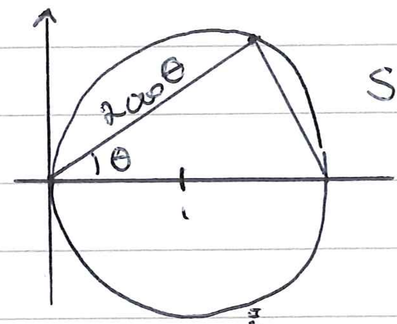


$S=[0,1]\times[0,\frac{\pi}{2}]$

$$= \int_0^{\frac{\pi}{2}} \left(\int_0^1 \sqrt{2-r^2}\, r\,dr \right) d\theta = \int_0^{\frac{\pi}{2}} \frac{2}{3\cdot 2}(2-r^2)^{\frac{3}{2}} \Big|_0^1 = -\frac{1}{3}\int_0^{\frac{\pi}{2}} (1-2\sqrt{2})\,d\theta =$$

$$\underline{\underline{(2\sqrt{2}-1)\cdot\frac{\pi}{6}.}}$$

g) $\iint_R (x^2+y^2)^{3/2}\,dx\,dy$



$S=\{(r,\theta);$
$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2},\ 0\le r\le 2\cos\theta\}$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\int_0^{2\cos\theta} r^3\cdot r\,dr \right) d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{1}{5} r^5\right]_0^{2\cos} d\theta$$

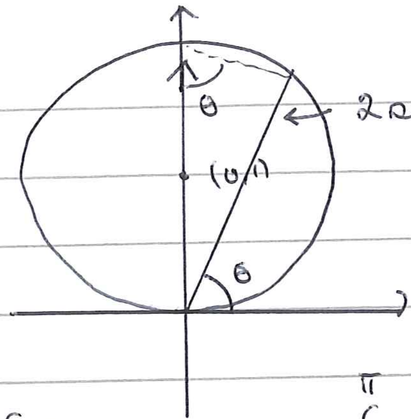
$$= \frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^5\theta\,d\theta = \frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta(1-\sin^2\theta)^2\,d\theta =$$

$$\frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta - 2\cos\theta\sin^2\theta + \cos\theta\sin^4\theta$$

$$= \frac{64}{5}\left(1-\frac{2}{3}+\frac{1}{5}\right) = \frac{64}{5}\cdot\frac{8}{15} = \frac{512}{75}$$

6.3.3

a)



$$\iint_R f\,dx\,dy = \int_0^{\pi}\left(\int_0^{2\sin\theta} f(r\cos\theta, r\sin\theta)\,r\,dr\right)d\theta$$

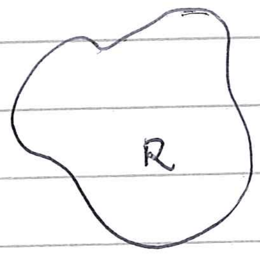
b)

$$\iint_R \sqrt{x^2+y^2}\,dx\,dy = \int_0^{\pi}\left(\int_0^{2\sin\theta} r\cdot r\,dr\right)d\theta = \int_0^{\pi}\frac{8}{3}\sin^3\theta\,d\theta$$

$$= \frac{8}{3}\int_0^{\pi}\sin\theta\,(1-\cos^2\theta)\,d\theta = \frac{8}{3}\left(2-2\cdot\frac{1}{3}\right) = \frac{8}{3}\cdot\frac{4}{3} = \underline{\underline{\frac{32}{9}}}$$

6.4. Anvendelser

Areal $A = \iint_R 1\cdot dx\,dy$

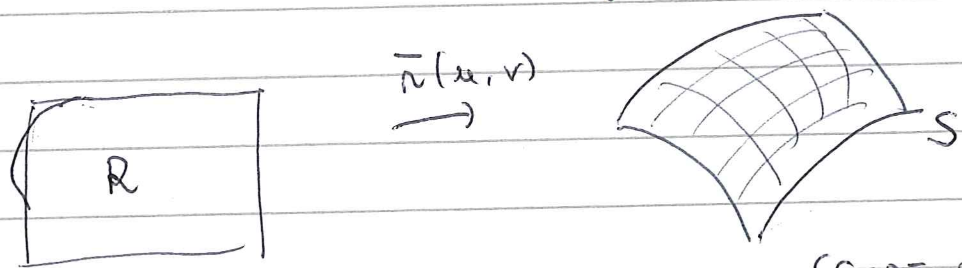


Massemiddelpunkt $\bar{x} = \frac{1}{A}\iint_R x\,dx\,dy$

$\bar{y} = \frac{1}{A}\iint_R y\,dx\,dy$

Massemiddelpunkt m/ tetthet $f(x,y)$:

$$\bar{x} = \frac{\iint xf\,dx\,dy}{\iint f\,dx\,dy} \qquad \bar{y} = \frac{\iint yf\,dx\,dy}{\iint f\,dx\,dy}.$$



