5.4.1 $x_1 = y_1 = 0$

$$x_{n+1} = 0.6x_n - 0.6y_n + 0.2$$
$$y_{n+1} = 0.6x_n + 0.6y_n + 1$$

Program og figur på side ②

Fikspunkt:

$$0.6x - 0.6y + 0.2 = x$$
$$0.6x + 0.6y + 1 = y$$

$$\Updownarrow$$

$$-0.4x - 0.6y = -0.2 \quad | \cdot -5$$
$$0.6x - 0.4y = -1 \quad | \cdot 5$$

$$\Updownarrow$$

$$2x + 3y = 1$$
$$3x - 2y = 5$$

Løsning $x = -1$, $y = 1$

Stemmer med figur.

5.4.2 To dyreslag, $x_1 = 20$, $y_1 = 2000$

$$x_{n+1} = 0.9x_n + 0.01y_n - 10$$
$$y_{n+1} = -1.01x_n + y_n + 300$$

Figur på side ③. Fikspunkt:

(I) $0.9x + 0.01y - 10 = x$

(II) $-1.01x + y + 300 = y$

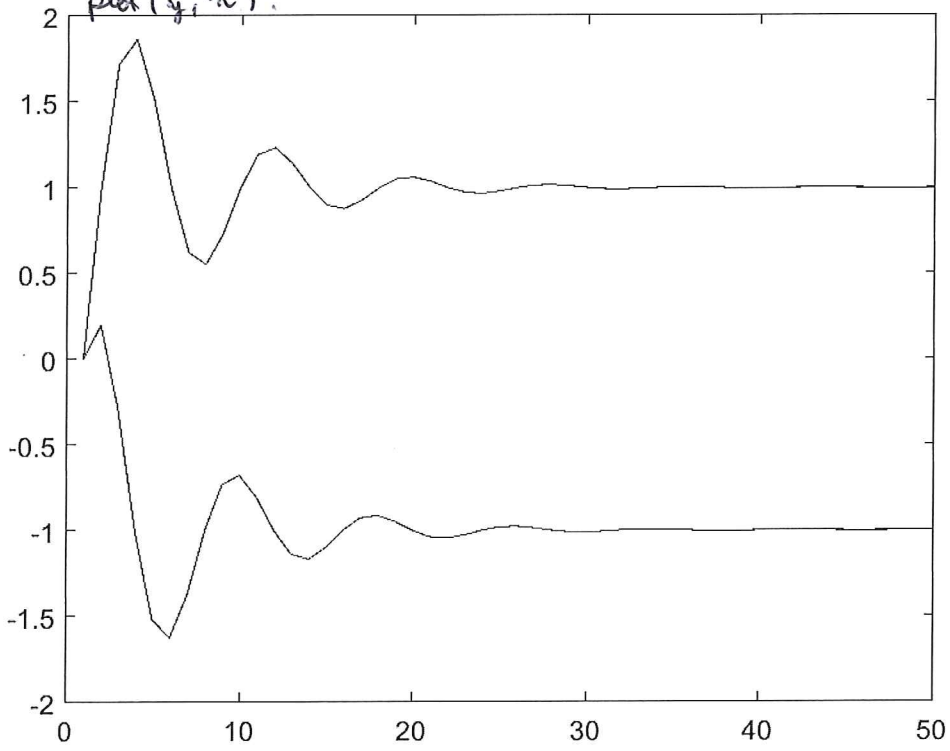
(II) gir $x = \frac{300}{1.01} \approx \underline{297}$

(I) gir $y = 1000 + 10x \approx \underline{3970}$

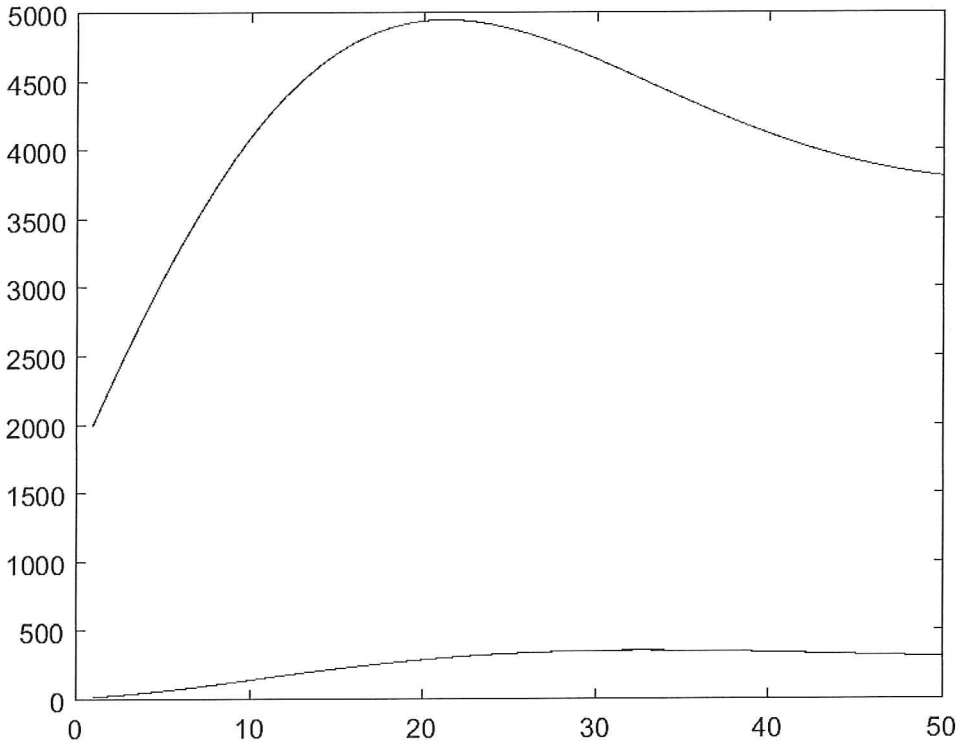
Stemmer med figur.

```
function [x,y] = sequence (m, k, N)
x = zeros (1,N);
y = zeros (1,N);
x(1) = m;
y(1) = k;
for n=1:N-1
x(n+1) = 0.6*x(n) - 0.6*y(n) + 0.2
y(n+1) = 0.6*x(n) + 0.6*y(n) + 1
end
```

```
[x,y] = sequence (0,0,50);
plot (x)
hold on
plot (y,'r')
```



Oppgave 5.4.1.


5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
10
20
30
40
50

Oppgave 5.4.2

5.4.4 Bensinprisen x_n og y_n

$$x_{n+1} = 1.01 \frac{x_n + y_n}{2}$$

$$y_{n+1} = \min(x_n, 1.1\, y_n)$$

$x_1 = 8$, $y_1 = 12$: Figur side (5)

$x_1 = 12$, $y_1 = 8$: Figur side (6)

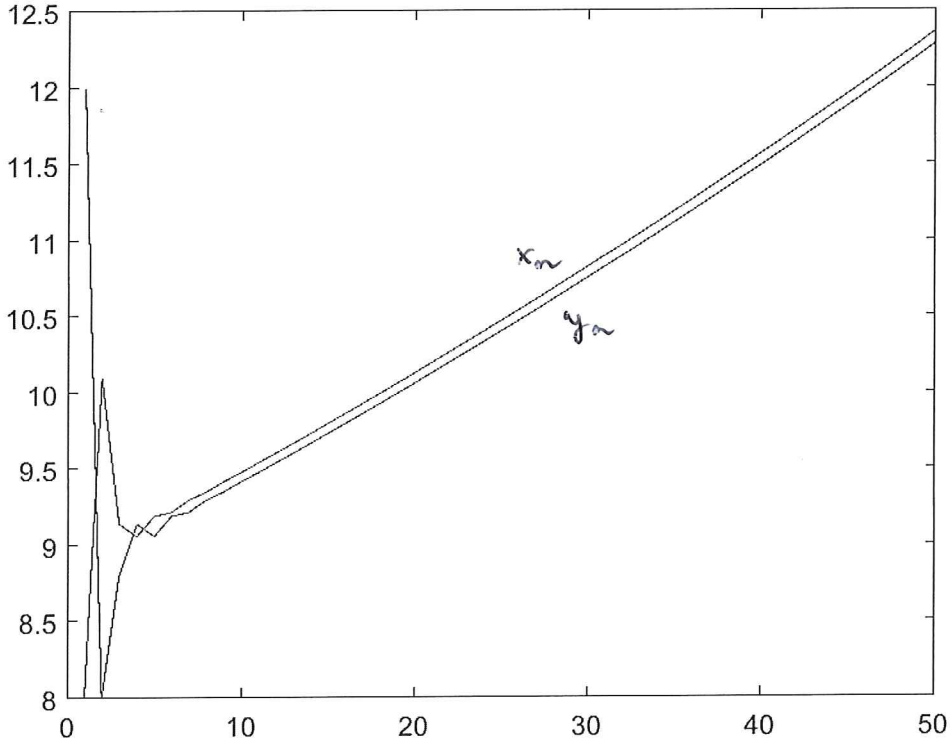
5.4.5 Insekter x_n og y_n (mål i millioner)

$$x_{n+1} = 2.2 x_n (1 - x_n) + 0.01 x_n y_n$$

$$y_{n+1} = 3.1 y_n (1 - y_n) - 0.02 x_n y_n$$

$x_1 = 0.5$ $y_1 = 0.5$ Figur side (7)