Areal: $\iint_R \left|\frac{\partial \bar{r}}{\partial u}\times\frac{\partial \bar{r}}{\partial v}\right| du\,dv. = \iint_S dS.$

Integral av skalarfelt: $\iint_S f\,dS$

Volum under graf $z = f(x,y) \geq 0$. $\quad V = \iint_R f\,dx\,dy.$

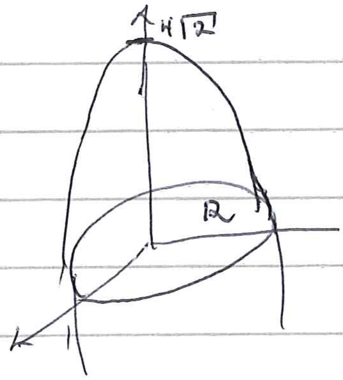
6.4.1 Beregn volum av E

a) $E = \{(x,y,z);\ 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le x + y^2\}$.

$$V = \int_0^2 \left(\int_0^1 x + y^2 \, dy \right) dx = \int_0^2 \left(x + \frac{1}{3}\right) dx = \frac{1}{2}x^2 + \frac{1}{3}x \Big|_0^2 = 2 + \frac{2}{3} = \frac{8}{3}.$$

d) E = området over xy under $z = \sqrt{32 - 2x^2 - 2y^2}$

$z = 0$ når $x^2 + y^2 = 16$, R = disk med radius 4



$$V = \iint_R z \, dx\, dy$$

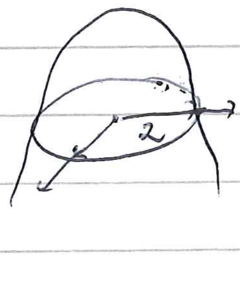
$$= \int_0^{2\pi} \left(\int_0^4 \sqrt{32 - 2r^2} \, r \, dr \right) d\theta$$

$$= \int_0^{2\pi} \left[(32 - 2r^2)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{-4} \right]_0^4 d\theta = \int_0^{2\pi} -\frac{1}{6}\left(0 - 32^{3/2}\right) d\theta =$$

$$\int_0^{2\pi} \frac{1}{6} \cdot 128\sqrt{2} \, d\theta = 2\pi \cdot \frac{1}{6} \cdot 128\sqrt{2} = \underline{\underline{\frac{128\sqrt{2}\,\pi}{3}}}$$

f) E = over xy planet og under grafen $z = 4 - (x-2)^2 - (y+1)^2$

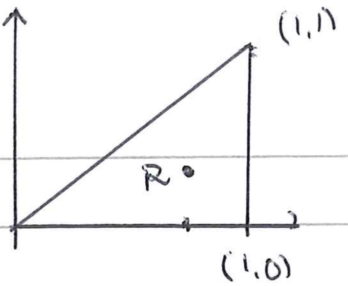
Flytter til origo $z = 4 - (x^2 + y^2)$



$$V = \int_0^2 \left(\int_0^{2\pi} (4 - r^2) \, r \, d\theta \right) dr = 2\pi \int_0^2 4r - r^3 \, dr$$

$$= 2\pi \left(2r^2 - \frac{1}{4}r^4\right)\Big|_0^2 = 2\pi(8 - 4) = \underline{\underline{8\pi}}$$

2.