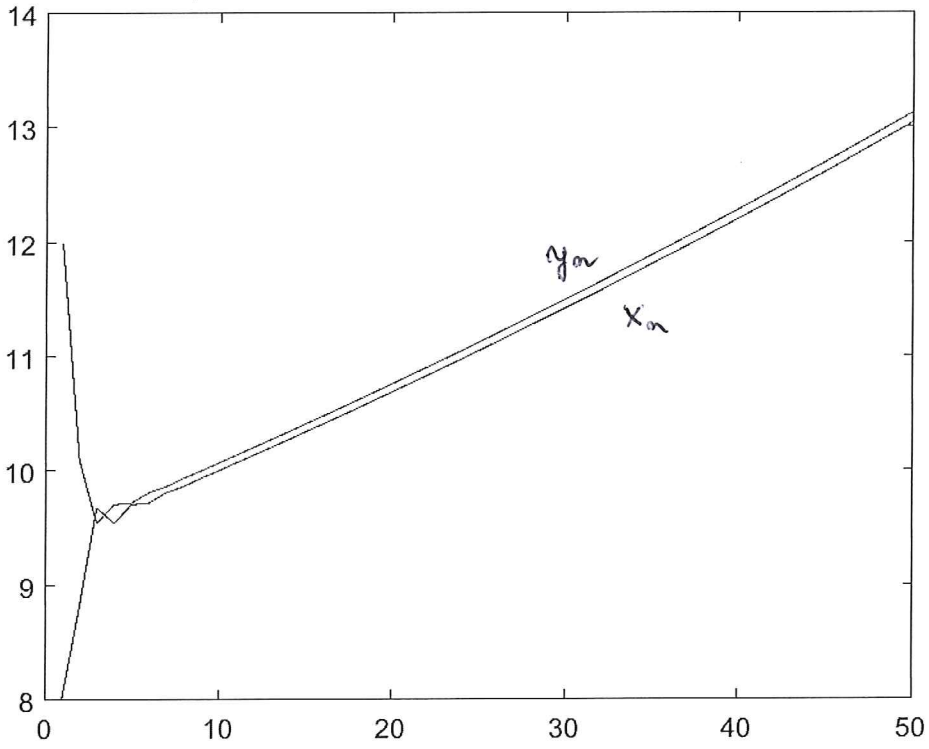
$x_1 = 0.1$ $y_1 = 0.8$ Figur side (8)



Oppgave 5.4.4.

$x_1 = 8$

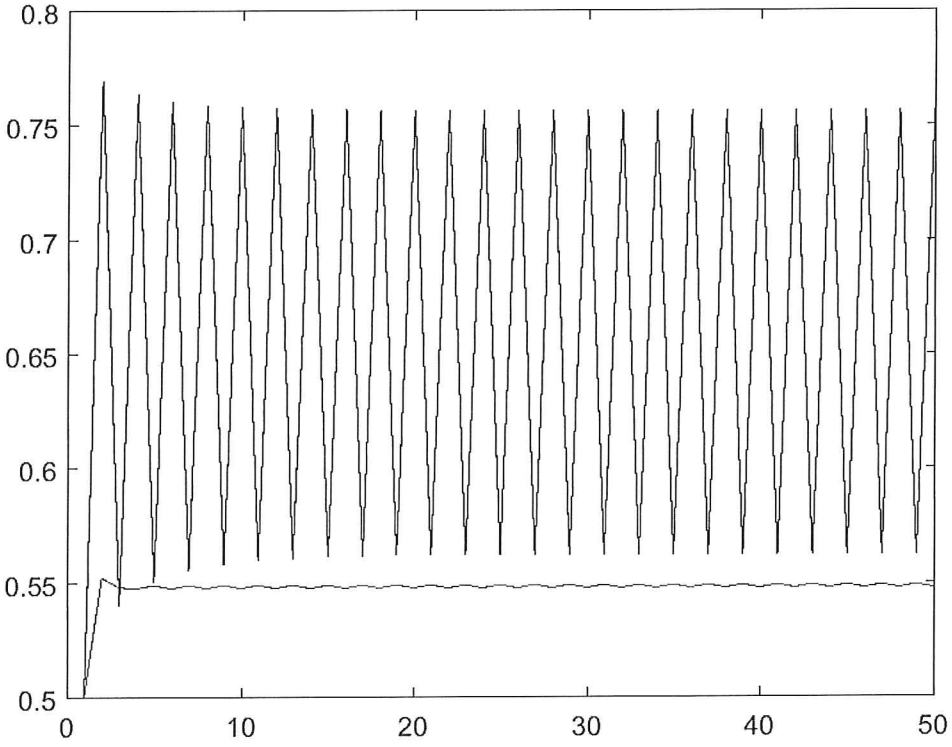
$y_1 = 12$



O ppgave 5.4.4.

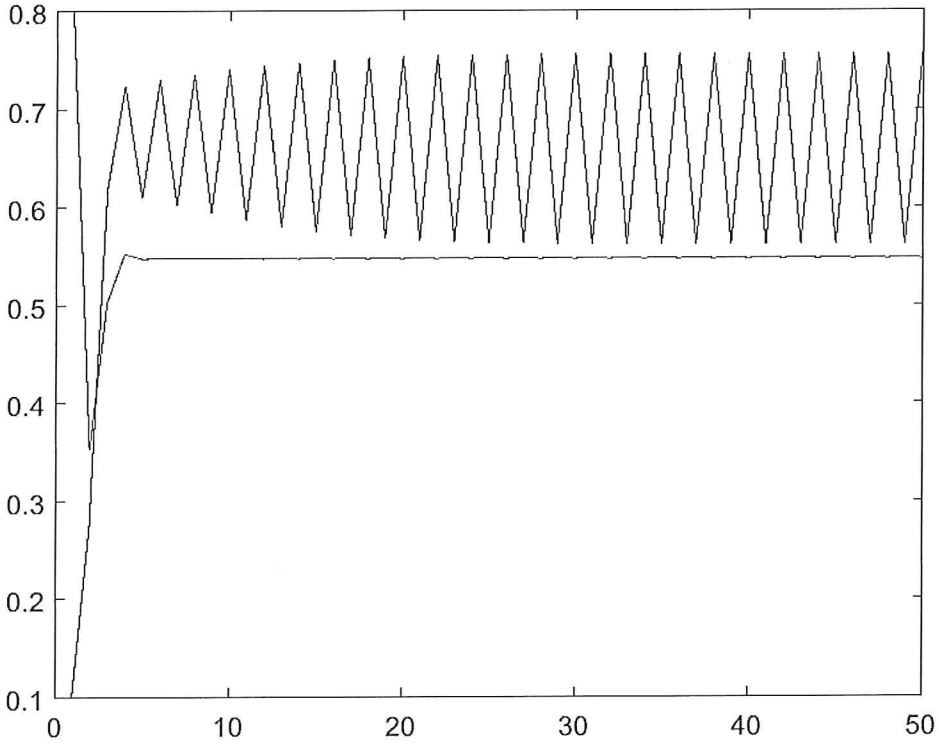
$x_1 = 12$

$y_1 = 8$



Oppgave 5.4.5

$X_1 = y_1 = 0.5$



Oppgave 5.4.5

$x_1 = 0.1 \quad y_1 = 0.8$

5.4.6.

Salg fra firma 1, pris p : $E_1(p,q) = 1000\, e^{-p/q - \alpha(p+q)}$

Salg fra firma 2, pris q : $E_2(p,q) = 1000\, e^{-q/p - \beta(p+q)}$

a) q fast.

Inntekt firma 1 : $I_1(p,q) = p E_1(p,q) = 1000\, p\, e^{-p/q - \alpha(p+q)}$

Maks inntekt når $0 = \frac{\partial I_1}{\partial p} = 1000\left(e^{-p/q-\alpha(p+q)} + p \cdot e^{-p/q-\alpha(p+q)}\left(-\frac{1}{q} - \alpha\right)\right)$

$= 1000\, e^{-p/q-\alpha(p+q)}\left(1 - p\left(\frac{1}{q} + \alpha\right)\right)$

Altså $1 - p\left(\frac{1}{q} + \alpha\right) = 0$

$$p = \frac{1}{\frac{1}{q} + \alpha} = \underline{\underline{\frac{q}{1+\alpha q}}}$$