Tetthet x.

$$\text{Total masse}: M = \iint_R x\,dx\,dy = \int_0^1 \Big(\int_0^x x\,dy \Big)\,dx = \int_0^1 x^2\,dx = \frac{1}{3}.$$

$$\text{Massemiddelp.}\quad \bar{x} = \frac{1}{M}\iint_R x\cdot x\,dx\,dy = 3\iint_R x^2\,dx\,dy = 3\int_0^1 x^3\,dx = \underline{\underline{\frac{3}{4}}}.$$

$$\bar{y} = \frac{1}{M}\iint_R yx\,dx\,dy = 3\int_0^1 \Big(\int_0^x xy\,dy \Big)\,dx = 3\cdot\frac{1}{2}\int_0^1 x^3\,dx = 3\cdot\frac{1}{2}\cdot\frac{1}{4} = \underline{\underline{\frac{3}{8}}}.$$

Eksamen juni 2015 – 4

V området avgrenset av

$$z = x^2 + 2x + y^2 - 4y = (x+1)^2 + (y-2)^2 - 5$$

$$\text{og}\quad z = 6 - (x^2 + 2x + y^2 + 4y) = 11 - \big((x+1)^2 + (y+2)^2\big)$$

a) Paraboloidene skjærer hverandre når

$$6 - x^2 - 2x - y^2 - 4y = x^2 + 2x + y^2 - 4y$$

$$6 - 2x^2 - 4x - 2y^2 = 0$$

$$2(3 - x^2 - 2x - y^2) = 0$$

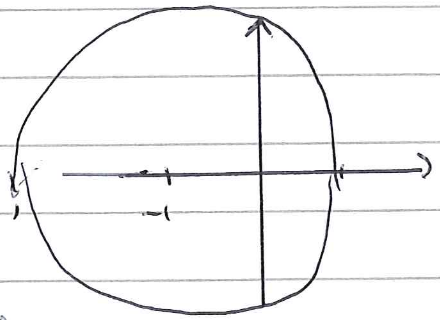
som gir

$$x^2 + 2x + y^2 = 3$$

$$(x+1)^2 - 1 + y^2 = 3$$

$$(x+1)^2 + y^2 = 4$$

Sirkel med sentrum $(-1,0)$ radius 2



Innenfor denne sirkelen er venstre side størst, altså er forskjellen $2(3 - x^2 - 2x - y^2)$

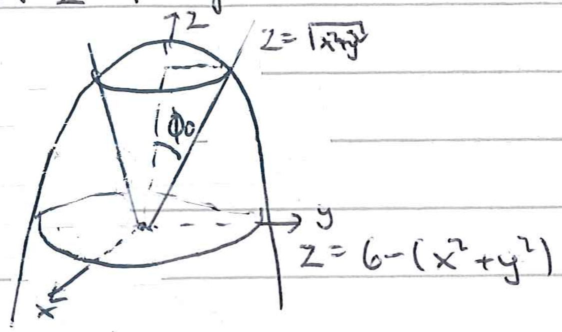
b) Polarkoordinater med sentrum i $(-1,0)$, $x = -1 + r\cos\theta$, $y = r\sin\theta$

$$V = 2\iint_D 3 - (x+1)^2 + 1 - y^2\,dx\,dy = 2\int_0^{2\pi}\!\!\int_0^2 (4 - r^2)\,r\,dr\,d\theta = 4\pi\int_0^2 4r - r^3\,dr$$

$$= 4\pi\Big(2r^2 - \frac{1}{4}r^4\Big)\Big|_0^2 = 4\pi\Big(8 - \frac{16}{4}\Big) = \underline{16\pi}.$$

Eksamen Juni 2016 - 3

Volumet avgrenset av paraboloiden $z = 6-(x^2+y^2)$ og kjeglen $z=\sqrt{x^2+y^2}$



I polarkoordinater

$z=\sqrt{x^2+y^2}=r$

$z=6-(x^2+y^2)=6-r^2$

a) De skjærer hverandre når

$$r=6-r^2 \text{, dvs}$$

$$r^2+r-6=0 \Rightarrow r=\frac{-1\pm\sqrt{1+24}}{2}=\frac{-1\pm 5}{2}=\begin{cases}2\\-3.\end{cases}$$

Altså for $r=2$.

Volumet ligger over disken $r\leq 2$. Der er $6-r^2$ størst, altså er

$$V=\iint_D (6-r^2-r)\,dx\,dy=\iint_D 6-x^2-y^2-\sqrt{x^2+y^2}\,dx\,dy$$

b) Vi beregner i polarkoordinater

$$V=\int_0^2\left(\int_0^{2\pi} 6-r^2-r\,d\theta\right)r\,dr=2\pi\int_0^2 6r-r^3-r^2\,dr=$$

$$2\pi\left(3r^2-\frac{1}{4}r^4-\frac{1}{3}r^3\right)\Big|_0^2=2\pi\left(12-4-\frac{8}{3}\right)=2\pi\cdot\frac{16}{3}=\underline{\underline{\frac{32}{3}\pi}}$$