Helt symmetrisk for firma 2. Maks inntekt når

$$q = \underline{\underline{\frac{p}{1+\beta p}}}$$

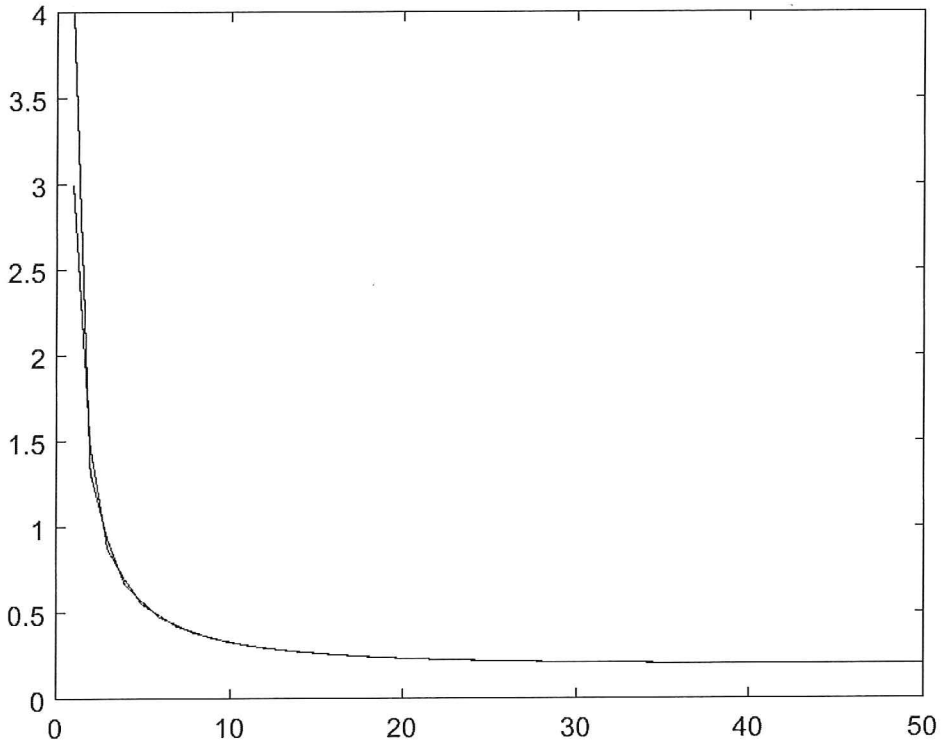
b) Priser første år: p_1 og p_2. Deretter

$$p_{n+1} = 1.1\, p_{n,\text{maks}} = \frac{1.1\, q_n}{1+\alpha q_n}$$

$$q_{n+1} = 1.1\, q_{n,\text{maks}} = \frac{1.1\, p_n}{1+\beta p_n}$$

c) $\alpha = \beta = 0.5$, $p_1 = 3$, $q_1 = 4$: Side 10, $p_1 = 1$, $q_1 = 1.3$ Side 11

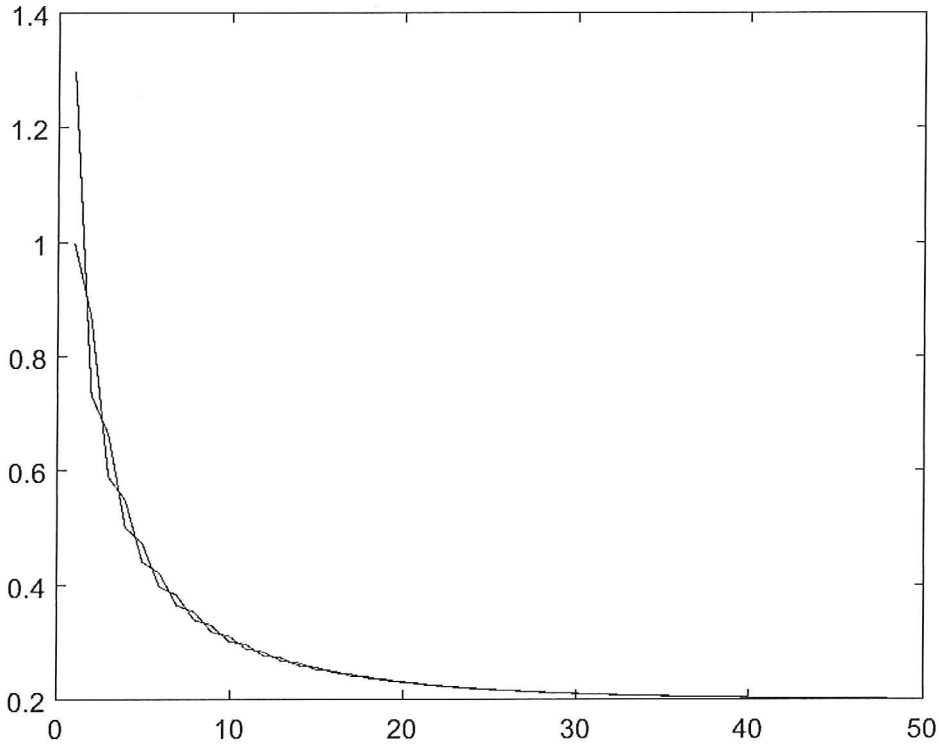
d) $\alpha = 0.05$, $\beta = 0.02$, $p_1 = 3$, $q_1 = 4$: Side 12, $p_1 = 1$, $q_1 = 1.3$ Side 13



Oppgave 5.4.6

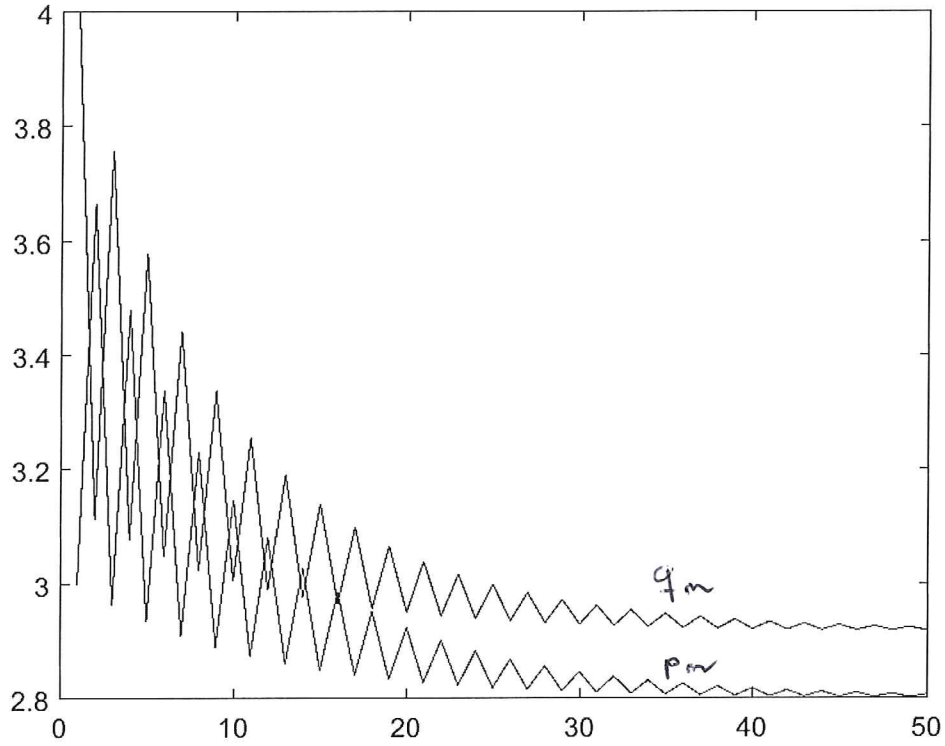
$\alpha = \beta = 0.5$

$(x_1, y_1) = (3, 4)$



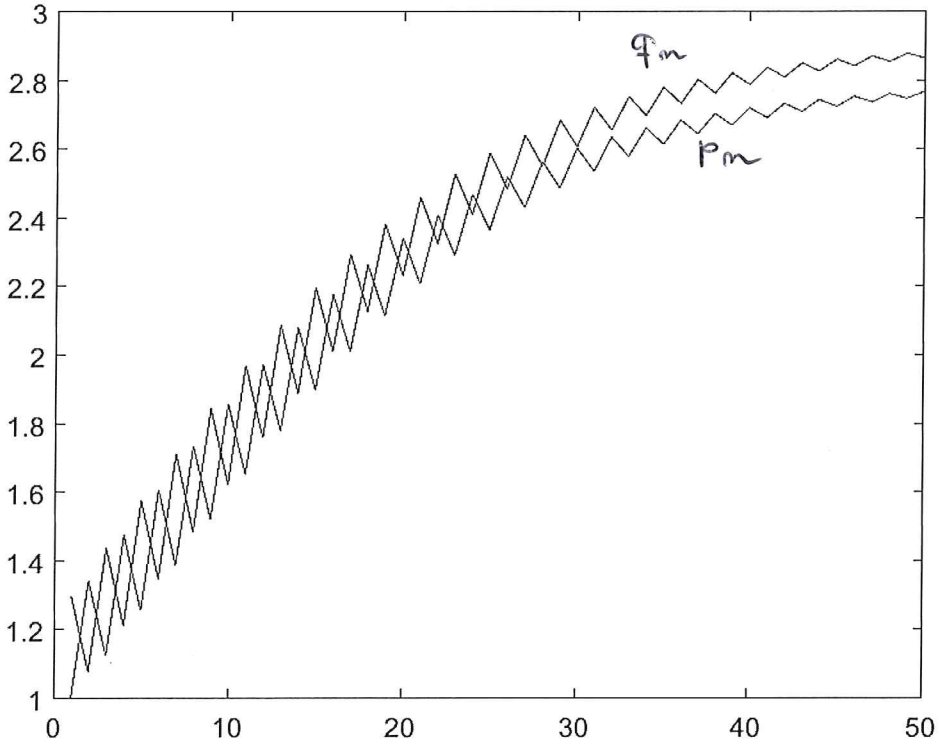
Oppgave 5.4.6.

$\alpha = \beta = 0.5$, $(p_1, q_1) = (1, 1.3)$



Oppgave 5.4.6 a)

$\alpha = 0.05$, $\beta = 0.02$ $(p_1, q_1) = (3, 4)$



Oppgave 5.4.6 a)

$\alpha = 0.05$, $\beta = 0.02$ $(p_1, q_1) = (1, 1.3)$

5.4.7 $f(x) = x^2 + x - 2$

$f(\sqrt{2}) = 2 + \sqrt{2} - 2 = \sqrt{2}$, så $\sqrt{2}$ er et fikspunkt.

Se figur side 15.

Vi ser at programmet divergerer raskt fra ca. 25 steg av. Dette kommer av at matlab ikke har eksakt verdi for $\sqrt{2}$ og at fikspunktet $\bar{x} = \sqrt{2}$ er "frastøtende", dvs. at vi ikke nærmer oss fikspunktet, men fjerner oss fra det. Fikspunktet er frastøtende fordi

$$f'(\sqrt{2}) = 2\sqrt{2} + 1 > 1.$$

5.5.1

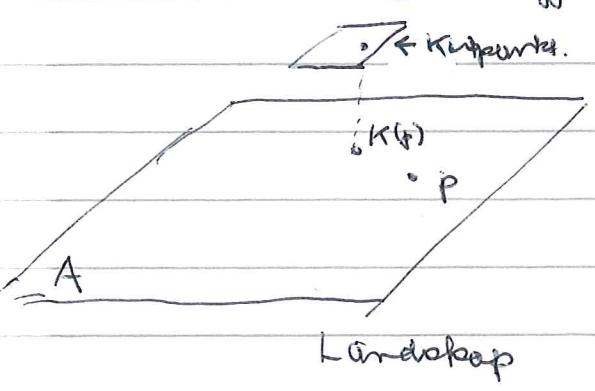
a) $x_{n+1} = \frac{1}{2}\sin(x_n + y_n)$

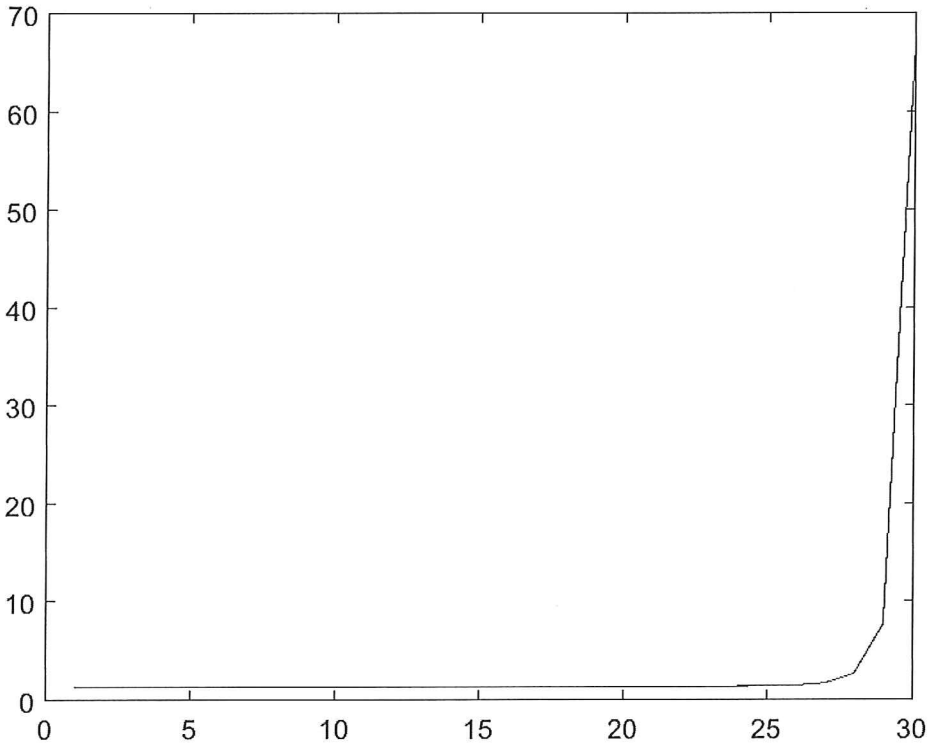
$y_{n+1} = \frac{1}{2}\cos(x_n - y_n)$

b) $x_1 = 1$, $y_1 = -1$, 30 iterasjoner: Figur side 16

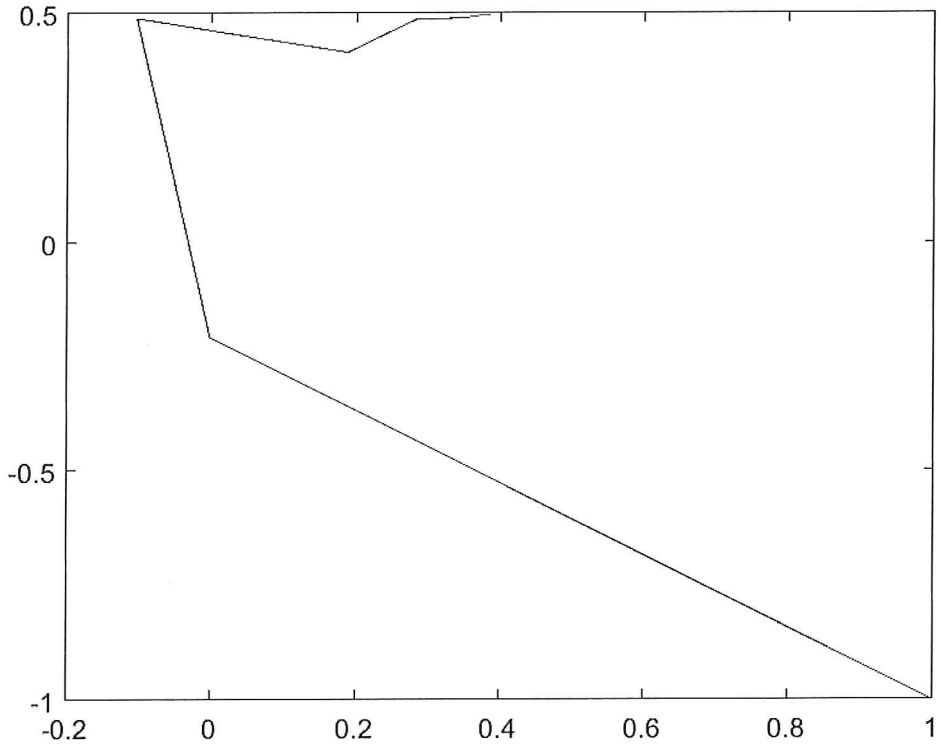
c) 4 Kjøringen med tilfeldig valg av x_1, y_1, 10 iterasjoner: Figur side 17.

5.5.4. La A betegne området i landskapet gjengitt på kartet. For $p \in A$, la $K(p)$ betegne det punktet i A som kartpunktet til p ligger over. Hvis kartet er i skala $1:M$ (M stort tall), så er $|K(p) - K(q)| = \frac{1}{M}|p - q|$, så K er en kontraksjon og har entydig fikspunkt. M er fikspunkt betyr jo at kartpunktet ligger rett over punktet.



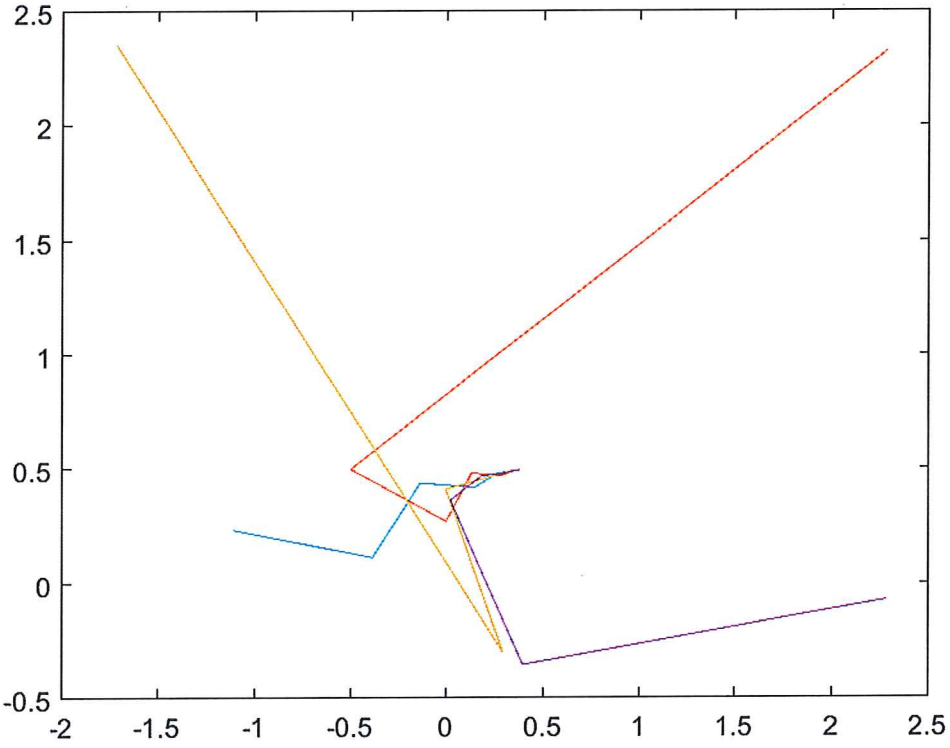


Oppgave 5.4.7.



Oppgave 5.5.1 b)

$(x_1, y_1) = (1, -1)$, $N = 30$



Oppgave 5.5.1. c)

4 random valg av x_i, y_i mellom -2.5 og 2.5.